

**MS2**  
**£2.00**

**WELSH JOINT EDUCATION COMMITTEE**  
**CYD-BWYLLGOR ADDYSG CYMRU**

**General Certificate of Education**  
**Advanced Subsidiary/Advanced**

**Tystysgrif Addysg Gyffredinol**  
**Uwch Gyfrannol/Uwch**

**MARKING SCHEMES**

**SUMMER 2005**

**MATHEMATICS**  
**(NEW SPECIFICATION)**

**WJEC**  
**CBAC**

## **INTRODUCTION**

The marking schemes which follow were those used by the WJEC for the 2005 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

**Mathematics C1**  
**Solutions and Mark Scheme**

1. (a) Gradient of  $AB = \frac{7+1}{1-5} = -2$  M1 (correct attempt to find gradients)
- Gradient of  $CD = \frac{3-7}{8-6} = -2$  A1 (both gradients)
- $\therefore$  Lines are parallel B1 (must involve two equal gradients)
- (b) Equation of  $AB$  is
- $$y - 7 = -2(x - 1)$$
- M1 (use of  $y - y_1 = m(x - x_1)$  (o.e.) with appropriate values)
- A1 (give mark here. F.T gradient if 2 Ms have been gained)
- $$2x + y - 9 = 0 \quad (1)$$
- (c) Gradient of  $L = \frac{1}{2}$  B1  $\left( -\frac{1}{\text{gradient of } AB}, \text{o.e.} \right)$
- Equation of  $L$  is
- $$y - 7 = \frac{1}{2}(x - 6)$$
- B1 (F.T. gradient of  $L$  if B1 gained in (c) and M2 gained in (a), (b))
- $$\begin{aligned} 2y - 14x &= x - 6 \\ x - 2y + 8 &= 0 \end{aligned} \quad (2)$$
- B1 (convincing)
- (d) Solve (1), (2)  $x = 2, y = 5$  M1, A1 (C.A.O.) (allow only for algebraic solution)
- (e) Mid-point of  $AB$  has coordinates  $\left( \frac{1+5}{2}, \left( \frac{7-1}{2} \right) \right)$  i.e. (3, 3) B1, B1
- $$EF = \sqrt{(3-2)^2 + (5-3)^2} = \sqrt{5} \ (\approx 2.24)$$
- M1 (correct formula)  
A1 (F.T. coordinates of  $E$  and  $F$ )

[14]

2. (a)  $3\sqrt{5} + 4\sqrt{5} - 5\sqrt{5} = 2\sqrt{5}$

M1(attempt to simplify/one correct answer)  
A1 (all correct)  
A1 (F.T. one slip with answer of form  $k\sqrt{5}$ )

(b) 
$$\frac{(6 + \sqrt{2})(2 - \sqrt{2})}{(2 + \sqrt{2})(2 - \sqrt{2})} = \frac{12 - 6\sqrt{2} + 2\sqrt{2} - 2}{4 - 2}$$

M1 (correct rationalising)

A1 (numerator with  $(\sqrt{2})^2 = 2$ , allow  $2 \times 6$ )

A1 (denominator with no ??)

$$= 5 - 2\sqrt{2} \text{ (allow } \frac{10 - 4\sqrt{2}}{2} \text{ )}$$

A1 (F.T. one slip)

[7]

3. (a)  $(f(1) = 0)$

M1 (any method)

$3 + 5 + a - 4 = 0$

A1

$a = -4$

Special Case B1 if  $a = -4$  assumed

(b)  $3x^3 + 5x^2 - 4x - 4 = (x - 1)(3x^2 + 8x + 4)$

M1 ( $3x^2 + ax + b$ ,  $a$  or  $b$  correct, any method)  
A1

$$= (x - 1)(3x + 2)(x + 2)$$

A1 (F.T. one slip)

Roots are  $1, -\frac{2}{3}, -2$

A1 (F.T. one slip)

(c) Remainder  $= 3(-1)^3 + 5(-1)^2 - 4(-1) - 4$

M1 (any method, division must have  $3x^2 + 2x + a$ )

$$= 2$$

A1

[8]

4.  $(1+2x)^6 = 1 + 6(2x) + \frac{6.5}{1.2}(2x)^2 + \frac{6.5.4}{1.2.3}(2x)^3 + \dots$  M1 (substitution of  $2x$ ,  $n=6$  in  $(1+x)^n$ )
- $= 1 + 12x + 60x^2 + 160x^3 + \dots$  A1  $(1+12x)$   
A1  $(60x^2)$   
A1  $(160x^3)$
- [4]
5.  $y + \Delta y = (x + \Delta x)^2 - 7(x + \Delta x) + 2$  B1
- $\Delta y = (x + \Delta x)^2 - 7(x + \Delta x) + 2 - (x^2 - 7x + 2)$  M1
- $= 2x\Delta x + (\Delta x)^2 - 7\Delta x$  A1
- $\frac{\Delta y}{\Delta x} = 2x + \Delta x - 7$  M1 (divide by  $\Delta x$  and let  $\Delta x \rightarrow 0$ .  
Method must involve  $\Delta x \rightarrow 0$  and some statement about answer being a limit)
- $\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x}$
- $= 2x - 7$  A1 (award for clear presentation and no abuse of notation)
- [5]
6. (a)  $16 \cdot \frac{1}{2\sqrt{x}}, -\frac{32}{x^2}$  (o.e) B1, B1
- $\frac{dy}{dx} = \frac{8}{2} - \frac{32}{16} = 2$  B1 (C.A.O.)
- (b) Slope of normal  $= -\frac{1}{2}$  B1  $\left( \frac{-1}{\text{candidate's } \frac{dy}{dx}} \right)$
- When  $x=4$ ,  $y = 16 \times 2 + \frac{32}{4} + 2$
- $= 42$  B1 (C.A.O.)

Equation is  $y - 42 = -\frac{1}{2}(x - 4)$

B1 (F.T. slope if first B1 in (c)  
gained, and candidate's value of  
 $y$ )

[6]

7. (a)  $\left(\frac{dy}{dx} = 0\right)$

$$3x^2 - 6x = 0$$

B1  $\left(\frac{dy}{dx}\right)$

M1  $\left(\frac{dy}{dx} = 0\right)$

$$3x(x - 2) = 0$$

$$x = 0, 2$$

A1 (either root)

When  $x = 0, y = 0$ ; when  $x = 2, y = -4$

A1 (both, C.A.O.)

$$\frac{d^2y}{dx^2} = 6x - 6$$

M1 (any method)

$$x = 0, \frac{d^2y}{dx^2} = -6 < 0$$

max. pt

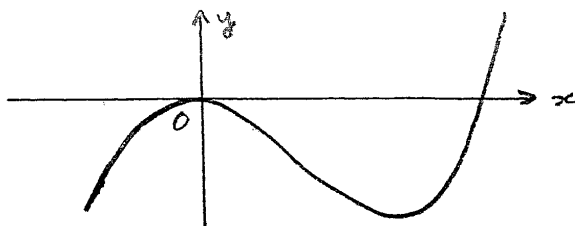
A1

$$x = 2, \frac{d^2y}{dx^2} = 6 > 0$$

min. pt

A1

(b)



