Contents

WELSH JOINT EDUCATION COMMITTEE | **CYD-BWYLLGOR ADDYSG CYMRU**

General Certificate of Education | **Tystysgrif Addysg Gyffredinol**

Advanced Level/Advanced Subsidiary | **Safon Uwch/Uwch Gyfrannol**

MATHEMATICS C1

Pure Mathematics

Specimen Mark Scheme

2005/2006

1. *(a)* Gradients: *AB* $\frac{10+2}{-12-4} = -\frac{3}{4}$, B1

BC $\frac{6-10}{10+12} = -\frac{4}{22}$, B1

CA $\frac{6+2}{10-4} = \frac{4}{3}$. B1

(b) gradient *AB* × gradient *AC* = −1 (o.e.) M1 ($m_1 m_2 = -1$)
(or Pythagoras[1] Theorem)

∴ AB ⊥^{r} AC
giving $B\hat{A}C = 90°$ A1

(c) Equation of *AB* is

$y + 2 = -\frac{3}{4}(x - 4)$ M1 ($y - y_1 = m(x - x_1)$)

so that $4y + 8 = -3x + 12$

or $3x + 4y - 4 = 0$ (1) A1 (convincing)

(d) Coordinates of *D*: (−1,8). B1, B1

∴ $AD = \sqrt{(4+1)^2 + (-2-8)^2}$ M1 (correct formula)

$= \sqrt{125}$ A1

2. $\frac{(2\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{10+2\sqrt{10}+\sqrt{10}+2}{5-2}$ M1 (rationalise)

A1 (numerator unsimplified)
A1 (denominator unsimplified)

$= \frac{12+3\sqrt{10}}{3}$

$= 4+\sqrt{10}$ A1

3. Equate *y*s

$\therefore\ x^2 - 3x + 2 = 3x - 7$ — M1

giving $x^2 - 6x + 9 = 0$ — M1 (standard form of quad and attempt to solve)

$\therefore\ (x-3)^2 = 0$

so that $x = 3, y = 2$ — A1

A1

The line $y = 3x - 7$ is a tangent to the curve $y = x^2 - 3x + 2$ at the point (3, 2) — B2

(Allow B1 for (3, 2) is point of intersection of line $y = 3x - 7$ and the curve $y = x^2 - 3x + 2$)

4. Condition for distinct real roots is — M1 (Use of $b^2 - 4ac$)

$16 - 4 \times 2k(k-1) > 0$ — A1

$\therefore\ 2 - k^2 + k > 0$

$k^2 - k - 2 < 0$ — A1 (convincing)

giving $(k-2)(k+1) < 0$ — B1 (fixed points)

then $-1 < k < 2$ or $(-1,2)$. — M1 (any method) A1

5. *(a)* $2(x^2 - 6x) + 25$

$= 2[(x-3)^2 - 9] + 25$

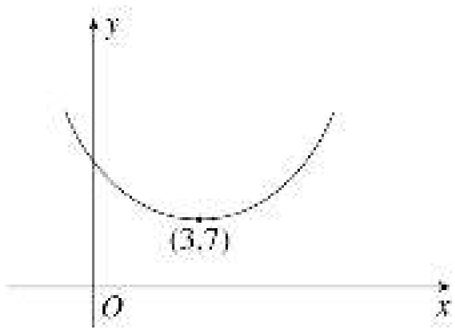
$= 2(x-3)^2 + 7$ — B1(*a*), B1(*b*), B1(*c*)

(*a*, *b*, *c* need not be displayed).

(b) Least value of $2x^2 - 12x + 25 = 7$ — B1

Corresponding value of $x = 3$ — B1

(c) — M1 (shape), A1 (location of minimum)



6.	*(a)*	$-8k+32-6-2=0$			M1 (use of factor theorem) A1 (correct)
		so that $k=3$			A1 (convincing)
	(b)	$3x^3+8x^2+3x-2$	=	$(x+2)(3x^2+2x-1)$	M1 A1
			=	$(x+2)(3x-1)(x+1)$.	A1
		Roots are		$-2, \frac{1}{3}, -1$.	A1
	(c)	Remainder	=	$3(3)^3+8(3)^2+9-2$	M1 (use of remainder theorem)
			=	160.	A1
7.	*(a)*	$(2x+3)^4=(2x)^4+4(2x)^3(3)+6(2x)^2(3)^2$ $+4(2x)(3)^3+3^4$			M1 (5 terms, 2 correct) A2 (unsimplified, −1 for each error)
		$=16x^4+96x^3+216x^2+216x+81$			A1 (simplified)
	(b)	${}^{n}C_2(3)^2$	=	54	B1
		so that $\frac{n(n-1)3^2}{1.2}$	=	54	B1
		$\therefore\ n^2-n-12$	=	0	M1 (standard form of quadratic and attempt to solve)
		$\therefore\ n=4$			A1

8. *(a)* $y+\delta y=(x+\delta x)^2-4(x+\delta x)+2.$ (o.e.) B1

$\delta y=(x+\delta x)^2-4(x+\delta x)+2$ (o.e.) M1

$-x^2+4x-2$

$=2x\delta x+(\delta x)^2-4\delta x.$ (o.e.) A1

$\therefore \dfrac{\mathrm{d}y}{\mathrm{d}x}=\lim_{\delta x\to 0}\dfrac{\delta y}{\delta x}=\lim_{\delta x\to 0}(2x+\delta x-4)$ (o.e.) M1

$=2x-4.$ A1

(b) $3(-4)x^{-5}+4\cdot\frac{1}{2}x^{-\frac{1}{2}}$ M1 (attempted differentiation of x^{-4}

A1

$= \quad -\dfrac{12}{x^5}+\dfrac{2}{\sqrt{x}}.$ M1 (attempted differentiation of $x^{\frac{1}{2}}$)

A1

9. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4x^3+1$ B1

$= 5$ at (1,3). B1

Equation is

$y-3=5(x-1)$ M1 (for using gradient)

A1

10. *(a)* $\frac{dy}{dx} = 3x^2 - 6x - 9.$ B1

$\frac{dy}{dx} = 0 \Rightarrow 3(x^2 - 2x - 3) = 0.$ M1

giving $x = 3, -1$ A1

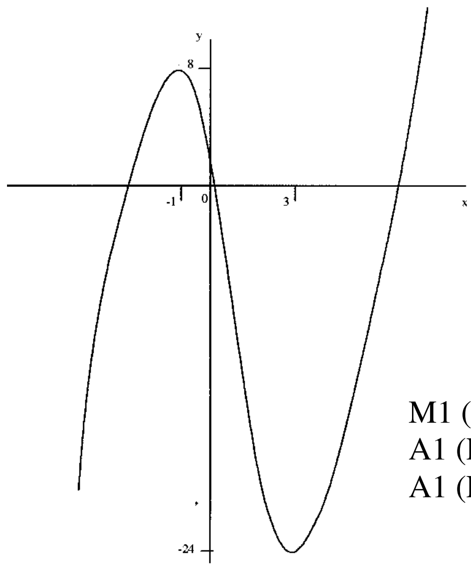
When $x = 3, y = -24,$
$x = -1, y = 8$ A1, A1

$\frac{d^2y}{dx^2} = 6x - 6.$ M1 (any method)

$x = -1,$ $\frac{d^2y}{dx^2} = -6 - 6 < 0.$ Max. point at $(-1, 8)$ A1

$x = 3,$ $\frac{d^2y}{dx^2} = 6 \times 3 - 6 > 0.$ Min. point at $(3, -24)$. A1

(b)



M1 (general shape)
A1 (Position of Max)
A1 (Position of Min)

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MATHEMATICS C2

Pure Mathematics

Specimen Mark Scheme

2005/2006

1. Integral $\approx \dfrac{0 \cdot 25}{2}$ $[0{\cdot}5 + 2\,(0{\cdot}3386243 + 0{\cdot}2285714$ — M1 (correct formula)

$+ 0{\cdot}1572482)$ — B1 (3 values)

$+ 0{\cdot}1111111]$ — B1 (other 2 values)

$\approx 0{\cdot}2575$ — A1 (F.T. one slip)

2. (*a*) $6\,(1 - \cos^2 \theta) + \cos \theta\ - 5 = 0$ — M1 (correct elimination of $\sin^2 x$)

$\therefore 6 \cos^2\theta - \cos \theta - 1 = 0$ — M1 (standard form of quadratic and an attempt to solve)

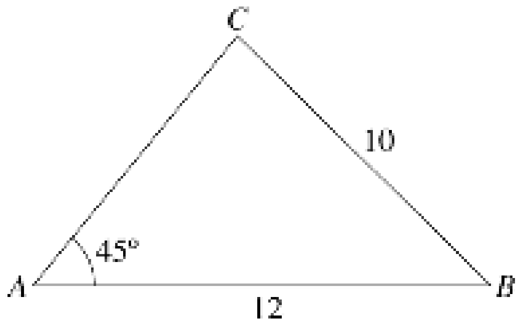
giving $\cos \theta = \frac{1}{2}, -\frac{1}{3}$. — A1

$\theta = 60°, 300°, 109{\cdot}5, 250{\cdot}5°$ — B1 (60°, 109·5°) B1, B1

(*b*) $3x = 135°, 315°, 495°$ — B1

$x = 45°, 105°, 165°$. — B1, B1, B1

3.



