

973/01

MATHEMATICS C1

Pure Mathematics

A.M. MONDAY, 23 May 2005

(1½ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A, B, C, D have coordinates $(1, 7), (5, -1), (8, 3), (6, 7)$ respectively.

(a) Show that AB and CD are parallel. [3]

(b) Find the equation of AB . [2]

(c) The line L passes through the point D and is perpendicular to AB . Show that L has equation

$$x - 2y + 8 = 0. \quad [3]$$

(d) The lines L and AB intersect at the point E . Find the coordinates of E . [2]

(e) Calculate the length of EF , where F is the mid-point of AB . [4]

2. Simplify each of the following, expressing your answers in surd form:

(a) $\sqrt{45} + \sqrt{80} - \sqrt{125}$; [3]

(b) $\frac{6 + \sqrt{2}}{2 + \sqrt{2}}$. [4]

3. (a) Given that $x - 1$ is a factor of $3x^3 + 5x^2 + ax - 4$, show that $a = -4$. [2]

(b) Solve the equation $3x^3 + 5x^2 - 4x - 4 = 0$. [4]

(c) Calculate the remainder when $3x^3 + 5x^2 - 4x - 4$ is divided by $x + 1$. [2]

4. Write down and simplify the first four terms in the binomial expansion of $(1 + 2x)^6$. [4]

5. Given $y = x^2 - 7x + 2$, find $\frac{dy}{dx}$ from first principles. [5]

6. The curve C has equation

$$y = 16\sqrt{x} + \frac{32}{x} + 2.$$

(a) Find the value of $\frac{dy}{dx}$ when $x = 4$. [3]

(b) Find the equation of the normal to C at the point where $x = 4$. [3]

7. The curve C has equation $y = x^3 - 3x^2$.

- (a) Find the coordinates of the stationary points of C and determine the nature of each of these points. [7]
- (b) Sketch C . [3]
- (c) Find the range of values of k for which there are three real and distinct solutions of the equation $x^3 - 3x^2 = k$. [2]

8. (a) Express the quadratic expression $x^2 - 6x + 16$ in the form $(x - a)^2 + b$, where the values of the constants a and b are to be determined. **Deduce** the least value of $x^2 - 6x + 16$. [3]

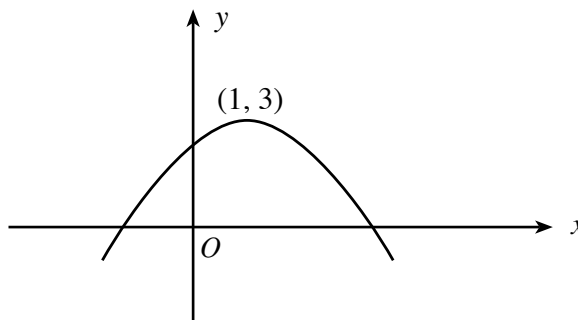
(b) Solve the inequality

$$(x + 1)^2 \leq 4x + 9. \quad [4]$$

9. The straight line $y = 2x + c$ is a tangent to the curve $y = x^2 + 6x + 7$.

- (a) Determine the value of the constant c . [4]
- (b) Find the coordinates of the point of contact of the tangent and the curve. [2]

10. The diagram shows the graph of $y = f(x)$. The graph has a maximum point at $(1, 3)$.



Sketch the following graphs, using a separate set of axes for each graph and indicating the coordinates of the stationary point in each case.

- (a) $y = 4f(x)$ (b) $y = f(x - 2)$ (c) $y = f\left(\frac{x}{2}\right)$ [2], [2], [2]