B1 (shape, allow graph not  
crossing  $x$  - axis  
B2 (stationary pts on a cubic)

(c) Three solutions for  $-4 < k < 0$

B2 (F.T candidate's stationary  
points)

Special Cases

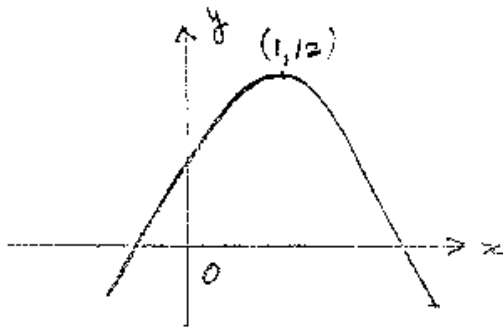
B1 for  $k < 0$  or  $k > -4$  (or  $k < 0, k < -4$ )

B1 for  $-4 \leq k \leq 0$

[12]

|    |                                                                                  |                                                           |
|----|----------------------------------------------------------------------------------|-----------------------------------------------------------|
| 8. | (a) $x^2 - 16x + 16 = (x - 3)^2 + 7$                                             | B1 $((x - 3)^2)$<br>B1 (7) (b)                            |
|    | Least value = 7                                                                  | B1 (F.T. candidate's $b$ , least value must be mentioned) |
|    | No marks for answer derived by calculus.                                         |                                                           |
|    | (b) $x^2 + 2x + 1 \leq 4x + 9$                                                   | M1 (correct method of rearranging)                        |
|    | $x^2 - 2x - 8 \leq 0$                                                            |                                                           |
|    | $(x - 4)(x + 2) \leq 0$                                                          | A1 (fixed points 4, -2 identified, C.A.O.)                |
|    | $-2 \leq x \leq 4$                                                               | M1 (any method)<br>A1 (F.T. fixed points)                 |
|    | Allow B1 for $x \geq x \geq 4$                                                   | M1 (any method)<br>A1 (F.T. fixed points)                 |
|    | Allow B1 for $x \geq -2$ , or $x \leq 4$<br>MO AO for $x \leq 4$ and $x \leq -2$ |                                                           |
|    |                                                                                  | [7]                                                       |
| 9. | (a) $2x + c = x^2 + 6x + 7$                                                      | B1                                                        |
|    | $x^2 + 4x + 7 - c = 0$                                                           | M1 (arranging quadratic)                                  |
|    | $4^2 - 4(7 - c) = 0$                                                             | M1 (condition for real roots)                             |
|    | $c = 3$                                                                          | A1 (C.A.O.)                                               |
|    | (b) Then $(x + 2)^2 = 0$ so that $x = -2$ (twice)                                | B1 (F.T. derived $c$ )                                    |
|    | Point of contact is $x = -2$ , $y = -4 + 3 = 1$                                  | B1 (F.T. derived $c$ )                                    |
|    | <u>or</u>                                                                        |                                                           |
|    | The line intersects the curve where gradient = 2                                 | M1 (equate to 2)                                          |
|    | $2x + 6 = 2$                                                                     | M1 (attempt to diff.)<br>A1 (correct)                     |
|    | $x = -2$                                                                         | A1 (C.A.O.)                                               |
|    | Point of contact is $x = -2$ , $y = -1$                                          | B1 (F.T. one slip)                                        |
|    | Then $-1 = 4 + c$ , $c = 3$                                                      | B1                                                        |
|    |                                                                                  | [6]                                                       |

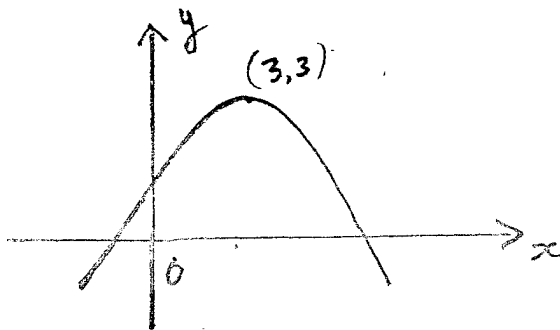
10. (a)



M1 (stationary point coordinates)

A1 (+ve y, -ve x intercepts)

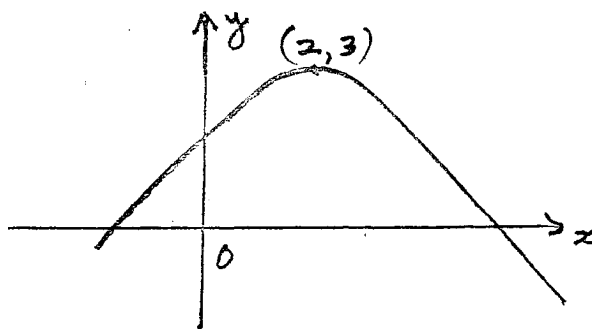
(b)



M1 ( $y = 3, x \neq 1$ )

A1 ( $x = 3$ )

(c)



M1 (stationary point correct)

A1 (+ve y, -ve x intercepts)

[6]

**Mathematics C2**  
**Solutions and Mark Scheme**

1.  $h = 0.2$

$$\text{Integral} \approx \frac{0.2}{2} \left[ 1 + 1.41421356 + 2(1.01980390 + 1.07703296 + 1.16619038 + 1.28062484) \right]$$

M1 (correct formula  $h=0.2$ )  
B1 (4 values)  
B1 (2 values)

$$\approx 1.150$$

A1 (F.T. one slip)

Special Case 7 ordinates (six intervals taken)

$$h = \frac{1}{6}$$

$$\text{Integral} \approx \frac{1}{12} \left[ 1 + 1.41421356 + 2(1.01379376 + 1.05409255 + 1.1803399 + 1.20185043 + 1.30170828) \right]$$

M1 (correct formula  $h=\frac{1}{6}$ )  
B1 (all values)

$$\approx 1.149$$

A1 (F.T. one slip)

[4]

2. (a)  $8(1 - \sin^2 x) + 2 \sin x - 7 = 0$

M1 (use of  $\sin^2 x + \cos^2 x = 1$ ,  
allow  
 $8 \cos^2 x + 2 \sqrt{1 - \cos^2 x} - 7 = 0$ )

$$8 \sin^2 x - 2 \sin x - 1 = 0$$

$$(4 \sin x + 1)(2 \sin x - 1) = 0$$

M1 (attempt to solve quadratic in  
 $\sin x$ , correct formula or  
 $(a \sin x + b)(c \sin x + d)$   
with  $ac = \sin^2 x$  coefft,  
 $bd = \text{constant term}$ )

$$\sin x = -\frac{1}{4}, \frac{1}{2}$$

A1 (C.A.O.)

$$x = 194.5^\circ, 345.5^\circ, 30^\circ, 150^\circ$$

B1 ( $194^\circ - 194.5^\circ$ )  
B1 ( $345.5^\circ - 346^\circ$ )  
B1 ( $30^\circ, 150^\circ$ )

Full F.T for  $\sin x = a, b$  (one +, -)

2 marks for  $\sin x = -, -$

1 mark for  $\sin x = +, +$

Subtract 1 for each additional angle in each branch within range. Ignore additional values outside range.