(*a*) By the Sine Rule,

$$\frac{10}{\sin 45°} = \frac{12}{\sin C}$$ — M1 (correct Sine Rule)

$$\therefore \sin C = \frac{12 \sin 45°}{10}$$

$\therefore C = 58°, 122°$ (correct to the nearest degree) A1, A1 (*b*) When $C = 58°$, $B = 180° - (45° + 58°) = 77°$

Sine Rule, $\dfrac{10}{\sin 45°} = \dfrac{AC}{\sin 77°}$ — M1 (any correct method)

A1 (correct B)

so that $AC = \dfrac{10\sin 77°}{\sin 45°} \approx 13{\cdot}8\text{cm}$ — A1

When $C = 122°$, $B = 180° - (45° + 122°) = 13°$

Sine Rule, $\dfrac{10}{\sin 45°} = \dfrac{AC}{\sin 13°}$ — A1

so that $AC = \dfrac{10\sin 13°}{\sin 45°} \approx 3{\cdot}2\text{cm}$ — A1

4. (*a*) $S_n = a + a\,r + \ldots + a\,r^{n-2} + a\,r^{n-1}$ — B1

$rS_n = \quad a\,r + a\,r^2 + \ldots + a\,r^{n-2} + a\,r^{n-1} + a\,r^n$ — M1

$\therefore\ S_n - rS_n = a - a\,r^n$

$\therefore\ S_n = \dfrac{a(1-r^n)}{1-r}$ — A1(convincing)

Sum to infinity $= \dfrac{a}{1-r}$ — B1

(*b*) $\dfrac{a}{1-r} = 4a$ — M1 ($\dfrac{a}{1-r} = k\,a$, $k = \dfrac{1}{4}$ or 4)

A1 (correct)

$\therefore\ 1 - r = \dfrac{1}{4}$ — M1 (eliminate a and attempt to solve)

giving $r = \dfrac{3}{4}$ — A1

5. (*a*)

$a + 3d = 11$ — B1

$a + 5d = 17$ — B1

$d = 3$, $a = 2$ — M1(attempt to solve)

A1

(*b*) $S_8 = \dfrac{8}{2}\ [2 \times 2 + 7 \times 3] = 100$ — B1

6.	$\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{2}{x} + C$	M1 (attempt to integrate $x^{\frac{1}{2}}$)
		A1
	$= \dfrac{2}{3}\ x^{\frac{3}{2}} + \dfrac{2}{x} + C$	M1 (attempt to integrate x^{-2})
		A1
7.	(*a*) Coordinates of *A* $\quad 12 - 2x = 2x^2$	M1 (equate *y*s)
	$x^2 + x - 6 = 0$	M1 (standard form of quadratic and attempt to solve)
	$\therefore\ x = 2$	A1
	$\therefore\ A\ (2, 8)$	A1
	Coordinates of *B* are (6,0)	B1
	(*b*) Integral = $\displaystyle\int_0^2 2x^2 dx + \int_2^6 (12 - 2x)\mathrm{d}x$	M1 (sum up difference or definite integrals)
		A1 (all correct)
	$= \left[\dfrac{2x^3}{3}\right]_0^2 + \left[12x - x^2\right]_2^6$	B1, B1, B1
	$= \dfrac{16}{3} - 0 + 72 - 36 - (24 - 4)$	M1 (use of limits)
	$= \dfrac{64}{3}$	A1 (C.A.O.)
8.	$A\hat{O}B = \theta,$	
	$2 \times 6{\cdot}8 + 6{\cdot}8\theta = 23{\cdot}12$	B1
	so that $\theta = 1{\cdot}4$	B1
	shaded area = $\dfrac{1}{2}r^2(1\cdot4 - \sin 1\cdot4)$	B1 (sector area)
		B1 (triangle area)
		M1 (difference of areas)
	$\approx 9{\cdot}58\text{cm}^2$	A1

9.	(*a*)	Let $x = a^y$, $y = \log_a x$	B1
		so that $x^n = (a^y)^n = a^{ny}$	B1
		and $\log_a x^n = ny = n \log_a x$	B1
	(*b*)	$(y + 1) \log 2 = \log 3$	B1
		$\therefore y = \dfrac{\log 3 - \log 2}{\log 2}$	M1 (attempt to isolate *y*)
			A1
		$\approx 0{\cdot}585$	B1
10.	(*a*)	(5, 7); 5	B1, B1
	(*b*)	Gradient of radius = $\dfrac{7-3}{5-2} = \dfrac{4}{3}$	M1 (correct method of finding gradient of radius)
		$\therefore$ Gradient of tangent = $-\dfrac{3}{4}$	M1
			A1
		$\therefore$ Equation of tangent is	
		$y - 3 = -\dfrac{3}{4}(x - 2)$	A1
	(*c*)	(i) $CQ^2 = (13 - 5)^2 + (13 - 7)^2 = 64 + 36 = 100$	M1 (attempt to find *CQ*)
		CQ > radius and *Q* lies outside circle	A1 (convincing)
		(ii) For circles touching externally, sum of radii = *CQ*	M1
		$\therefore$ Radius of required circle = $\sqrt{100} - 5 = 5$	A1
		Equation of circle is	
		$(x - 13)^2 + (y - 13)^2 = 25$ (o.e.)	A1

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MATHEMATICS C3

Pure Mathematics

Specimen Mark Scheme

2005/2006

1.

x	$x^3 + 10x - 4$	
0	-4	
1	7	M1 (attempt to find values)

change of sign of $x^3 + 10x - 4$ indicates that there is a root between 0 and 1.
A1 (correct values and conclusion)

$x_0 = 0{\cdot}3$, $x_1 = 0{\cdot}3973$, $x_2 = 0{\cdot}39372873$ B1 (x_1)

$x_3 = 0{\cdot}39389633$, $x_4 = 0{\cdot}39388853$

$x_4 \approx 0{\cdot}39389$. B1 ($x_4$)

x	$x^3 + 10x - 4$		
0·393885	−0·00004	Change of sign indicates	M1
0·393895	0·00006	presence of root which	A1
		is 0·39389, correct to	A1
		five decimal places	

2. Integral $\approx \dfrac{0\cdot 25}{3}$ [1·41421356 + 4·12310563 + 2(2·46221445) +4 (1·85510275 + 3·22163099)] M1 (formula with $h = 0{\cdot}25$)

B1 (3 values)

B1 (2 other values)

$\approx$ 2·56 A1 (C.A.O.)

3. $9 > 2x - 5 > -9$. B1, B1

$\therefore \quad 14 > 2x > -4$

giving $7 > x > -2$ or $(-2,7)$ B1 ($7 > x$)
B1 ($x > -2$)

4. *(a)* $3y^2\frac{\mathrm{d}y}{\mathrm{d}x} - 2x^2y\frac{\mathrm{d}y}{\mathrm{d}x} - 2xy^2 = 2x + 3$

B1 $\left(3y^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)$
M1 $\left(-2x^2y\frac{\mathrm{d}y}{\mathrm{d}x} \pm 2xy^2\right)$
A1

$\therefore \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + 3 + 2xy^2}{3y^2 - 2x^2y}$

A1 (all correct)

(b) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2t}{3t^2} = \frac{2}{3t}$

M1 $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} / \frac{\mathrm{d}x}{\mathrm{d}t}\right)$
A1

$\therefore \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\frac{\mathrm{d}x}{\mathrm{d}t}}$

M1 (correct formula)

$= -\frac{\frac{2}{3t^2}}{3t^2} = -\frac{2}{9t^4}$

A1, A1 (convincing)

5. $\operatorname{cosec}^2\theta - 1 = 7 - 2\operatorname{cosec}\theta$

M1 (correct elimination of cot θ)

$\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta - 8 = 0$

M1 (standard form of quadratic and attempt to solve)

giving $\operatorname{cosec}\theta = 2, -4,$

A1

so that $\sin\theta = \frac{1}{2}, -\frac{1}{4}$

B1

$\therefore \theta = 30°, 150°, 194{\cdot}5°, 345{\cdot}5°$

B1 (30°, 150°)
B1 (194·5°) B1 (345·5°)

6.	*(a)*	$e^{2x}\cos x + 2e^{2x}\sin x$	M1 ($e^{2x}f(x) + g(x)\sin x$) A1 ($f(x) = \cos x$), A1 ($g(x) = ke^{2x}$) A1 ($k = 2$)
	(b)	$\dfrac{(x^2+3)(4x)-(2x^2-4)(2x)}{(x^2+3)^2} = \dfrac{20x}{(x^2+3)^2}$	M1 $\left(\dfrac{(x^2+3)f(x)-(2x^2-4)g(x)}{(x^2+3)^2}\right)$ A1 ($f(x) = 4x$, $g(x) = 2x$) A1 (simplification)
	(c)	$8x\sec^2(4x^2+3)$	B1 ($f(x)\sec^2(4x^2+3)$) B1 ($f(x) = 8x$)
7.	*(a)*	(i) $-\frac{1}{4}e^{-4x+1}$	B1 (ke^{-4x+1}) B1 ($k = -\frac{1}{4}$)
		(ii) $\frac{1}{2}\ln\lvert 2x+1\rvert - \dfrac{1}{6(3x+7)^2}$	M1 ($k\ln\lvert 2x+1\rvert$) A1 ($k = \frac{1}{2}$) M1 $\left(\dfrac{k}{(3x+7)^2}\right)$ A2 $\left(k = -\dfrac{1}{6}\right)$ (A1 for $-\frac{1}{2}$)
	(b)	$\left[\dfrac{-\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$	M1 ($k\cos 2x$, $k = \pm\frac{1}{2}, -2$) A1 ($k = -\frac{1}{2}$)
		$= -\dfrac{\cos\pi}{2} + \dfrac{1}{2} = 1$	A1

8. *(a)* $y = \tan^{-1}x$

so that $\tan y = x$ B1

then $\sec^2 y = \dfrac{\mathrm{d}x}{\mathrm{d}y}$ B1

$\therefore \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\sec^2 y} = \dfrac{1}{1+\tan^2 y} = \dfrac{1}{1+x^2}$ B1

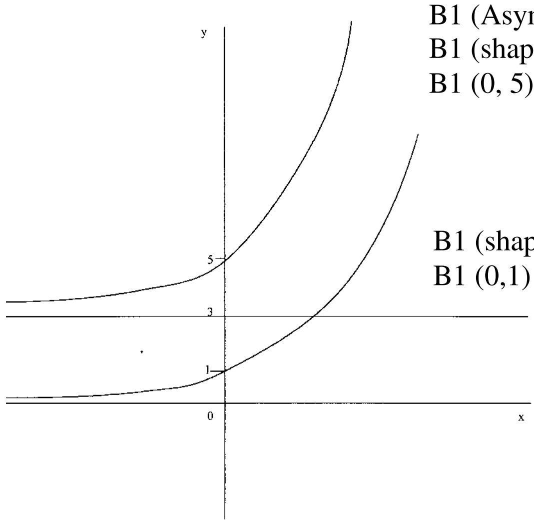
(b) $\dfrac{2x}{x^2+1}$ M1 $\left(\dfrac{f(x)}{x^2+1}\right)$ A1 $\left(f(x) = 2x\right)$

(c) $\displaystyle\int \frac{3+x}{1+x^2}\mathrm{d}x = 3\int \frac{1}{1+x^2}\mathrm{d}x + \frac{1}{2}\int \frac{2x}{1+x^2}\mathrm{d}x$ M1 (attempt to rewrite)

A1 (correct)

$= 3\tan^{-1} x + \dfrac{1}{2}\ln(1+x^2) + C$ B1, B1

9.



