

974/01

MATHEMATICS C2

Pure Mathematics

A.M. MONDAY, 23 May 2005

(1½ hours)

NEW SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with six ordinates to find an approximate value for

$$\int_0^1 \sqrt{1+x^2} \, dx .$$

Show your working and give your answer correct to three decimal places. [4]

2. (a) Find the values of x in the range $0 \leq x \leq 360^\circ$ satisfying

$$8 \cos^2 x + 2 \sin x - 7 = 0. \quad [6]$$

- (b) Find the values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan 2x = 1. \quad [3]$$

3. (a) An arithmetic series has first term a and common difference d . Write down the n th term and prove that the sum of the first n terms is given by

$$S_n = \frac{n}{2} [2a + (n-1)d] . \quad [4]$$

- (b) The seventh term of an arithmetic series is twice the third term. The sum of the first ten terms of the series is 195.

(i) Find the common difference of the series.

(ii) Find the sum of the first sixty terms. [7]

4. The sum of the first two terms of a geometric series is 6.4 , and the sum to infinity of the series is 10 .

(a) Given that the common ratio is positive, find its value. [5]

(b) Find, correct to three decimal places, the sum of the first eleven terms of the series. [3]

5. The circle C is given by the equation

$$x^2 + y^2 - 8x + 4y - 5 = 0.$$

(a) Find the radius and the coordinates of the centre of C . [3]

(b) (i) Show that $P(1, -6)$ lies on C . [1]

(ii) Find the equation of the tangent to C at P . [4]

6. (a) Given that $x > 0, y > 0$, show that

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y . \quad [3]$$

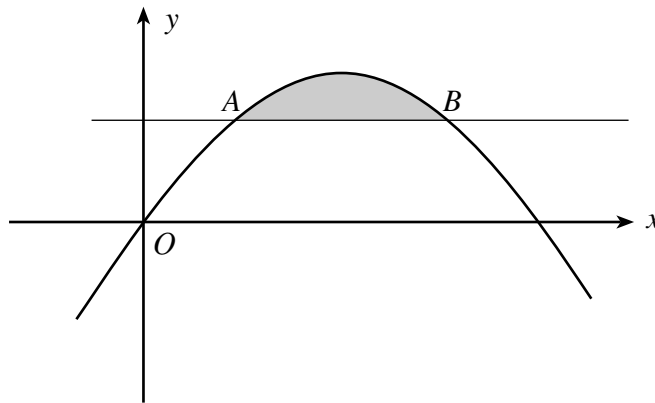
- (b) (i) Solve the equation

$$5^{2x+1} = 7,$$

giving your answer correct to four decimal places.

- (ii) Express $\log_{10} 2 + 2 \log_{10} 18 - \frac{3}{2} \log_{10} 36$ as a single logarithm in its simplest form. [8]

7. (a) Find $\int \left(2x^{\frac{3}{4}} + \frac{7}{x^{\frac{1}{2}}} \right) dx$. [2]
- (b)



The diagram shows a sketch of the curve $y = 6x - x^2$ and the line $y = 5$. The line and the curve intersect at the points A and B .

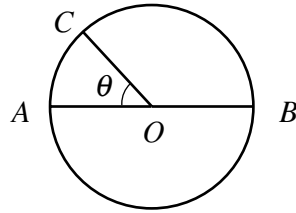
- (i) Showing your working, find the coordinates of A and B .
- (ii) Find the area of the shaded region. [10]

8. The triangle ABC is such that $AB = x$ cm, $BC = (x - 3)$ cm, $CA = (x - 1)$ cm and $\hat{ABC} = 60^\circ$.

- (a) Use the cosine rule to show that $x = 8$. [4]
- (b) Find the area of triangle ABC , giving your answer in surd form. [2]

TURN OVER.

9.



The diagram shows three points A, B, C on a circle with centre O and radius 4 cm , such that AB is a diameter of the circle and $\widehat{AOC} = \theta$ radians. Given that the area of the sector BOC is 5 cm^2 more than the area of the sector AOC ,

(a) show that $\theta = \frac{8\pi - 5}{16}$, [3]

(b) calculate the difference between the arc length BC and the arc length AC . [3]