- (b)  $2x = 45^\circ, 225^\circ$  B1 (one value)  
 $x = 22\frac{1}{2}^\circ, 112\frac{1}{2}^\circ$  B1, B1

Subtract 1 for each additional value in the range. Ignore additional values outside range.

[9]

3. (a)  $n^{\text{th}} \text{ term} = a + (n - 1)d$  (must be displayed)

$$S_n = a + (a + d) + \dots + a - (n - 2)d + a + (n - 1)d$$

B1 (at least 3 terms, one at each end)

$$S_n = a + (n - 1)d + a + (n - 2)d + \dots + (a + d) + a$$

M1 (reverse and add)

$$2S_n = (2a + (n - 1)d) + (2a + (n - 1)d) + \dots + (2a + (n - 1)d) + (2a + (n - 1)d)$$

( $n$  terms)

$$= n[2a + (n - 1)d]$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

A1 (convincing)

- (b)  $a + 6d = 2(a + 2d)$  (1)

M1 ( $a + 6d = k(a + 2d)$ )

( $k = \frac{1}{2}, 2$ )

A1 ( $k = 2$ )

$$\frac{10}{2}[2a + 9d] = 195 \quad (2)$$

B1

Solve (1), (2)  $d = 3$

M1 (reasonable attempt to solve equations)

A1 (C.A.O.)

$$a = 6$$

A1 (F.T. if either of first M1 or B1 gained)

$$S_{60} = \frac{60}{2} [2 \times 6 + 59 \times 3] = 5670$$

B1 (F.T. candidate's values if either first M1 or B1 gained) [11]

4. (a)  $a + ar = 6.4, \frac{a}{1-r} = 10$

B1, B1

Eliminate  $a$   $10(1-r)(1+r) = 6.4$

M1 (reasonable attempt to eliminate  $a$ )

A1 (C.A.O.) (any correct expression in  $r$  or  $a$ )

$$1 - r^2 = 0.64$$

$$r = 0.6$$

A1 (F.T. one slip if one B earned)

(b)  $a = 10 \times (1 - 0.6) = 4$

A1 (F.T. value of  $r$  if one B earned)

$$S_{11} = \frac{4}{0.4} (1 - (0.6)^{11}) = 9.964$$

M1 (use of correct formula with derived  $a, r$ )

A1 (C.A.O.)

[8]

5. (a) Centre  $(4, -2)$ ; radius  $= \sqrt{4^2 + 2^2 + 5}$   
 $= 5$

B1 (centre)  
M1 (correct attempt to find radius)  
A1 (radius)

(b) (i)  $1^2 + 36 - 8 - 24 - 5 = 0$   
(so that  $P$  lies on  $C$ )

B1

(ii) Gradient of radius  $= \frac{-2+6}{4-1} = \frac{4}{3}$

B1

$$\text{Gradient of tangent} = -\frac{3}{4}$$

M1

$$\left( \frac{-1}{\text{candidate's gradient of radius}} \right) \text{o.e.}$$

A1 (correct simplified, F.T. one slip)

$$\text{Equation is } y + 6 = -\frac{3}{4}(x-1)$$

A1 (F.T. one slip)

Alternative for gradient

$$2x + 2y \frac{dy}{dx} - 8 + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{3}{4}$$

M1 (attempt to diff.)

A1 (all correct)

A1(F.T. one slip) [8]

6. (a)  $\left. \begin{array}{l} x = a^m, y = a^n \\ \log_a x = m, \log_a y = n \end{array} \right\}$

$$\frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$$

B1 (properties of  $a^n = x$  and  $\log_a x$ )

B1 (laws of indices)

$$\log_a \left( \frac{x}{y} \right) = m - n = \log_a x - \log_a y$$

B1 (convincing)

(b) (i)  $\log 5^{2x+1} = \log 7$

M1 (take logs, one correct)

$$(2x + 1) \log 5 = \log 7$$

A1 (all correct)

$$2x + 1 = \frac{\log 7}{\log 5}$$

m1 (attempt to start isolating  $x$ )

$$x = \frac{\log 7 - \log 5}{2 \log 5} \approx 0.1045$$

A1 (C.A.O.)

(ii)  $\log_{10} = \frac{2 \times 18^2}{36^{3/2}} = \log_{10} 3$

B1 (use of addition law)

B1 (subtraction law)

B1 (power law)

B1 (C.A.O., simplified answer)

[11]

7. (a)  $\frac{2x^{7/4}}{7/4} + \frac{7x^{1/2}}{1/2} (+ C)$

B1, B1

(b) (i)  $6 - x^2 = 5$

M1 (equating ys)

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

M1 (correct method of solving quadratic equations)

$$\therefore x = 1, 5$$

$$A(1, 5), B(5, 5)$$

A1

$$(ii) \quad \text{area} = \int_1^5 (6x - x^2) dx$$

M1 (use of integration to find area)

$$- \int_1^5 5 dx$$

M1 (substitution of correct areas)

$$= \left[ 3x^2 - \frac{x^3}{3} \right]_1^5 - [5x]_1^5$$

B3 (integration)

$$= 75 - \frac{125}{3} - 3 + \frac{1}{3} - 20$$

M1 (use of candidate's limits, any order)

$$= \frac{32}{3}$$

A1 (C.A.O.)

[12]

$$8. \quad (a) \quad (x-1)^2 = (x-3)^2 + x^2 - 2x(x-3) \cdot \frac{1}{2}$$

M1 (correct cosine rule with  $\cos 60^\circ$ )

A1 ( $\cos 60^\circ = \frac{1}{2}$ )

$$x^2 - 2x + 1 = x^2 - 6x + 9 + x^2 - x^2 + 3x$$

M1 (both binomial expansions correct, all terms present)

$$x = 8$$

A1 (convincing)

Special Case If  $x = 8$  assumed and

either  $\cos 60 = \frac{1}{2}$  derived

or  $\cos 60^\circ = \frac{1}{2}$

also assumed and work shown to be consistent, allow

M1 (correct use of cosine rule)  
A1

$$(b) \quad \text{area of triangle} = \frac{1}{2} \times 5 \times 8 \times \sin 60^\circ = 10\sqrt{3}$$

M1 (use of formula)

A1 (C.A.O.)