B1 (Asymptote $y = 3$)
B1 (shape)
B1 (0, 5)

B1 (shape)
B1 (0,1)

10.	*(a)*	(i)	Let $y = \ln(2x - 3) + 4$	M1 (attempt to isolate x)
			$\therefore \quad y - 4 = \ln(2x - 3)$	
			giving $e^{y-4} = 2x - 3$	A1
			$x = \dfrac{e^{y-4} + 3}{2}$	A1
			so that $f^{-1}(x) = \dfrac{e^{x-4} + 3}{2}$	A1
		(ii)	domain $[4,\infty)$, range $[2,\infty)$	B1, B1
	(b)	$gh(x) = (2x + 2)^2 + 3$, $hg(x) = 2(x^2 + 3) + 2$.		B1, B1
		$\therefore (2x + 2)^2 + 3 = 2[2(x^2 + 3) + 2] + 15$		M1 (unsimplified)
		$\therefore 4x^2 + 8x + 4 + 3 = 4x^2 + 16 + 15$		M1 (attempt to solve)
			and $x = 3$	A1

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MATHEMATICS C4

Pure Mathematics

Specimen Mark Scheme

2005/2006

1. $1+\left(-\tfrac{1}{2}\right)(2x)+\dfrac{\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)(2x)^2}{1.2}$

$+\dfrac{\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\left(-\tfrac{5}{2}\right)(2x)^3}{1.2.3}+\ldots$

$= 1-x+\tfrac{3}{2}x^2-\tfrac{5}{2}x^3+\ldots$ — B1 $(1-x)$
B1 $\left(\tfrac{3}{2}x^2\right)$ B1 $\left(-\tfrac{5}{2}x^3\right)$

$\dfrac{(1-x)^2}{(1+2x)^{\frac{1}{2}}}=\left(1-2x+x^2\right)\left(1-x+\tfrac{3}{2}x^2-\tfrac{5}{2}x^3+\ldots\right)$

$= 1-x+\tfrac{3}{2}x^2-\tfrac{5}{2}x^3+\ldots$

$-2x+2x^2-3x^3+\ldots$

$+x^2-x^3+\ldots$

$= 1-3x+\tfrac{9}{2}x^2-\tfrac{13}{2}x^3+\ldots$ — B1 $(1-3x)$
B1 $\left(\tfrac{9}{2}x^2\right)$
B1 $\left(-\tfrac{13}{2}x^3\right)$

Valid for $|x|<\tfrac{1}{2}$ — B1

2. *(a)* Substitution of appropriate value of θ — B1
Convincing conclusion — A1

(b) $3(1-2\sin^2\theta)=1-\sin\theta$ — M1 (correct elimination of $\cos^2\theta$)

$\therefore\ 6\sin^2\theta-\sin\theta-2=0$ — M1 (Standard form of quadratic and attempt to solve)

$\therefore\ (3\sin\theta-2)(2\sin\theta+1)=0$

so that $\sin\theta=\tfrac{2}{3},-\tfrac{1}{2}$ — A1

$\therefore\ \theta=41{\cdot}8^\circ, 138{\cdot}2^\circ, 210^\circ, 330^\circ$ — B1 ($41{\cdot}8^\circ$, $138{\cdot}2^\circ$)
B1 (210°) B1 (330°)

3.	$R\cos\alpha = 5$, $R\sin\alpha = 4$	B1 (both)
	$R = \sqrt{41}$, $\alpha = 38{\cdot}66°$	M1 (any method of funding R or α) A1 (R) A1 (38.7°)
	$\therefore \sin(\theta + 38{\cdot}7°) = \dfrac{3}{\sqrt{41}}$	M1
	$\therefore \theta + 38{\cdot}66° = 152{\cdot}06°, 387{\cdot}94°$	A1 (any value)
	$\therefore \theta = 113{\cdot}4°, 349{\cdot}3°$	A1 (both)
4.	*(a)* Let $\dfrac{3x^2+2x+1}{x^2(x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$	M1 (correct form)
	$\therefore 3x^2 + 2x + 1 \equiv Ax(x-1) + B(x-1) + Cx^2$	m1 (clearing fraction and attempt to find constants)
	so that $A = -3$, $B = -1$, $C = 6$	A2 (3 constants) A1 (2 constants)
	(b) $\displaystyle\int \frac{3x^2+2x+1}{x^2(x-1)}\mathrm{d}x = -3\ln\lvert x\rvert + \frac{1}{x}$	B1, B1
	$+ 6\ln\lvert x-1\rvert$	B1
	($\lvert\quad\rvert$ may be omitted)	
5.	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2a}{2at} = \dfrac{1}{t}$	M1 $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} / \dfrac{\mathrm{d}x}{\mathrm{d}t}\right)$ A1
	For normal at P, gradient $= -p$	B1
	$\therefore y - 2ap = -p(x - ap^2)$	M1
	giving $px + y - 2ap - ap^3 = 0$	A1 (convincing)
	Q: $y = 0$, $x = 2a + ap^2$	B1
	R: $y = 0$, $x = ap^2$	B1
	$\therefore QR = 2a$	B1

6. *(a)* $\frac{\mathrm{d}x}{\mathrm{d}t} = -kx$ B1

Now at $t = 0$, $x = 2$, $\frac{dx}{dt} = -0 \cdot 064$ M1 (use of data)

Then $k = \frac{-0.064}{-2} = 0 \cdot 032$

$\therefore \frac{dx}{dt} = -0 \cdot 032x$ A1

(b) $\int \frac{dx}{x} = \int -0 \cdot 032\mathrm{d}t$ M1 (separation of variables and integration of r.h.s.)

$\therefore \ln x = -0{\cdot}032t + A$ A1

$t = 0$, $x = 2$

$\therefore A = \ln 2$ M1

$\therefore \ln x = -0{\cdot}032t + \ln 2$

$\therefore 0.032t = \ln 2 - \ln \underline{x}$

$= \ln \frac{2}{x}$ A1

$\therefore t = \frac{1}{0 \cdot 032} \ln \frac{2}{x}$

$= \frac{125}{4} \ln \frac{2}{x}$ A1

(c) $x = 1, t = \frac{125}{4} \ln 2$ M1

$\approx 21{\cdot}66$ years A1

7.	Volume	$= \pi\int_1^e x^3 \ln x \, dx$	B1
		$= \pi\left\{\left[\frac{x^4}{4}\ln x\right]_1^e - \int_1^e \frac{x^4}{4}\frac{1}{x}dx\right\}$	M1 (parts, correct choice of *u, v*)
			A1
		$= \pi\left\{\left[\frac{x^4}{4}\ln x\right]_1^e - \int_1^e \frac{x^3}{4}dx\right\}$	m1 (division)
		$= \pi\left\{\left[\frac{x^4}{4}\ln x\right]_1^e - \left[\frac{x^4}{16}\right]_1^e\right\}$	A1
		$= \frac{\pi}{16}(3e^4 + 1)$	A1
8.	*(a)*	$\int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta = \int_0^{\frac{\pi}{4}} \frac{1+\cos 2\theta}{2} d\theta$	M1 ($a + b\cos 2\theta$)
			A1 ($a = ½$, $b = ½$)
		$= \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{4}}$	B1
		$= \frac{\pi}{8} + \frac{1}{4} - \frac{0}{2} - \frac{0}{4}$	
		$= \frac{\pi}{8} + \frac{1}{4}$	A1 (convincing)
	(b)	$x = 3\tan\theta$, $dx = 3\sec^2\theta \, d\theta$	M1 (substitution for d*x*)
		$x = 0,\ \theta = 0,\ x = 3,\ \theta = \frac{\pi}{4}$	B1 (limits)
		$27\int_0^{\frac{\pi}{4}} \frac{3\sec^2\theta}{(9\tan^2\theta + 9)^2} d\theta = 27\int_0^{\frac{\pi}{4}} \frac{3\sec^2\theta}{81\sec^4\theta} d\theta$	A1 (unsimplified)
			A1 ($1 + \tan^2\theta = \sec^2\theta$)
		$= \frac{3\times 27}{81}\int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta$	A1 (simplified)
		$= \left(\frac{\pi}{8} + \frac{1}{4}\right)$	A1

9. *(a)* Where the lines intersect,

$2\mathbf{i} + \mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	M1 (attempt to equate **i, j, k** terms)
$= 2\mathbf{i} + 2\mathbf{j} + t\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	
$\therefore 2 + \lambda = 2 + \mu$, $1 + \lambda = 2 + 2\mu$, $2\lambda = t + \mu$.	A1 (correct)
Then $\lambda = \mu = -1$ $t = -1$	M1 (attempt to solve) A1 (λ, μ) A1 (convincing)
Position vector of point of intersection is $\mathbf{i} - 2\mathbf{k}$	B1
(b) $\cos\theta = \dfrac{(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(\mathbf{i}+2\mathbf{j}+\mathbf{k})}{\lvert\mathbf{i}+\mathbf{j}+2\mathbf{k}\rvert\,\lvert\mathbf{i}+2\mathbf{j}+\mathbf{k}\rvert}$	B1 (identification of appropriate vectors) B1 (scalar product)
Now $(\mathbf{i} + \mathbf{j} + 2\mathbf{k}).\ (\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ $= 1 + 2 + 2 = 5$ $\lvert\mathbf{i} + \mathbf{j} + 2\mathbf{k}\rvert = \sqrt{6}$ $\lvert\mathbf{i} + 2\mathbf{j} + \mathbf{k}\rvert = \sqrt{6}$ $\cos\theta = \dfrac{5}{6}$	M1 (correct method) A1 B1 (for one)
$\theta \approx 34°$.	B1

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MATHEMATICS FP1

Further Pure Mathematics

Specimen Mark Scheme

2005/2006

1. If 2 - i is a root, so is 2 + i. M1A1

Also, $x^2 - 4x + 5$ s a factor of the quartic. A1

Using long division, M1A1A1

$$
\begin{array}{r}
x^2 - 2x + 5 \\
x^2 - 4x + 5 \;)\; x^4 - 6x^3 + 18x^2 - 30x + 25 \\
x^4 - 4x^3 + 5x^2 \qquad\qquad\qquad \\
\hline
-2x^3 + 13x^2 - 30x \qquad \\
-2x^3 + 8x^2 - 10x \qquad \\
\hline
5x^2 - 20x + 25 \\
5x^2 - 20x + 25 \\
\hline
. \quad . \quad .
\end{array}
$$

The other roots are

$$\frac{2 \pm \sqrt{4-20}}{2}, \text{ ie } 1 \pm 2\text{i}.$$ M1A1

2.

$$\text{f}(x+h) - \text{f}(x) = \frac{1}{(x+h)^3} - \frac{1}{x^3}$$ M1A1

$$= \frac{x^3 - (x+h)^3}{x^3(x+h)^3}$$ A1

$$= \frac{-h(3x^2 + 3hx + h^2)}{x^3(x+h)^3}$$ A1

$$\text{f}'(x) = \lim_{h \to 0} \frac{-(3x^2 + 3hx + h^2)}{x^3(x+h)^3}$$ M1

$$= -\frac{3}{x^4}$$ A1

3.

$$\frac{8-6\text{i}}{1-2\text{i}} = \frac{(8-6\text{i})(1+2\text{i})}{(1-2\text{i})(1+2\text{i})}$$ M1A1

$$= \frac{20+10\text{i}}{5} = 4 + 2\text{i}$$ A1

$3(a + \text{i}b) - (a - \text{i}b) = 4 + 2\text{i}$ M1

$2a = 4$ and $4b = 2$ A1

$a = 2$ and $b = 1/2$ A1

4. *(a)*

$$T_n = n^2 + 2n - (n-1)^2 - 2(n-1)$$ M1A1

$$= n^2 + 2n - n^2 + 2n - 1 - 2n + 2$$ A1

$= 2n + 1$ (AG)

(b)

$$S_n = 4\sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$ M1

$$= \frac{4n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} + n$$ A1A1A1

$$= \frac{n}{3}(4n^2 + 6n + 2 + 6n + 6 + 3)$$ A1

$$= \frac{n}{3}(4n^2 + 12n + 11)$$ A1

5. Assume the proposition is true for $n = k$. M1

Letting $T_k = 3^{2k} + 7$, consider

$$T_{k+1} - T_k = 3^{2k+2} + 7 - (3^{2k} + 7)$$ m1

$$= 3^{2k}(9 - 1)$$ A1

$$= 3^{2k}.8$$ A1

So, if the proposition is true for $n = k$, it is also true for $n = k + 1$. A1

It is true for n = 1 since $T_1 = 16$ which is divisible by 8. A1

The result is therefore proved by mathematical induction A1

[The final A1 is for a concluding statement plus satisfactory lay-out]

6. $$\ln f(x) = \frac{1}{2}\ln(x^3 + 1) - \frac{3}{2}\ln(x^2 + 1)$$ M1A1

$$\frac{f'(x)}{f(x)} = \frac{1}{2}.\frac{3x^2}{(x^3+1)} - \frac{3}{2}.\frac{2x}{(x^2+1)}$$ M1A1A1

Putting $x = 2$,

$$\frac{f'(2)}{3/5\sqrt{5}} = \frac{1}{2}.\frac{12}{9} - \frac{3}{2}.\frac{4}{5}$$ M1

$$f'(2) = -\frac{8}{25\sqrt{5}} \quad (-0.143)$$ A1

7. $\alpha + \beta = -3$ and $\alpha\beta = 3$ B1

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \alpha\beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} + \alpha\beta$$ M1 A1 A1

$$= 4$$ A1

$$\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha} + \frac{\alpha}{\beta}\cdot\alpha\beta + \frac{\beta}{\alpha}\cdot\alpha\beta = 1 + (\alpha+\beta)^2 - 2\alpha\beta$$ M1 A1

$$= 4$$ A1

$$\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha}\cdot\alpha\beta = \alpha\beta = 3$$ M1 A1

Required equation is

$x^3 - 4x^2 + 4x - 3 = 0$ M1 A1

8.	*(a)*	Det = 1(10 – 1) – 1(5 – λ) +2(1 – 2λ) = 6 – 3λ	M1A1
		Matrix is singular when λ = 2.	A1
	(b)	(i) Using reduction to echelon form,	
		$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ \mu - 4 \end{bmatrix}$$	M1A1A1
		Comparing the second and third rows, μ = 4 for consistency.	M1A1
		(ii) Put $z = a$ so that $y = a$ and $x = 2 - 3a$.	M1A1A1
9.	*(a)*	Matrix of translation = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$	B1
		Matrix of rotation = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1
		T - Matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$	M1A1
		$= \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$	A1
	(b)	Consider the image of the point (a, a^2). This is given by	M1
		$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ a^2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 - a^2 \\ a + 2 \\ 1 \end{bmatrix}$$	A1
		So, $x = 1 - a^2, y = a + 2$. Eliminating *a*, the required equation is	M1
		$$1 - x = (y - 2)^2.$$	A1

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MATHEMATICS FP2

Further Pure Mathematics

Specimen Mark Scheme

2005/2006

1. For the continuity of g at $x = 1$,

$$1 + b = 2 + a$$ M1A1

Since $g'(x) = 1 + 2bx$ for $x < 1$ and $3ax^2$ for $x > 1$, it follows that M1

$$1 + 2b = 3a$$ A1

The solution is $a = 3$, $b = 4$. M1A1

2. The equation can be rewritten

$$-\sin 3\theta + 2\sin 3\theta \cos 2\theta = 0$$ M1A1

$$\sin 3\theta(2\cos 2\theta - 1) = 0$$ A1

Either

$$\sin 3\theta = 0$$ M1

from which $\theta = 0°, 60°, 120°, 180°$. A2 (−1 each error or extra root in the range)

Or

$$\cos 2\theta = 1/2$$ M1

from which $\theta = 30°, 150°$. A1

3. Converting to trigonometric form

$3 - 2i = r(\cos\theta + i\sin\theta)$ M1

where $r = \sqrt{13}$ and $\theta = \tan^{-1}\left(\frac{-2}{3}\right)$ A1, A1

$(r(\cos\theta + i\sin\theta))^{1/3} = r^{1/3}(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3})$ M1

So $(3 - 2i)^{1/3} = (\sqrt{13})^{1/3}(\cos\frac{-0.5880}{3} + i\sin\frac{-0.5880}{3})$ A1, A1

$= 1{\cdot}50 - 0{\cdot}30i$ A1

or $= (\sqrt{13})^{1/3}(\cos\frac{2\pi - 0.5880}{3} + i\sin\frac{2\pi - 0.5880}{3})$ A1

$= -0{\cdot}49 + 1{\cdot}45i$ A1

or $= (\sqrt{13})^{1/3}(\cos\frac{4\pi - 0.5880}{3} + i\sin\frac{4\pi - 0.5880}{3})$ A1

$= -1{\cdot}01 - 1{\cdot}15i$ A1

4. *(a)* $f'(x) = \frac{2(x-1) - (2x+1)}{(x-1)^2} = -\frac{3}{(x-1)^2}$ M1A1

This is < 0 for all x in the domain so f is strictly decreasing. A1

(b) The range is $(2,\infty)$. B1B1

(c) (i) $f(3) = 7/2$ and $f(4) = 3$ so $f(S) = [3,7/2]$. M1A1

(ii) $f(x) = 3 \Rightarrow x = 4$ (from above) and $f(x) = 4 \Rightarrow x = 5/2$. B1M1A1

So $f^{-1}(S) = [5/2,4]$. A1

5. Using de Moivre's Theorem,

$z^n = \cos n\theta + \mathrm{i}\sin n\theta$ M1A1

$z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$ A1

So, $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$ A1

$\left(z - \dfrac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \dfrac{10}{z} + \dfrac{5}{z^3} - \dfrac{1}{z^5}$ M1A1

$= z^5 - \dfrac{1}{z^5} - 5\left(z^3 - \dfrac{1}{z^3}\right) + 10\left(z - \dfrac{1}{z}\right)$ A1

$(2\mathrm{i}\sin\theta)^5 = 2\mathrm{i}\sin 5\theta - 10\mathrm{i}\sin 3\theta + 20\mathrm{i}\sin\theta$ M1

whence

$\sin^5\theta = \dfrac{1}{16}(\sin 5\theta - 5\sin 3\theta + 10\sin\theta)$ A1

6. *(a)* Completing the square,

$(y+2)^2 = 8(x-1)$ M1A1

(i) Vertex is (1, –2). A1

(ii) Focus is (3, –2) A1

(b) (i) For the point P,

$(y+2)^2 = 16p^2$

and $8(x - 1) = 16p^2$. M1A1

confirming that P lies on the parabola.

(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4}{4p} = \dfrac{1}{p}$ M1A1

Equation of tangent is

$y - 4p + 2 = \dfrac{1}{p}(x - 2p^2 - 1)$ M1A1

(iii) This passes through the origin if

$-4p^2 + 2p = -2p^2 - 1$

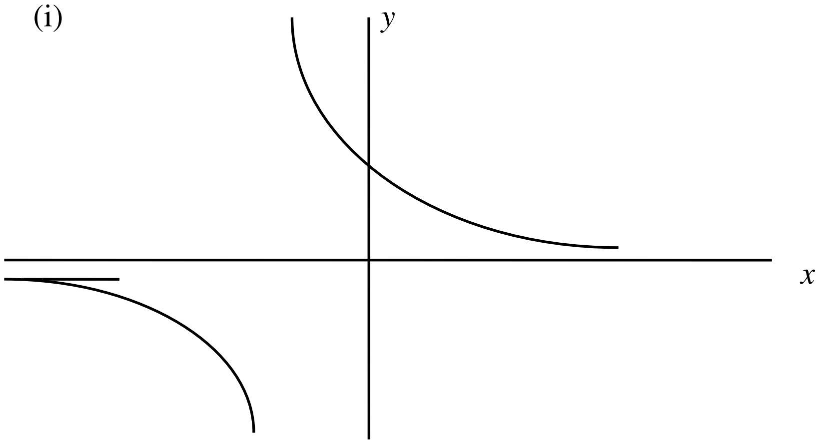
$2p^2 - 2p - 1 = 0$ M1A1

$p = \dfrac{2 \pm \sqrt{4+8}}{4} = \dfrac{1 \pm \sqrt{3}}{2}$ A1

In view of line 1 in (ii),

Gradients of tangents = $\dfrac{2}{1 \pm \sqrt{3}}$ A1

7. *(a)* (i)



G1 (shape)
G1 (asymptotes)

(ii) The asymptotes are x = -1 and y = 0. B1B1

(b) Let $\frac{1}{(x+1)(x^2+4)} \equiv \frac{A}{(x+1)} + \frac{Bx+C}{(x^2+4)} = \frac{A(x^2+4)+(x+1)(Bx+C)}{(x+1)(x^2+4)}$ M1

x = -1 gives $A = \frac{1}{5}$. A1

Coeff of x^2 gives $B = -\frac{1}{5}$. A1

Const term gives $C = \frac{1}{5}$. A1

(c) Int = $\frac{1}{5}\int_0^1 \frac{1}{(x+1)}\mathrm{d}x + \frac{1}{5}\int_0^1 \frac{1}{(x^2+4)}\mathrm{d}x - \frac{1}{5}\int_0^1 \frac{x}{(x^2+4)}\mathrm{d}x$ M1

$= \frac{1}{5}\left[\ln(1+x)\right]_0^1 + \frac{1}{10}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^1 - \frac{1}{10}\left[\ln(x^2+4\right]_0^1$ A1A1A1