[6]

9. (a)  $\frac{1}{2}4^2(\pi - \theta) - \frac{1}{2}.4^2\theta = 5$  (o.e.) B1 (one correctly identified area)
- $8\pi - 8\theta - 8\theta = 5$  B1 (correct equation)
- $\theta = \frac{8\pi - 5}{16}$  B1 (convincing)
- (b) Difference =  $4(\pi - \theta) - 4\theta$  B1 (arc  $AOC = 4\theta$ )
- Diff =  $\frac{5}{2}$  (allow any answer rounding to 2.50) B1 (arc  $BOC = 4(\pi - \theta)$ )
- B1

[6]

**Mathematics C3**  
**Solutions and Mark Scheme**

1.  $h = 0.25$

$$\text{Integral} \approx \frac{0.25}{3} [1 + 1.4142136 + 4(1.0004882 + 1.1123420) + 2 \times 1.0155049]$$

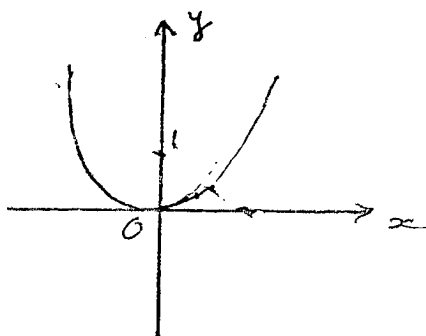
$$\approx 1.075$$

(accept any answers rounding to 1.075)

M1 (formula with  $h = 0.25$ )  
B1 (3 values)  
B1 (2 values)  
A1

[4]

2. (a)



G1 (straight line with +ve intercept, -ve slope)  
G2 (correct shape and position of curve)  
(Two intersections), two roots E1

(b)

|     |                |
|-----|----------------|
| $x$ | $x^4 + 3x - 1$ |
| 0   | -1             |
| 1   | 3              |

M1 (attempt to find signs or values)  
A1 (correct signs or values, correct conclusion)

Change of sign indicates presence of root

$$x_0 = 0.3, \quad x_1 = 0.330633 \text{ (accept 0.331)}$$

B1 ( $x_1$ )

$$x_2 = 0.329350, \quad x_3 = 0.32941$$

$$x_4 = 0.32941$$

(accept any answer rounding to 0.32941)

B1 ( $x_4$ )

Try 0.329405, 0.329415

|          |                |
|----------|----------------|
| $x$      | $x^4 + 3x - 1$ |
| 0.329405 | -0.00001       |
| 0.329415 | 0.00002        |

M1 (attempt to find signs or values)  
A1 (correct signs or values)

(Change of signs indicates presence of root which is 0.32941, correct to five decimal places)

A1 (conclusion)

[10]

3. (a)  $\theta = \frac{\pi}{4}$ , for example (45°) B1
- $\cot^2\theta = 1$
- $1 + \operatorname{cosec}^2\theta = 1 + 2 = 3$  B1
- $(\therefore \cot^2\theta \neq \operatorname{cosec}^2\theta + 1)$
- (b)  $10(\tan^2\theta + 1) = 11\tan\theta + 16$  M1 ( $\sec^2\theta = 1 + \tan^2\theta$ )
- $10\tan^2\theta - 11\tan\theta - 6 = 0$  M1 (grouping terms and attempt to solve quadratic  
( $a\tan\theta + b$ )( $c\tan\theta + d$ )  
with  $ac = \text{coefficient of } \tan^2\theta$   
 $bd = \text{constant term, or correct formula}$ )
- $(2\tan\theta - 3)(5\tan\theta + 2) = 0$
- $\tan\theta = \frac{3}{2}, \tan\theta = -\frac{2}{5}$  A1
- $\theta = 56.3^\circ, 236.3^\circ, 158.2^\circ, 338.2^\circ$  B1 ( $56.3^\circ, 236.3^\circ$ )
- $(56 - 56.5) (236 - 236.5) (158 - 158.5) (338 - 338.5)$  B1 ( $158.2^\circ$ )
- B1 ( $338.2^\circ$ )

Subtract 1 for each additional value in each branch

[8]

4. (a)  $2x + 2x\frac{dy}{dx} + 2y + 6y\frac{dy}{dx} = 0$  B1 ( $2x\frac{dy}{dx} + 2y$ )
- B1 ( $6y\frac{dy}{dx}$ )
- $\frac{dy}{dx} = -\frac{x+y}{x+3y}$  (o.e) B1 (all correct)
- (b)  $\frac{dy}{dx} = \frac{6t}{8t^3} = \frac{3}{4t^2}$  M1  $\left(\frac{\dot{y}}{\dot{x}}\right)$  A1 (correct)
- $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \bigg/ \frac{dx}{dt} = \frac{-\cancel{3}/2t^3}{8t^3}$  M1 (correct formula)
- $= -\frac{3}{16t^6}$  A1 (F.T one slip in  $\frac{dy}{dx}$ )

[7]

Alternative scheme for Q. 4(b)

(b) (i)  $y^2 = \frac{9}{2}x$

$$2y \frac{dy}{dx} = \frac{9}{2}$$

$$\frac{dy}{dx} = \frac{9}{4y} = \frac{9}{12t^2}$$

$$= \frac{3}{4t^2}$$

M1 (correct cartesian form and attempt to differentiate)

(or differentiate  $y = \sqrt{\frac{9}{2}x^{\frac{1}{2}}}$ )

A1 (unsimplified form)

(ii)  $\frac{dy}{dx} = \frac{9}{4y} = \frac{9}{4\sqrt{\frac{9}{2}x^{\frac{1}{2}}}} = \frac{1}{2}\sqrt{\frac{9}{2}}x^{-\frac{1}{2}}$

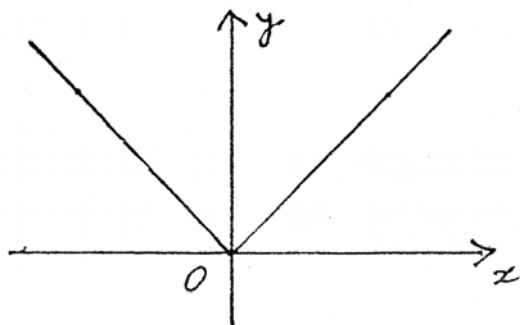
M1(correct cartesian and attempt to diff.)

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{\frac{9}{2}}x^{-\frac{3}{2}} = -\frac{1}{4}\sqrt{\frac{9}{2}}\frac{1}{2^{\frac{3}{2}}t^6}$$

A1 (unsimplified F.T. form; one slip in  $\frac{dy}{dx}$ )

$$= -\frac{3}{16t^6}$$

5. (a)



B1 (left hand side)

B1 (right hand side, award only if whole graph above x-axis)

(b)  $x = \pm \frac{1}{2}$

B1 (both)

(c)  $x > \frac{1}{3}, x < -3$

B1 ( $x > \frac{1}{3}$ ) M1 ( $3x + 4 < -5$ )