$= \frac{1}{5}\ln 2 + \frac{1}{10}\tan^{-1}\left(\frac{1}{2}\right) - \frac{1}{10}(\ln 5 - \ln 4)$ A1A1A1

$= 0.163$ A1

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MATHEMATICS FP3

Further Pure Mathematics

Specimen Mark Scheme

2005/2006

1. $1+\sinh^2 x = 3+\sinh x$ M1

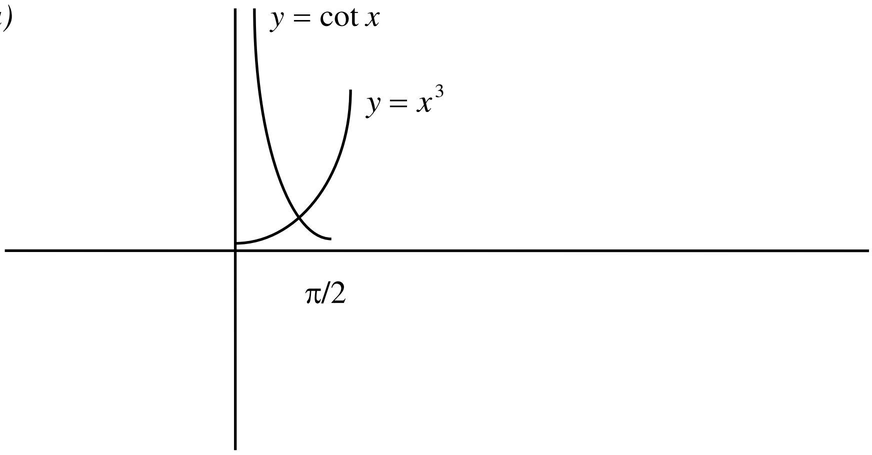
leading to $\sinh^2 x - \sinh x - 2 = 0$ A1

$(\sinh x - 2)(\sinh x + 1) = 0$ M1

$x = \sinh^{-1}(2) = \ln(2+\sqrt{5})$ A1A1

or $x = \sinh^{-1}(-1) = \ln(-1+\sqrt{2})$ A1A1

2. *(a)*



G1G1

The one point of intersection confirms that there is only one root in $(0,\pi/2)$. B1

(b) The Newton-Raphson iteration is

$$x \to x - \frac{x^3 - \cot x}{3x^2 + \operatorname{cosec}^2 x}$$ M1A1

The iterates are

1

0.9188838545 M1

0.9158148137

0.9158112781 A1

$x_3 = 0.915811$ correct to 6 decimal places. A1

Let f(x) = $x^3 - \cot x$ M1

f(0.9158105) = negative

f(0.9158115) = positive A1

So the root is 0.915811 correct to 6 decimal places. A1

3. *(a)*

Area = $2\pi\int_0^1 y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ M1

$= 2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx$ A1

(b)

$u = 1+9x^4, du = 36x^3dx$ B1

$[0,1] \to [1,10]$ B1

Area = $2\pi.\frac{1}{36}\int_1^{10}\sqrt{u}du$ M1A1

$= \frac{\pi}{18}\left[\frac{2}{3}u^{3/2}\right]_1^{10}$ A1

$= \frac{\pi}{27}(10\sqrt{10}-1)$ A1

4.

$I = \int_0^{\pi/2} e^{-2x} d\sin x$ M1

$= \left[e^{-2x}\sin x\right]_0^{\pi/2} - \int_0^{\pi/2}\sin x.{-2}e^{-2x}dx$ A1A1

$= e^{-\pi} + 2J$ A1

$J = \int_0^{\pi/2} e^{-2x} d(-\cos x)$ M1

$= -\left[e^{-2x}\cos x\right]_0^{\pi/2} + \int_0^{\pi/2}\cos x.{-2}e^{-2x}dx$ A1A1

$= 1 - 2I$ A1

Substituting,

$I = e^{-\pi} + 2(1-2I)$

so $I = \frac{2}{5} + \frac{1}{5}e^{-\pi}$ M1A1

It follows that

$J = \frac{1}{5} - \frac{2}{5}e^{-\pi}$ A1

5. *(a)*

$f(x) = \ln(1 + \sin x) : f(0) = 0$ B1

$f'(x) = \frac{\cos x}{1+\sin x}$: $f'(0) = 1$ B1B1

$f''(x) = -\frac{1}{1+\sin x}$: $f''(0) = -1$ B1B1

$f'''(x) = \frac{\cos x}{(1+\sin x)^2}$: $f'''(0) = 1$ B1B1

The series is

$x - \frac{x^2}{2} + \frac{x^3}{6}$ M1A1

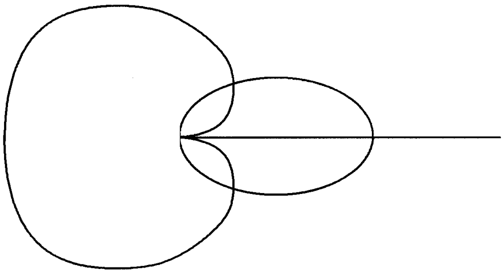
(b)

Int $\approx \left[\frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24}\right]_0^{1/3}$ M1A1

$= \frac{1}{18} - \frac{1}{27\times 6} + \frac{1}{81\times 24}$ A1

$= 0.05$ A1

6. *(a)*



G1G1

(b) Area = $\frac{1}{2}\int_0^{2\pi}(1-\cos\theta)^2\,d\theta$ M1

$= \frac{1}{2}\int_0^{2\pi}(1-2\cos\theta+\cos^2\theta)d\theta$ A1

$= \frac{1}{2}\int_0^{2\pi}\left(\frac{3}{2}-2\cos\theta+\frac{1}{2}\cos 2\theta\right)d\theta$ A1

$= \frac{1}{2}\left[\frac{3\theta}{2}-2\sin\theta+\frac{1}{4}\sin 2\theta\right]_0^{2\pi}$ A1

$= \frac{3\pi}{2}$ A1

(c) The curves meet where

$\cos 2\theta = 1 - \cos\theta$ M1

$2\cos^2\theta+\cos\theta-2=0$ A1

$\cos\theta = \frac{-1\pm\sqrt{17}}{4} = 0.780776...$ A1

giving (0.219, 0.675) and (0.219, -0.675) A1A1

The curves also intersect at the pole (origin). B1

7. *(a)* $\sin n\theta - \sin(n-2)\theta = 2\cos(n-1)\theta\sin\theta$ M1A1

The given result follows by division by $\sin\theta$.

(b) Using this result,

$\int_0^{\pi}\frac{\sin n\theta-\sin(n-2)\theta}{\sin\theta}d\theta = 2\int_0^{\pi}\cos(n-1)\theta\,d\theta$ M1

$I_n - I_{n-2} = \frac{2}{n-1}\left[\sin(n-1)\theta\right]_0^{\pi}$ A1A1

$= 0$ A1

(c) (i) If n is even,

$I_n = I_0 = \int_0^{\pi}0.\,d\theta$ M1A1

$= 0$ A1

(ii) If n is odd,

$I_n = I_1 = \int_0^{\pi}d\theta$ M1A1

$= [\theta]_0^{\pi}$

$= \pi$ A1

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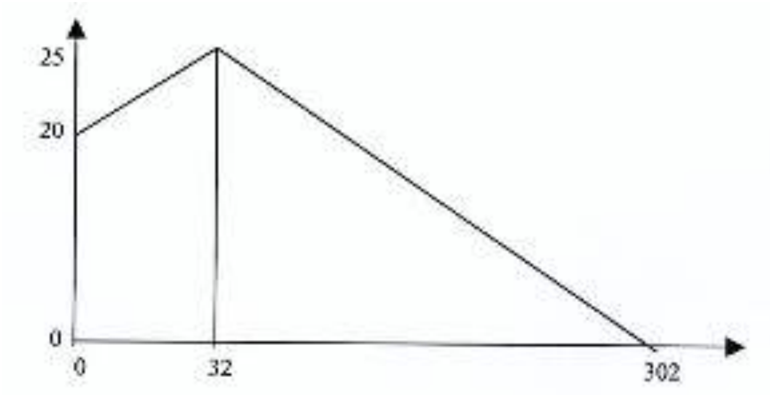
MATHEMATICS M1

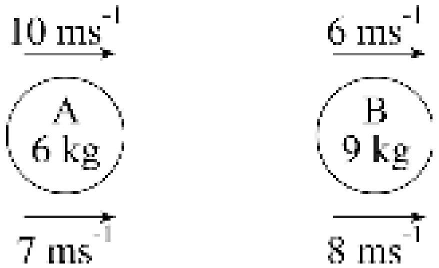
Mechanics

Specimen Mark Scheme

2005/2006

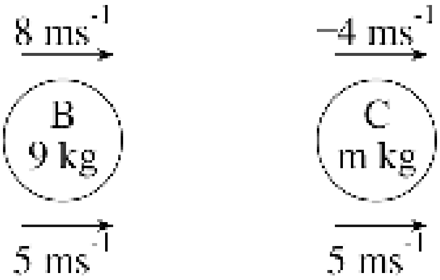
1.	*(a)*	Attempt to apply N2L to lift, dimensionally correct equation	M1
		$600g - T = 600a$	A1
		$a = 0.4$, $T = 600(9.8 - 0.4) = 5640$ N	A1
	(b)	$a = 0$, $T = 600 \times 9.8 = 5880$ N	B1
2.	*(a)*	Attempted use of $s = \frac{1}{2}(u+v)t$ with $u = 20$, $v = 25$, $s = 720$	M1
		$720 = 0.5(20 + 25)t$	A1
		time taken during acceleration = $t = 32$ s	A1
	(b)		M1 (v–t graph) G1 (1st 2 points correct) G1 (last part correct)
	(c)	Use of distance = area under graph	M1
		Distance = $720 + 0{\cdot}5 \times 270 \times 25$	A1
		$= 4095$ m	A1
3.	*(a)*	Use of $v^2 = u^2 + 2as$ with $u = 0$, $v = 1.4$, $a = 9.8$, $s = h$	M1
		$14^2 = 0^2 + 2 \times 9.8h$	A1
		$h = 10$ m	A1
	(b)	Use of $v = u + at$ with $u = 0$, $v = 1.4$, $a = 9.8$	M1
		$14 = 9.8t$	A1
		$t = \frac{10}{7}$ s	A1
	(c)	Required speed = 0.6×14	M1
		$= 8.4$ ms^{-1}	A1

4. *(a)*



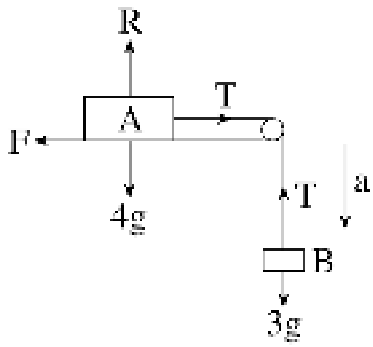
Attempt at restitution equation	M1
$8 - 7 = - e(6 - 10)$	A1
$e = 0.25$	A1

(b)



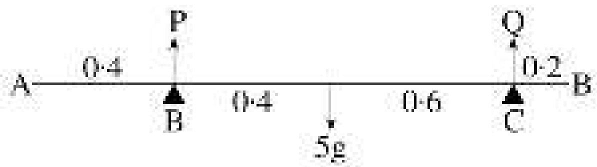
Attempt at use of conservation of momentum, dimensionally correct equation	M1
$9 \times 8 - 4m = (9 + m) \times 5$	A1
$m = 3$ kg	A1

5.



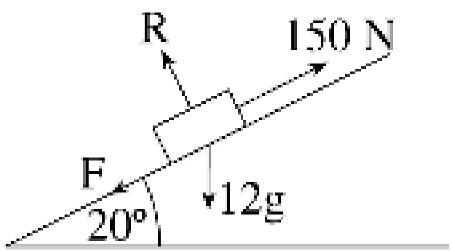
(a)	N2L applied to each particle, dimensionally correct equation	
	For A, $T = 4a$	M1 A1
	For B, $3g - T = 3a$	M1 A1
	Substitute for T, $3g - 4a = 3a$	m1
	$a = \dfrac{3\times 9\cdot 8}{7} = 4.2 \text{ ms}^{-2}$	A1
	$T = 4 \times 4.2 = 16.8$ N	A1
(b)	Vertically for B, $T = 3g$	B1
	Vertically for A, $R = 4g$	B1
	Horizontally for a, $F = T = 3g$	B1
	Use of $F \le \mu R$	M1
	$\mu \ge \dfrac{F}{R} = \dfrac{3g}{4g} = 0.75$	A1

6.



(a)	Attempt to take moments about C/B to obtain dimensionally correct equation	M1
	$-P \times 1 + 5g \times 0.6 = 0$	A1
	$P = 3g = 29.4$ N	A1
	Attempt at second equation with P and Q	M1
	$P + Q = 5g$ (o.e.)	A1
	$Q = 5g - 3g = 2g = 19.6$ N	A1
(b)	Attempt to take moments about C, dimensionally correct equation	M1
	$-P \times 1 + 5g \times 0.6 = W \times 0.2$	A1
	Plank does not tilt iff $P = 3g - 0.2W \ge 0$	m1
	Greatest $W = \dfrac{3g}{0.2} = 15g = 147$ N	A1

7.



(a)	Resolve perpendicular to plane $R = 12g\cos 20°$	M1 A1
	$F = 0.6\,R$	M1
	$F = 0.6 \times 12 \times 9.8 \cos 20°$	A1
	Attempt to apply N2L // to plane, dimensionally correct	M1
	$150 - F - 12g\sin 20° = 12a$	A2 (−1 each error)
	$a = 3.62(28) \text{ ms}^{-2}$	A1

(b) e.g. Body modelled as particle
No air resistance — B1

8. *(a)* attempt at use of Impulse = change of momentum — M1

$-2 = 0.25(v - 5)$ — A1

$v = -3$ — A1

(b) Attempt at use of Impulse = force × time — M1

$-2 = F \times 0.2$ — A1

Magnitude of force = 10 N — A1

9. *(a)*

	Area	Dist of c of m from *AE*	Dist of c of m from *AB*		
DEF	36	2	10	B1	Areas
ABCF	12	1	3	B1	Distance from *AE*
Lamina	48	x	y	*B1*	Distances from *AB*

Take moments — M1

$48x = 36 \times 2 + 12 \times 1$ — A1

$x = 1{\cdot}75$ — A1

$48y = 36 \times 10 + 12 \times 3$ — A1

$y = 8{\cdot}25$ — A1

(b) Required angle identified — M1

$\tan\theta = \dfrac{1\cdot 75}{18 - 8\cdot 25}$ — A1

$\theta = 10{\cdot}18°$ — A1

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MATHEMATICS M2

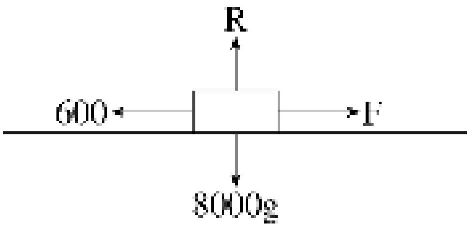
Mechanics

Specimen Mark Scheme

2005/2006

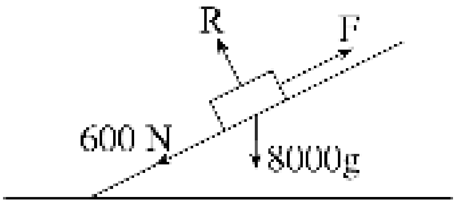
1.	*(a)*	Use of $a = \frac{dv}{dt}$	M1
		$a = 6t + 40t^3$	A1
	(b)	Use of $x = \int v \, dt$	M1
		$x = t^3 + 2t^5 + c$	A1
		Use of $t = 0$, $x = -3$	m1
		$x = t^3 + 2t^5 - 3$	
		when $t = 2$, $x = 8 + 64 - 3 = 69$ m	A1

2. *(a)*



Use of $F = \frac{P}{25}$	B1
N2L $F - 600 = 0$	M1
$\frac{F}{25} = 600$	
$P = 600 \times 25 = 15000$ W	A1

(b)



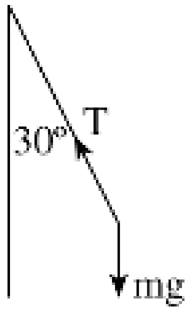
$F = \frac{36 \times 1000}{3} = 12000$	B1
$F - 600 - 8000g \sin\alpha = 8000a$	M1 A1
$12000 - 600 - 8000 \times 9 \cdot 8 \times \frac{1}{14} = 8000a$	A1
$a = 0.725$ ms^{-2}	A1

3.	*(a)*	Tension in string, $T = \frac{19 \cdot 6 \times 0 \cdot 5}{0 \cdot 7}$	M1
		$T = 14$ N	A1
	(b)	Energy stored in string = $\frac{1}{2} \times \frac{19 \cdot 6 \times 0 \cdot 5^2}{0 \cdot 7}$	M1
		= 3.5 J	A1

	(c)	Attempt at conservation of energy (at least 3 terms)	M1
		Gain in potential energy = $0.4 \times 9.8 \times 0.5 \sin\alpha$	M1
		$= 0.4 \times 9.8 \times 0.5\ 0.6 = 1.176$ J	A1
		Gain in kinetic energy = $0.5 \times 0.4\, v^2$	B1
		$1.176 + 0.2v^2 = 3.5$	A1
		$v = 3.4(088)$ ms^{-1}	A1
4.	*(a)*	$y = ut - \frac{1}{2}gt^2$	M1
		$0 = 24{\cdot}5 \sin 30°t - \frac{1}{2}9\cdot 8t^2$	A1,A1
		$t = 2{\cdot}5$ seconds	A1
	(b)	$AB = 24{\cdot}5 \cos 30° \times 2{\cdot}5$	M1 FT student's time
		$= 53{\cdot}0$ m	A1
	(c)	Greatest Height = $24{\cdot}5 \sin 30° \times 1{\cdot}25 - \frac{1}{2}.\ 9{\cdot}8(1{\cdot}25)^2$	M1 FT student's time
		$= 7{\cdot}656$ m	A1
	(d)	$\dot{y} = 24{\cdot}5 \sin 30° - 9{\cdot}8 \times 2 = -7{\cdot}35$	B1 cao
		$\dot{x} = 24{\cdot}5 \cos 30° = 21{\cdot}218$	B1 cao
		Speed = $\sqrt{(-7\cdot 35)^2 + (21\cdot 218)^2} = 22\cdot 45$ m/s	M1,A1
		Angle with horizontal = $\tan^{-1}\ (\frac{-7\cdot 35}{21\cdot 218} = 19\cdot 1°$	M1,A1
5.	*(a)*	$\mathbf{a} = \frac{1}{2}((3\mathbf{i} - 2\mathbf{j}) - (\mathbf{i} + 2\mathbf{j}))$	M1
		$\mathbf{a} = \mathbf{i} - 2\mathbf{j}$	A1
	(b)	Use of $\mathbf{v} = \int \mathbf{a}\, dt$	M1
		$\mathbf{v} = t\mathbf{i} - 2t\mathbf{j} + \mathbf{c}$	A1
		When $t = 0$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$	m1
		$\mathbf{c} = \mathbf{i} + 2\mathbf{j}$	A1
		So $\mathbf{v} = (t + 1)\mathbf{i} - (2t - 2)\mathbf{j}$ (o.e.)	
	(c)	Use of $\mathbf{a.v} = 0$	M1
		$(t + 1) - 2(2 - 2t) = 0$	A1
		$t = 0.6$	A1

	(d)	Use of $\mathbf{r} = \int \mathbf{v}\,dt$	M1
		$\mathbf{r} = \frac{1}{2}t^2\,\mathbf{i} - t^2\mathbf{j} + t\mathbf{i} + 2t\mathbf{j} + \mathbf{c}$	A1
		when $t = 0$, $\mathbf{r} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{c} = 2\mathbf{i} - \mathbf{j}$	m1
		$\mathbf{r} = (\frac{1}{2}t^2 + t + 2)\mathbf{i} + (-t^2 + 2t - 1)\mathbf{j}$	A1
	(e)	when $t = 2$, $\mathbf{r} = (2 + 2 + 2)\mathbf{i} + (-4 + 4 - 1)\mathbf{j}$	M1
		$\mathbf{r} = 6\mathbf{i} - \mathbf{j}$	
		$\lvert\mathbf{r}\rvert = \sqrt{6^2 + 1^2}$	M1
		$\lvert\mathbf{r}\rvert = \sqrt{37}$	A1
6.	*(a)*	Attempt at conservation of energy	M1
		$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg(a - a\cos\theta)$	A2 (-1 each error)
		$v^2 = u^2 + 2ag(1 - \cos\theta)$	A1
	(b)	Attempt at N2L towards centre	M1
		$R - mg\cos\theta = m \times$ acceleration	A1
		$R - mg\cos\theta = -\frac{mv^2}{a}$	A1
		$R = mg\cos\theta - \frac{m}{a}(u^2 + 2ag - 2ag\cos\theta)$	m1
		$R = mg\cos\theta - 2mg + 2mg\cos\theta - \frac{mu^2}{a}$	
		$R = mg(3\cos\theta - 2) - \frac{mu^2}{a}$	A1
	(c)	$R = 0$ when ball bearing leaves bowl.	M1
		$0.05 \times 9.8(3\cos\theta - 2) - \frac{0\cdot05 \times 2^2}{0\cdot5} = 0$	A1
		$\cos\theta = 0.938(77551)$	
		$\theta = 20.15(3)°$	A1