A1

[6]

|    |     |       |                                                                            |                                                              |
|----|-----|-------|----------------------------------------------------------------------------|--------------------------------------------------------------|
| 6. | (a) | (i)   | $2e^{2x-5}$                                                                | M1 ( $ke^{2x-5}$ ) A1 ( $k = 2$ )                            |
|    |     | (ii)  | $x^2 \cdot \frac{1}{x} + 2x \ln x$                                         | M1 ( $x^2 f(x) + \ln x g(x)$ )                               |
|    |     |       | $= x + 2x \ln x$                                                           | A1 ( $f(x) = \frac{1}{x}, g(x) = 2x$ )                       |
|    |     | (iii) | $4(3x^2 + 2)^3 \cdot 6x$                                                   | A1                                                           |
|    |     |       | $= 24x(3x^2 + 2)^3$                                                        | M1 ( $4(3x^2 + 2)^3 f(x), f(x) \neq 1$ )                     |
|    |     |       |                                                                            | A1 ( $f(x) = 6x$ )                                           |
|    |     |       |                                                                            | A1                                                           |
|    | (b) |       | $\frac{d}{dx}(\tan x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$ | M1 $\left( \frac{\cos f(x) - \sin x g(x)}{\cos^2 x} \right)$ |
|    |     |       | $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$                                   | A1 ( $f(x) = \cos x, g(x) = -\sin x$ )                       |
|    |     |       | $= \frac{1}{\cos^2 x} = \sec^2 x$                                          | A1 (convincing)                                              |
|    | (c) |       | $x = \tan y, \frac{dx}{dy} = \sec^2 y$                                     | B1                                                           |
|    |     |       | $\frac{dy}{dx} = \frac{1}{\sec^2 y}$                                       | B1                                                           |
|    |     |       | $= \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$                             | B1 (convincing, must see<br>$1 + \tan^2 y$ for $\sec^2 y$ )  |

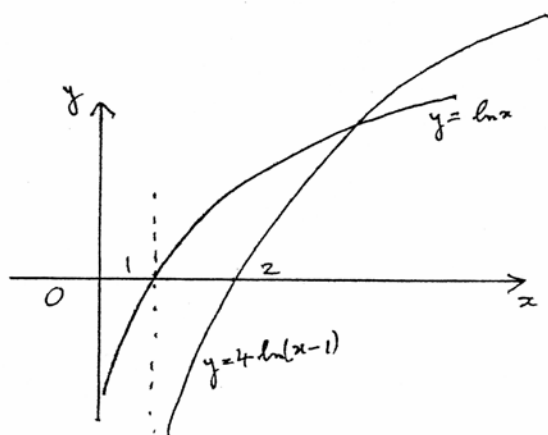
[14]

|                    |                                                |                                            |
|--------------------|------------------------------------------------|--------------------------------------------|
| <u>Alternative</u> | $1 = \sec^2 y \frac{dy}{dx}$                   | B1                                         |
|                    | $\frac{dy}{dx} = \frac{1}{\sec^2 y}$           | B1 ( $\frac{dy}{dx}$ , subject of formula) |
|                    | $= \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$ | B1 (convincing)                            |

7. (a) (i)  $\frac{1}{3} \ln|3x+7|$  (+ C) M1 ( $k \ln|3x+7|$ , | | may be omitted)  
A1 ( $k = \frac{1}{3}$ )
- (ii)  $\frac{1}{3} e^{3x+2}$  (+ C) M1 ( $k e^{3x+2}$ ) A1 ( $k = \frac{1}{3}$ )
- (iii)  $-\frac{1}{5(5x+2)^3}$  (+ C) M1  $\left(\frac{k}{(5x+2)^3}, (o.e) k \neq 3\right)$  A1  
 $\left(k = -\frac{1}{5}\right)$
- (b)  $\left[-\frac{1}{4} \cos\left(4x + \frac{\pi}{6}\right)\right]_0^{\frac{\pi}{6}}$  M1  
 $\left(k \cos\left(4x + \frac{\pi}{6}\right), k = -1, -4, \frac{1}{4}, -\frac{1}{4}\right)$   
A1 ( $k = -\frac{1}{4}$ )
- $= \left(-\frac{1}{4} \cos \frac{5\pi}{6} + \frac{1}{4} \cos \frac{\pi}{6}\right)$  M1  
 $\left(k \cos \frac{5}{6} - k \cos \frac{\pi}{6}, \text{allowable } k\right)$
- $= \frac{1}{4} \frac{\sqrt{3}}{2} + \frac{1}{4} \frac{\sqrt{3}}{2}$
- $= \frac{\sqrt{3}}{4}$  ( $\approx 0.433$ , correct to 3 decimal places) A1 (C.A.O)

[10]

8.



$\frac{\ln x}{x}$

B1 (y asymptote)

B1 (1,0)

$\frac{4 \ln(x-1)}{x-1}$

B1 (translation to right, necessary to see (2,0))

B1 (intersection of asymptote at  $x=1$ )

B1 (intersection in first quadrant)

[5]

9. Let  $y = \ln(x - 2) + 3$
- $y - 3 = \ln(x - 2)$  B1 ( $y - 3 = \ln(x - 2)$ , o.e.)
- $e^{y-3} = x - 2$  M1 (attempt to exponentiate and isolate  $x$ )
- $x = e^{y-3} + 2$  A1
- $f^{-1}(x) = e^{x-3} + 2$  A1 (F.T. one slip)
- [4]

10. (a) Range of  $f, g$   $(1, \infty), (7, \infty)$  respectively (o.e.) B1, B1 (penalise equality once)
- (b)  $f(1) = 2$  and 2 is not in the domain of  $g$ . B1
- (c)  $fg(x) = (2x - 3)^2 + 1$  M1 (correct composition)
- $(2x - 3)^2 + 1 = 3x^2 - 6x + 17$
- $x^2 - 6x - 7 = 0$  A1
- $x = -1, 7$  A1 (F.T. one slip)
- $x = 7$  ( $-1$  not in domain of  $g$ ) A1 (F.T. one slip)
- [7]

Special Case  $gf(x) = 3x^2 - 6x + 17$

gives  $x^2 - 6x + 18 = 0$

and attempt to solve

B1

# Mathematics C4

## Solutions and Mark Scheme

1. (a) Let  $\frac{8x^2 + x - 5}{(2x-1)^2(x+2)} \equiv \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{(x+2)}$  M1
- $\therefore 8x^2 + x - 5 \equiv A(2x-1)(x+2) + B(x+2) + C(2x-1)^2$  M1
- $x = \frac{1}{2} \quad B = -1, \quad x = -2, \quad C = 1$  A1
- Equate coefficients of  $x^2$ :  $2A + 4C = 8$   
 $A = 2$  A1
- (b)  $\int \frac{2}{2x-1} dx - \int \frac{1}{(2x-1)^2} dx + \int \frac{1}{x+2} dx$
- $= \ln|2x-1| + \frac{1}{2(2x-1)} + \ln|x+2| \quad (+ C)$  B1, B1, B1
- (no need for modulus) [7]
2.  $(1-2x)^{\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-2x) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2!}(-2x)^2 + \dots$
- $= 1 + x + \frac{3}{2}x^2 + \dots$  B1 (1 + x) B1 ( $\frac{3}{2}x^2$ )
- Expansion valid for  $|x| < \frac{1}{2}$  B1
- $\left(1 - \frac{1}{4}\right)^{\frac{1}{2}} \approx 1 + \frac{1}{8} + \frac{3}{128} = \frac{147}{128}$  B1 (F.T one slip)
- $\left(\frac{3}{4}\right)^{\frac{1}{2}} = \frac{128}{147}$
- $\therefore \sqrt{3} \approx \frac{256}{147}$  B1 (C.A.O)
- [5]

3.  $8x + 3y + 3x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$

B1 ( $2y \frac{dy}{dx}$ )

B1 ( $3y + 3x \frac{dy}{dx}$ )

$$\frac{dy}{dx} = \frac{8x + 3y}{2y - 3x}$$

$$= -\frac{19}{4}$$

B1 (C.A.O.)