7. *(a)*



Resolve vertically. $T\cos 30° = mg$	M1 A1
$T = \dfrac{0 \cdot 2 \times 9 \cdot 8}{\cos 30^{\circ}} = 2.26$	A1
(b) N2L towards centre	M1
$T\sin 30° = \dfrac{mu^2}{0 \cdot 8 \sin 30^{\circ}}$	A1 L.H.S.
	A1 R.H.S.
Substitute for T	m1
$u = 1.50(43979)\ \text{ms}^{-1}$	A1
(c) Tension is constant throughout the rope.	B1

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MATHEMATICS M3

Mechanics

Specimen Mark Scheme

2005/2006

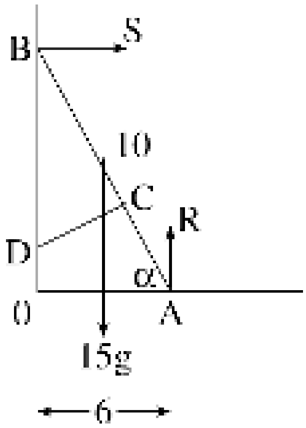
1.	*(a)*	N2L applied to particle	M1
		$4a = -4g - \dfrac{gv^2}{9}$	A1
		Magnitude of retardation = $\dfrac{g}{36}(36 + v^2)$	A1 (convincing)
	(b)	$\dfrac{dv}{dt} = -\dfrac{g}{36}\left(36 + v^2\right)$	M1
		$36\int \dfrac{dv}{36 + v^2} = -g\int dt$	A1
		$36 \times \dfrac{1}{6}\tan^{-1}\dfrac{v}{6} = -gt + c$	A1
		When $t = 0$, $v = 6$	m1
		$C = 6 \times \dfrac{\pi}{4} = \dfrac{3\pi}{2}$	A1
		At maximum height, $v = 0$	M1
		$t = \dfrac{3\pi}{2g} = 0 \cdot 48\,\text{s}$	A1
	(c)	$v\dfrac{dv}{dx} = -\dfrac{g}{36}\left(36 + v^2\right)$	M1
		$36\int \dfrac{v\,dv}{36 + v^2} = -g\int dx$	A1
		$10 \ln(36 + v^2) = -gx + c$	A1
		When $t = 0$, $v = 6$, $x = 0$	m1
		$c = 18 \ln(72)$	A1
		At maximum height. $v = 0$	M1
		$x = \dfrac{18}{9 \cdot 8}\ln(2) = 1.27$ m	A1
2.	*(a)*	Max. speed $= a\omega$	M1
		$a\omega = 8$	A1
		$\omega = \dfrac{8}{5}$	A1
		Period $= \dfrac{2\pi}{\omega} = \dfrac{5\pi}{4}$	B1 (F.T.)
		Time for 9 oscillations = $\dfrac{45\pi}{4}$ (or 35.34s)	B1

(b)	Using $v^2 = \omega^2(a^2 - x^2)$ with $a = 5, x = 4, \omega = \frac{8}{5}$	o.e.	M1
	$v^2 = 1\cdot6^2(5^2 - 4^2)$		A1
	$v = 4\cdot8\text{ms}^{-1}$		A1
(c)	$\lvert a\rvert = \omega^2 x$		M1
	$= 1\cdot6^2 \times 3$		A1
	$= 7\cdot68\text{ms}^{-2}$		A1
(d)	$x = 5\sin(1\cdot6t)$		B1
	When $x = -2\cdot4$, $t_A = -0\cdot3129$		B1
	When $x = 3\cdot6$, $t_B = 0\cdot5024$		B1
	Required time $= t_B - t_A$		M1 (overall strategy)
	$= 0\cdot8153$		A1 (C.A.O.)

3.

Auxiliary equation $m^2 + 8m + 12 = 0$		M1
$(m + 2)(m + 6) = 0$		
$m = -2, -6$		A1
Complementary function $x = Ae^{-2t} + Be^{-6t}$		A1
For particular integral, try $x = at + b$		M1
$\frac{dx}{dt} = a$		
Substitute into differential equation		
$8a + 12(at + b) = 12t + 20$		A1
Compare coefficients		m1
$12a = 12$		
$a = 1$		
$8a + 12b = 20$		
$b = 1$	both values	A1
General solution $x = Ae^{-2t} + Be^{-6t} + t + 1$		A1
When $t = 0$, $x = 0$		M1
$0 = A + B + 1$		B1
When $t = 0$, $\frac{dx}{dt} = 3$		
$\frac{dx}{dt} = -2Ae^{-2t} - 6Be^{-6t} + 1$		B1
$-2A - 6B + 1 = 3$		A1
Solving $A = -1$, $B = 0$		A1
General solution is $x = -e^{-2t} + t + 1$		
When $t = 2$, $x = 2\cdot98$ m		A1

4. *(a)*



Resolve horizontally	M1
$S = T\sin\alpha$	A1
Moments about A, dimensionally correct	M1
$15g \times 3 + 4\times T = S \times 8$	A1
Substitute for $S = 0{\cdot}8T$	m1
$45g + 4T = 6{\cdot}4T$	
$T = 183{\cdot}75$ N	A1
$S = 147$ N	A1
Resolve vertically	M1
$R = 15g + T\cos\alpha$	A1
$R = 15 \times 9{\cdot}8 + 183{\cdot}75 \times 0{\cdot}6 = 257{\cdot}25$ N	A1

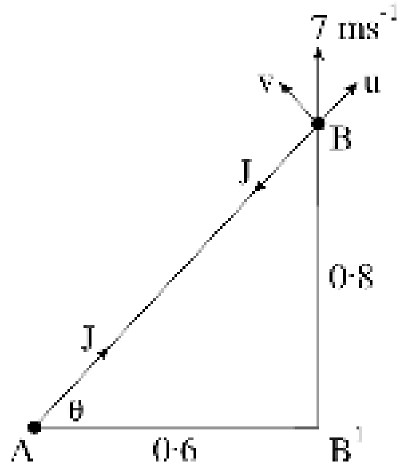
(b)

Using $T = 2000$ N and adding extra term in moment equation	M1
$441 + 4T + 80g \times x\cos\alpha = 6{\cdot}4\,T$	A1
$470{\cdot}4x = (6{\cdot}4 - 4) \times 2000 - 441$	
$x = 9{\cdot}27$ m	A1
Man will not reach top of ladder	A1

(c)

Any reasonable assumption (e.g. man is a particle, ladder is a rigid rod)	B1

5. *(a)* B1



$v = 7\cos\theta = 4.2\ \text{ms}^{-1}$	B1
For B, $-J = 5 \times 7\sin\theta - 5u$	M1 A1
For A, $J = 2u$	B1
$-J = 28 - 5u = -2u$	
$u = 4\ \text{ms}^{-1}$	A1
Speed of $A = 4\ \text{ms}^{-1}$	
Speed of $B = \sqrt{4^2 + 4\cdot2^2}$	M1
Speed of $B = \sqrt{33.64} = 5\cdot8\ \text{ms}^{-1}$	A1
$J = 2u = 8$ Ns	A1

(b)

Loss in K.E. = Initial K.E. - Final K.E.	M1
Loss in K.E. = $0.5 \times 5 \times 7^2 - 0.5 \times 5 \times 33.64 - 0.5 \times 2 \times 4^2$	m1 A1
Loss in K.E. = 22.4 J	A1

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MATHEMATICS S1

Statistics

Specimen Mark Scheme

2005/2006

1. *(a)* Prob = $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\times 6 = \frac{2}{7}$ (or $\dfrac{\binom{4}{1}\times\binom{3}{1}\times\binom{2}{1}}{\binom{9}{3}}$) M1M1A1

(b) P(3 reds) = $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7} = \frac{1}{21}$ B1

P(3 blues) = $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7} = \frac{1}{84}$ B1

P(3 yellow) = 0 B1

P(3 same colour) = Sum = $\frac{5}{84}$ B1

(or $\dfrac{\binom{4}{3}+\binom{3}{3}}{\binom{9}{3}}$)