Equation is  $y - 1 = -\frac{19}{4}(x - 2)$

B1 (F.T one slip)

[4]

4. (a)  $2\sin\theta \cos\theta = \cos\theta$

M1

$$\cos\theta = 0, \sin\theta = \frac{1}{2}$$

$$\theta = 90^\circ, 270^\circ, 30^\circ, 150^\circ$$

A3 (−1 for each omission)  
(−1 for each additional value)

(b) Using  $R \sin(\theta + \alpha)$  with  $R = \sqrt{17}$ ,  $\alpha = 14.04^\circ$

M1 A1 A1

$$\sin(\theta + 14.04^\circ) = \frac{2}{\sqrt{17}}$$

$$\theta + 14.04^\circ = 29.02^\circ, 150.98^\circ$$

B1 (1 value)

$$\theta = 14.98^\circ, 136.94^\circ$$

B1, B1

[10]

5. Volume =  $\pi \int_1^4 \left( \sqrt{x} + \frac{4}{\sqrt{x}} \right)^2 dx$

B1

$$= \pi \int_1^4 \left( x + 8 + \frac{16}{x} \right) dx$$

B1

$$= \pi \left[ \frac{x^2}{2} + 8x + 16 \ln x \right]_1^4$$

B1

$$= \pi \left( 31\frac{1}{2} + 16 \ln 4 \right)$$

M1

$$\approx 168.6$$

A1 (C.A.O.)

[5]

6. (a)  $\frac{dy}{dx} = \frac{2p}{2} = p$  M1 ( $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$ , one correct)  
A1
- Equation is
- $y - (p^2 + 3) = p(x - 2p - 1)$  M1
- $px - y = p^2 + p - 3$  A1 (convincing)
- (b)  $2p + 3 = p^2 + p - 3$  M1
- $p^2 - p - 6 = 0$  A1
- $p = 3, -2$  A1 (FT one slip)
- Choose  $p = -2$  (2<sup>nd</sup> quadrant)
- Tangent is  $2x + y = 1$  A1 (FT candidate's values)
- [8]
7. (a)  $u = 2x - 1, du = 2 dx$
- when  $x = 0, u = -1; x = 1, u = 1$
- $\int_{-1}^1 \frac{(u+1)}{2} u^9 \frac{du}{2}$  M1 A1 (no limits required)
- $= \frac{1}{4} \int_{-1}^1 (u^{10} + u^9) du$  m1
- $= \frac{1}{4} \left[ \frac{u^{11}}{11} + \frac{u^{10}}{10} \right]_{-1}^1$  A1
- $= \frac{1}{22}$  A1 (F.T. one slip)
- (b) (i)  $\int x \cos 2x dx = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx$  M1 A1 A1
- $= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} (+C)$  A1 (F.T. one slip)
- (ii)  $\int x \cos^2 x dx = \int x \frac{(1 + \cos 2x)}{2} dx$  M1 A1
- $= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} (+C)$  A1 (F.T. first result) [12]

|    |     |                                                                                                             |                                           |
|----|-----|-------------------------------------------------------------------------------------------------------------|-------------------------------------------|
| 8. | (a) | $\frac{dP}{dt} = kP$                                                                                        | B1                                        |
|    | (b) | $\int \frac{dP}{P} = \int k dt$                                                                             | M1                                        |
|    |     | $\ln P = kt (+C)$                                                                                           | A1                                        |
|    |     | $t = 0, P = P_0 \quad \ln P_0 = C$                                                                          | M1                                        |
|    |     | $\ln P = kt + \ln P_0$                                                                                      |                                           |
|    |     | $\ln\left(\frac{P}{P_0}\right) = kt$                                                                        | A1                                        |
|    |     | $\frac{P}{P_0} = e^{kt}$                                                                                    |                                           |
|    |     | $P = P_0 e^{kt}$                                                                                            | A1 (convincing)                           |
|    | (c) | $1.2 P_0 = P_0 e^{2k}$                                                                                      | M1 (attempt to find k)                    |
|    |     | $2k = \ln 1.2$                                                                                              |                                           |
|    |     | $k = \frac{1}{2} \ln 1.2$                                                                                   | A1                                        |
|    |     | $T = \frac{\ln 2}{\frac{1}{2} \ln 1.2} \approx 7.6$                                                         | m1 A1 (F.T. one slip)                     |
|    |     |                                                                                                             | [10]                                      |
| 9. | (a) | (i) $\mathbf{OP} = \mathbf{OA} + \lambda \mathbf{AB}$                                                       | M1                                        |
|    |     | $= 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda (-12\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$            | B1 ( <b>AB</b> )                          |
|    |     | $\mathbf{r} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda (-12\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$ | A1 (must have <b>r</b> , F.T. <b>AB</b> ) |
|    |     | (ii) Point of intersection:                                                                                 |                                           |
|    |     | $5 - 12\lambda = -1 + 2\mu$                                                                                 | M1                                        |
|    |     | $1 + 3\lambda = 7 - 5\mu$                                                                                   | A1                                        |
|    |     | $\lambda = \frac{1}{3}, \mu = 1$                                                                            | M1 A1 (CAO)                               |
|    |     | Position vector = $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$                                                   | B1 (candidate's parameters)               |

(b)  $(3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \cdot (\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$

M1

$$= 3 - 8 + 5 = 0$$

$\therefore$  Vectors are perpendicular

A1 (value and conclusion)  
[10]

10.  $x^2 - 10x + 25 < 0$

B1

$$(x - 5)^2 < 0$$

B1

$x - 5$  not real

B1

( $x$  not real)

Contradiction

B1

so  $x + \frac{25}{x} \geq 10$

[4]

**A/AS Level Mathematics - FP1 – June 2005 - Markscheme**

|   |                                               |      |
|---|-----------------------------------------------|------|
| 1 | $ x + 1 + iy  = 2 x + i(y - 2) $              | M1   |
|   | $(x + 1)^2 + y^2 = 4[x^2 + (y - 2)^2]$        | M1A1 |
|   | $x^2 + 2x + 1 + y^2 = 4x^2 + 4y^2 - 16y + 16$ | A1   |
|   | $3x^2 + 3y^2 - 2x - 16y + 15 = 0$             | A1   |

|   |                                               |      |
|---|-----------------------------------------------|------|
| 2 | $S_n = 4 \sum_{r=1}^n r^3 - 4 \sum_{r=1}^n r$ | M1   |
|   | $= \frac{4n^2(n+1)^2}{4} - \frac{4n(n+1)}{2}$ | A1A1 |
|   | $= n(n+1)(n^2 + n - 2)$                       | M1A1 |
|   | $= (n-1)n(n+1)(n+2)$                          | A1   |

|   |                                                                                   |      |
|---|-----------------------------------------------------------------------------------|------|
| 3 | $f(x+h) - f(x) = \frac{1}{(x+h)^2 + x + h} - \frac{1}{x^2 + x}$                   | M1A1 |
|   | $= \frac{x^2 + x - [(x+h)^2 + x + h]}{[(x+h)^2 + x + h](x^2 + x)}$                | m1   |
|   | $= \frac{-h(2x + h + 1)}{[(x+h)^2 + x + h](x^2 + x)}$                             | A1   |
|   | $f'(x) = \lim_{h \rightarrow 0} \frac{-(2x + h + 1)}{[(x+h)^2 + x + h](x^2 + x)}$ | M1   |
|   | $= -\frac{2x + 1}{(x^2 + x)^2}$                                                   | A1   |

|   |                                                                                                                                                                                                                      |    |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 4 | <p>(a) Matrix of translation = <math>\begin{bmatrix} 1 &amp; 0 &amp; 1 \\ 0 &amp; 1 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p>                                                                            | B1 |
|   | <p>Matrix of reflection = <math>\begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p>                                                                                 | B1 |
|   | <p>T - Matrix = <math>\begin{bmatrix} 1 &amp; 0 &amp; 1 \\ 0 &amp; 1 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> | M1 |
|   | <p>= <math>\begin{bmatrix} 0 &amp; 1 &amp; 1 \\ 1 &amp; 0 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p>                                                                                                      | A1 |

(b) Fixed points satisfy

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{M1}$$

giving  $y + 1 = x$  and  $x + 2 = y$  A1

These equations are inconsistent so no fixed points. A1

5 The proposition is true by inspection for  $n = 1$ . B1

Assume the proposition is true for  $n = k$ . M1

Consider

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2^k - 1 \\ 0 & 2^k \end{bmatrix} \quad \text{m1}$$

$$= \begin{bmatrix} 1 & 2^k - 1 + 2^k \\ 0 & 2 \cdot 2^k \end{bmatrix} \quad \text{A1}$$

$$= \begin{bmatrix} 1 & 2^{k+1} - 1 \\ 0 & 2^{k+1} \end{bmatrix} \quad \text{A1}$$

So, if the proposition is true for  $n = k$ , it is also true for  $n = k + 1$ . A1

Extra mark for goods presentation A1

6 METHOD 1

If  $1 + i$  is a root, so is  $1 - i$ . B1

So,  $x^2 - 2x + 2$  is a factor of the cubic. B1

Using long division,

$$\begin{array}{r} x + 4 \\ x^2 - 2x + 2 \ ) \ x^3 + 2x^2 + \lambda x + \mu \\ \underline{x^3 - 2x^2 + 2x} \phantom{+ \mu} \\ 4x^2 + (\lambda - 2)x + \mu \\ \underline{4x^2 - 8x + 8} \phantom{+ \mu} \end{array} \quad \begin{array}{l} \text{M1} \\ \\ \text{A1} \\ \text{A1} \end{array}$$

It follows that  $\lambda - 2 = -8$  so  $\lambda = -6$  M1A1

And  $\mu = 8$ . A1

Since the sum of the roots is  $-2$ , the other root is  $-4$ . M1A1

METHOD 2

Substituting  $1 + i$ , M1

$$(1 + i)^3 + 2(1 + i)^2 + \lambda(1 + i) + \mu = 0 \quad \text{A1}$$

$$(1 + 3i + 3i^2 + i^3) + 2(1 + 2i + i^2) + \lambda(1 + i) + \mu = 0 \quad \text{M1A1}$$

$$-2 + \lambda + \mu = 0 \text{ and } \lambda + 6 = 0 \quad \text{A1}$$

$$\lambda = -6 \text{ and } \mu = 8 \quad \text{M1A1}$$

Since  $1 + i$  is a root, so is  $1 - i$ . B1

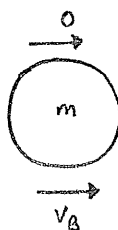
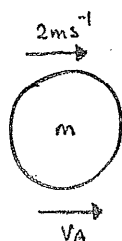
Since the sum of the roots is  $-2$ , the other root is  $-4$ . M1A1

|    |        |                                                                                                                                                                        |            |
|----|--------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|
| 5  | 7      | (a) Consider $z = \frac{1}{w}$ giving $x + iy = \frac{1}{u + iv}$                                                                                                      |            |
| M1 |        | $= \frac{u - iv}{u^2 + v^2}$                                                                                                                                           | M1A1       |
|    |        | Taking real and imaginary parts,                                                                                                                                       | M1         |
|    |        | $x = \frac{u}{u^2 + v^2} \quad \text{and} \quad y = \frac{-v}{u^2 + v^2}$                                                                                              | A1         |
|    |        | (b) Substituting,                                                                                                                                                      |            |
|    |        | $\frac{u^2}{(u^2 + v^2)^2} + \frac{v^2}{(u^2 + v^2)^2} = 2$                                                                                                            | M1A1       |
|    |        | giving $u^2 + v^2 = 1/2$                                                                                                                                               | A1         |
| 6  | 8      | We note that $\alpha + \beta + \gamma = 2$<br>$\beta\gamma + \gamma\alpha + \alpha\beta = 3$<br>and $\alpha\beta\gamma = -3$ .                                         | B1         |
|    |        | Consider $\frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$                                  | M1A1       |
|    |        | $= -2/3$                                                                                                                                                               | A1         |
|    |        | Consider $\frac{1}{\gamma\alpha} \cdot \frac{1}{\alpha\beta} + \frac{1}{\alpha\beta} \cdot \frac{1}{\beta\gamma} + \frac{1}{\beta\gamma} \cdot \frac{1}{\gamma\alpha}$ | M1         |
|    |        | $= \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha^2\beta^2\gamma^2}$                                                                                           | A1         |
|    |        | $= 1/3$                                                                                                                                                                | A1         |
|    |        | Consider $\frac{1}{\beta\gamma} \cdot \frac{1}{\gamma\alpha} \cdot \frac{1}{\alpha\beta} = 1/9$                                                                        | M1A1       |
|    |        | The required equation is                                                                                                                                               |            |
|    |        | $x^3 + \frac{2}{3}x^2 + \frac{1}{3}x - \frac{1}{9} = 0$                                                                                                                | M1A1       |
| 9  | (a)    | Det = $1(\lambda - 15) - 2(\lambda - 9) + (5 - 3) = -\lambda + 5$<br>Matrix is singular when $\lambda = 5$ .                                                           | M1A1<br>A1 |
|    | (b)(i) | Using reduction to echelon form,                                                                                                                                       |            |
|    |        | $\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$            | M1A1       |
|    |        | Put $z = \alpha$ .                                                                                                                                                     | M1         |
|    |        | $y = 2\alpha - 1$                                                                                                                                                      | A1         |
|    |        | $x = 3 - 5\alpha$                                                                                                                                                      | A1         |

|      |                                                                                                                                                                            |      |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| (ii) | $\text{Det}(\mathbf{A}) = 2$                                                                                                                                               | B1   |
|      | Cofactor matrix = $\begin{bmatrix} -12 & 6 & 2 \\ -1 & 0 & 1 \\ 5 & -2 & -1 \end{bmatrix}$                                                                                 | M1A1 |
|      | Inverse matrix = $\frac{1}{2} \begin{bmatrix} -12 & -1 & 5 \\ 6 & 0 & -2 \\ 2 & 1 & -1 \end{bmatrix}$                                                                      | M1A1 |
|      | $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -12 & -1 & 5 \\ 6 & 0 & -2 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ | M1   |
|      | $= \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$                                                                                                                             | A1   |