2. E(Y) = 4 × 4 + 1 = 17 M1A1

Var (X) = 4 B1

Var (Y) = 16 × 4 = 64 M1A1

SD(Y) = 8 A1

3. *(a)* $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

= 0.7 + 0.4 - 0.28 = 0.82 (or = $1 - P(A'\cap B') = 1 - 0.18$) M1A1A1

(b) (i) Prob = $P(A)P(B') + P(A')P(B)$ M1A1

= 0.7 × 0.6 + 0.3 × 0.4 = 0.54 A1

(ii) $P(A'$ and $B')$ = 0.3 × 0.6 = 0.18 M1A1A1

4. *(a)* (i) P($X \geq 4$) = 0.8488 (or 1 - 0.1512) M1A1

(ii) P(X = 6) = 0.6063 - 0.4457 (or 0.5543 - 0.3937) M1A1A1

= 0.1606

(b) (i) P(Y = 2) = $e^{-1.12}\times\frac{1.12^2}{2} = 0.205$ M1A1

(ii) P(Y ≥ 2) = 1 - P(Y ≤ 1) M1

= 1 - $e^{-1.12}\times(1+1.12)$ A1

= 0.308 A1

5. *(a)* P(White) = 0.5 × 0.15 + 0.3 × 0.2 + 0.2 × 0.25 = 0.185 M1A1A1

(b) P(A | White) = $\frac{0.5\times 0.15}{0.185} = 0.405$ M1A1A1

6.	*(a)*	Mean = $10 \times 0.8 = 8$; SD = $\sqrt{10 \times 0.8 \times 0.2} = 1.26$		B1M1A1
	(b)	(i)	$P(X = 8) = \binom{10}{8} \times 0.8^8 \times 0.2^2 = 0.302$	M1A1
			(or using tables 0.6242 - 0.3222 or 0.6778 - 0.3758)	
		(ii)	Y is B(10,0.2) $\rightarrow$ $P(4 \le X \le 7) = P(3 \le Y \le 6)$	M1A1
			= 0.9991 - 0.6778 = 0.321	m1A1
			(or 0.3222 - 0.0009)	
7.	*(a)*	Mean = $1\times 0.2 + 2 \times 0.3 + 3 \times 0.2 + 4 \times 0.3 = 2.6$		M1A1
		$E(X^2) = 1 \times 0.2 + 4 \times 0.3 + 9 \times 0.2 + 16 \times 0.3 = 8.0$		M1A1
			Var = $8 - 2.6^2 = 1.24$	A1
	(b)	$E\left(\frac{1}{X}\right) = 1 \times 0.2 + \frac{1}{2} \times 0.3 + \frac{1}{3} \times 0.2 + \frac{1}{4} \times 0.3 = 0.492$		M1A1A1
	(c)	Possibilities are (2,2), (1,3), (3,1). (si)		B1
		Prob = $0.3 \times 0.3 + 2 \times 0.2 \times 0.2$		M1A1
		= 0.17		A1
8.	*(a)*	$E(X) = \frac{1}{12}\int_0^2 x(8x - x^3)dx$		M1A1
			$= \frac{1}{12}\left[\frac{8}{3}x^3 - \frac{x^5}{5}\right]_0^2$	A1
			= 1.24	A1
	(b)	(i)	$F(x) = \frac{1}{12}\int_0^x (8y - y^3)dy$	M1A1
			$= \frac{1}{12}\left[4y^2 - \frac{y^4}{4}\right]_0^x$	A1
			$= \frac{1}{12}\left(4x^2 - \frac{x^4}{4}\right)$	A1
		(ii)	$P(X \ge 1) = 1 - F(1) = 0.6875$	M1A1A1
		(iii)	The median satisfies F(m) = 0.5, ie	M1
			$4m^2 - \frac{m^4}{4} = 6$ leading to the given equation.	A1
			Solving,	
			$m^2 = \frac{16 \pm \sqrt{256 - 96}}{2}$	M1A1
			giving $m = 1.29$	A1

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MATHEMATICS S2

Statistics

Specimen Mark Scheme

2005/2006

1.	*(a)*	(i)	$z = \frac{200-195}{5} = 1$	M1A1
			$\Rightarrow$ P(overflow) = 0.1587	A1
		(ii)	$\mu = 200 - 2.326 \times 5 = 188.4$	M1A1A1
	(b)		T is N(980,125)	M1A1
			$z = \frac{1000-980}{\sqrt{125}} = 1.79$	m1A1
			Prob = 0.0367	A1
2.	*(a)*		Distribution of T is Poi(2.5).	B1
			Prob = $e^{-2.5}.\frac{2.5^2}{2} = 0.257$	M1A1
	(b)		T is now Poi(125) $\approx$ N(125,125)	M1A1
			$z = \frac{99.5-125}{\sqrt{125}} = -2.28$	M1A1A1
			Prob = 0.0113	A1
3.	*(a)*		$\bar{x} = 73.5$	M1A1
			95% confidence limits are	
			$73.5 \pm 1.96\sqrt{\frac{16}{10}}$	M1A1A1
			giving [71.0, 76.0].	A1
	(b)		A 95% confidence interval is an interval determined by a method which would ensure that the parameter lies within the interval 95% of the time.	B2 (Allow B1 if not completely convinced)
4.	*(a)*		$P(\pi R^2 > 36\pi) = P(R > 6)$	M1
			$= \frac{(10-6)}{(10-4)}$	A1
			$= \frac{2}{3}$	A1
	(b)	(i)	The density of R is f(r) = 1/6 (si)	B1
			$E(A) = \pi\int_4^{10} r^2.\frac{1}{6}dr$	M1A1
			$= \frac{\pi}{18}\left[r^3\right]_4^{10}$	A1
			$= 52\pi$	A1

(ii) $E(A^2) = \pi^2 \int_4^{10} r^4 . \frac{1}{6} dr$ M1A1

$= \frac{\pi^2}{30}\left[r^5\right]_4^{10}$ A1

$= 3299.2\pi^2$ A1

$\text{Var}(A) = 3299.2\pi^2 - 52^2\pi^2$

$= 595.2\pi^2$ A1

5. The appropriate test statistic is

$$\text{TS} = \frac{\bar{x} - \bar{y}}{\sigma\sqrt{\frac{1}{m} + \frac{1}{n}}}$$ M1

$$= \frac{52.6 - 49.8}{5\sqrt{\frac{1}{10} + \frac{1}{10}}}$$ A1A1

$= 1.25$ A1

EITHER

p-value = 2 × 0.1056

= 0.2112 M1A1

This is greater than 0.01 so accept that concentrations are equal. A1

OR

Critical value = 2.576 M1A1

The calculated value is less than this so accept that concentrations are equal. A1

6. *(a)* (i) X is B(20,p) (si) B1

Sig level = $P(X \geq 14 \mid p = 0.5)$ M1A1

= 0.0577 A1

(ii) We require

$P(X \geq 14 \mid p = 0.7) = P(Y \leq 6 \mid p = 0.3)$ M1A1

= 0.608 A1

(b) Under H_0, X is B(200,0.5) ≈ N(100,50) M1A1

$$z = \frac{119.5 - 100}{\sqrt{50}}$$ m1A1

= 2.76 A1

p-value = 0.00289 A1

Strong evidence to support Dafydd's theory. B1

7.	*(a)*	(i)	$H_0: \mu = 3$ versus $H_1: \mu > 3$	B1
		(ii)	In 5 days, number sold Y is Poi(15) under H_0.	B1
			p-value = $P(Y \geq 20)$	M1
			= 0.1248	A1
			We cannot conclude that the mean has increased.	B1
	(b)	Under H_0 the number sold in 100 days is Poi(300) ≈ N(300,300)		B1B1
			$z = \dfrac{329.5 - 300}{\sqrt{300}}$	M1A1
			= 1.70	A1
			p-value = 0.0446	A1
		Significant at the 5% level because 0.0446 < 0.05.		B1B1

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MATHEMATICS S3

Statistics

Specimen Mark Scheme

2005/2006

1. *(a)* The possible combinations are given in the following table.

Combination	Sum
1 2	3
1 3	4
1 3	4
2 3	5
2 3	5
3 3	6

M1A1 (combs)A1(sum)

The sampling distribution is

Sum	3	4	5	6
Prob	1/6	2/6	2/6	1/6

M1A1

(b) The possibilities are

Possibility	Sum
1 1	2
1 2	3
1 3	4
1 3	4
2 1	3
2 2	4
2 3	5
2 3	5
3 1	4
3 2	5
3 3	6
3 3	6
3 1	4
3 2	5
3 3	6
3 3	6

M1A2(Poss)A1(sum)
(−1 each error)

The distribution is

Sum	2	3	4	5	6
Prob	1/16	2/16	5/16	4/16	4/16

M1A1

2.	*(a)*	$\hat{p} = \frac{930}{1500} = 0.62$	B1
	(b)	$\text{SE} \approx \sqrt{\frac{0.62 \times 0.38}{1500}}$	M1
		$= 0.0125...$	A1
		The value of p is not known and has to be estimated.	B1
	(c)	90% confidence limits are	
		$0.62 \pm 1.645 \times 0.0125$	M1A1
		giving [0.60,0.64].	A1
	(d)	No we are not. The confidence interval has a level of confidence associated with it and it is not certain.	B1B1
3.	*(a)*	$H_0 : \mu_x = \mu_y$ versus $H_1 : \mu_x \neq \mu_y$	B1
	(b)	Test statistic $= \dfrac{2.51 - 2.45}{\sqrt{\frac{0.082}{120} + \frac{0.094}{80}}}$	M1A1A1
		$= 1.39..$	A1
		p-value $= 0.0823 \times 2$	M1A1
		$= 0.1646$	A1
		We conclude that there is no difference in mean weight.	B1
4.	*(a)*	(i) $\Sigma x = 30.4$; $\bar{x} = 3.8$	B1B1
		$\sum x^2 = 117.14$,	B1
		$\hat{\sigma}^2 = \frac{117.14}{7} - \frac{30.4^2}{7 \times 8}$	M1
		$= 0.231(428..)$	A1
		(ii) 99% confidence limits are	
		$3.8 \pm 2.365 \times \sqrt{\frac{0.231428..}{8}}$	M1A1A1
		giving [3.4, 4.2].	A1
	(b)	$\bar{x} = \frac{431.2}{110} = 3.92$	B1
		$\hat{\sigma}^2 = \frac{1712.1}{109} - \frac{431.2^2}{110 \times 109}$	M1
		$= 0.2$	A1
		95% confidence limits are	
		$3.92 \pm 1.96 \times \frac{0.2}{110}$	M1A1
		giving [3.84,4.00]	A1

5. *(a)*

$$b = \frac{8 \times 17882.5 - 220 \times 638.1}{8 \times 7100 - 220^2}$$ M1A1

$= 0.319$ A1

$$a = \frac{638.1 - 220 \times 0.319}{8}$$ M1

$= 71.0$ A1

(b) Test statistic $= \dfrac{0.319 - 0.3}{0.2}\sqrt{7100 - 220^2 / 8}$ M1A1A1

$= 3.08$ A1

p-value $= 2 \times 0.001$ M1

$= 0.002$ A1

Very strong evidence that the value of β is not 0.3. B1

(c) (i) Est length $= 71.0 + 0.319 \times 35$ M1

$= 82.165$ A1

$$\text{SE} = 0.2\sqrt{\frac{1}{8} + \frac{(35 - 27.5)^2}{7100 - 220^2 / 8}}$$ M1

$= 0.085$ A1

(ii) 95% confidence limits are

$82.165 \pm 1.96 \times 0.085$ M1

giving [82.0,82.3] A1

6. *(a)*

$$\text{E}\left(\hat{\theta}\right) = 2\text{E}(\overline{X})$$ M1

$= 2\text{E}(X)$ A1

$= 2.\dfrac{\theta}{2} = \theta$ A1

$$\text{Var}\left(\hat{\theta}\right) = 4\text{Var}(\overline{X})$$ M1

$= 4.\dfrac{\text{Var}(X)}{48}$ A1

$= \dfrac{\theta^2}{144}$ A1

$\text{SE}(\hat{\theta}) = \dfrac{\theta}{12}$ A1

(b)

$$\hat{\theta} - \theta \approx \text{N}\left(0, \frac{\theta^2}{144}\right)$$ B1

We require

$$\text{P}\left(\left|\hat{\theta} - \theta\right| > 0.05\theta\right)$$ M1

$z = \dfrac{0.05\theta}{\theta / 12}$ m1A1

$= 0.6$ A1

Prob $= 0.2743 \times 2 = 0.5286$ A1