Marks Scheme

1.



$$e = 0.6$$

Conservation of momentum

M1

$$2m + 0.m = m v_A + m v_B$$

A1

$$v_A + v_B = 2$$

Restitution

M1

$$v_B - v_A = -0.6(0 - 2)$$

A1

$$-v_A + v_B = 1.2$$

Subtract

$$2v_A = 0.8$$

<sup>M1</sup>  
(dep on both M's)

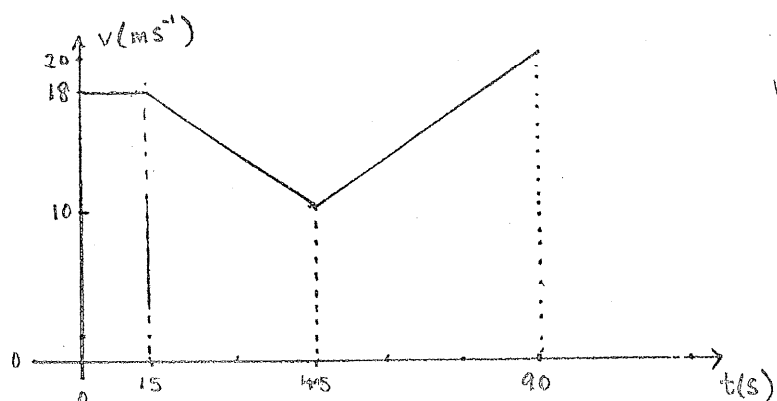
$$\text{Speed of first ball} = v_A = \underline{\underline{0.4 \text{ ms}^{-1}}}$$

$$\text{Speed of 2nd ball} = v_B = \underline{\underline{1.6 \text{ ms}^{-1}}}$$

both A1

2.

(a)



v-t graph

M1

A3

(b)

$$a = \frac{20 - 10}{45}$$

M1

$$= \frac{2}{9} \text{ ms}^{-2}$$

A1

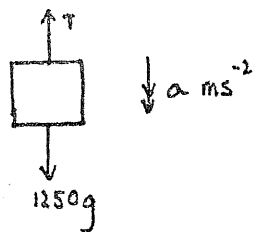
$$\begin{aligned} \text{(c) Distance AB} &= (18 \times 15) + \frac{1}{2} \times 30 (18 + 10) + \frac{1}{2} \times 45 (10 + 20) \\ &= 270 + 420 + 675 \\ &= \underline{1365 \text{ m.}} \end{aligned}$$

M1 A1

B1

600 A1

3.



(a) N2L

$$mg - T = ma$$

M1

$$1250 \times 9.8 - 11625 = 1250a$$

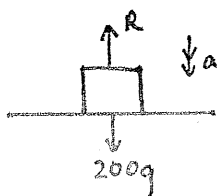
A1

$$a = \underline{0.5 \text{ ms}^{-2}}$$

cao

A1

(b)



$$200g - R = 200a$$

B1

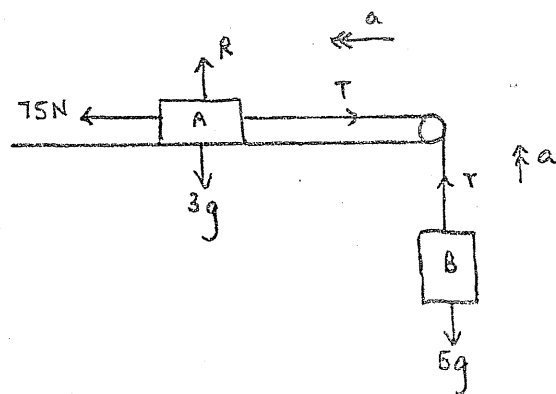
$$R = 200(9.8 - 0.5)$$

$$= \underline{1860 \text{ N}}$$

ft c/s a

B1

H.



|     |       |               |    |    |
|-----|-------|---------------|----|----|
| (a) | For A | $15 - T = 3a$ | MI | AI |
|     | For B | $T - 5g = 5a$ | MI | AI |

|     |                |                               |    |
|-----|----------------|-------------------------------|----|
| (b) | $15 - T = 3a$  | attempt to solve<br>same T, a | MI |
|     | $-5g + T = 5a$ |                               |    |

$$8a = 15 - 5 \times 9.8$$

$$a = \underline{3.25 \text{ ms}^{-2}} \quad \text{AI}$$

$$T = 5 \times 9.8 + 5 \times 3.25$$

$$= 65.25 \text{ N} \quad \text{AI}$$

5.

(a) using  $v^2 = u^2 + 2as$  with  $v = 10$ ,  $a = (\pm)9.8$ ,  $s = (\pm) 0.4$  MI

$$10^2 = u^2 + 2 \times 9.8 \times 0.4$$
 AI

$$u = \underline{9.6 \text{ ms}^{-1}}$$
 CAO AI

(b)  $10 \times e = 3.5$

$$e = \underline{0.35}$$
 BI

(c) 'Impulse  $|I| = 0.7 (10 + 3.5)$  MI

$$= \underline{9.45 \text{ Ns}}$$
 CAO AI

Direction of impulse is upwards BI

(d) using  $v = u + at$  with  $u = 3.5$ ,  $a = (\pm)9.8$ ,  $t = 0.5$  MI

$$v = 3.5 - 9.8 \times 0.5$$
 AI

$$= -1.4$$

Speed is  $1.4 \text{ ms}^{-1}$ , moving downwards AI

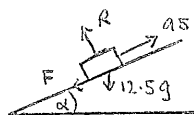
(e) using  $s = ut + \frac{1}{2}at^2$  with  $s = 0$ ,  $u = 3.5$ ,  $a = (\pm)9.8$  MI

$$0 = 3.5t - 4.9t^2$$
 AI

$$t = \underline{\frac{5}{7} \text{ s}}$$
 AI

$$(= 0.7143)$$

6.



$$\alpha = 20^\circ$$

(a)

$$R = 12.5g \cos \alpha$$

M1 A1

$$F = 0.4 \times 12.5 \times 9.8 \cos 20^\circ$$

B1

$$= \underline{46.0(45) \text{ N}}$$

ft R

B1✓

(b) N2L

draw correct, all forces

M1

$$95 - F - 12.5g \sin \alpha = 12.5a$$

A2 (-1 each error)

$$a = \underline{0.56(4.6) \text{ m s}^{-2}}$$

cao A1

7. Resolve to the right

$$x = 56 - 60 \sin \alpha$$
$$(\quad = 20 \text{ N})$$

MI AI

Resolve up the page

$$y = 78 + 60 \cos \alpha - 27$$
$$(\quad = 99 \text{ N})$$

MI AI

$$\text{Magnitude} = \sqrt{20^2 + 99^2}$$
$$= \underline{101 \text{ N}}$$

MI

AI ✓

ft x, y

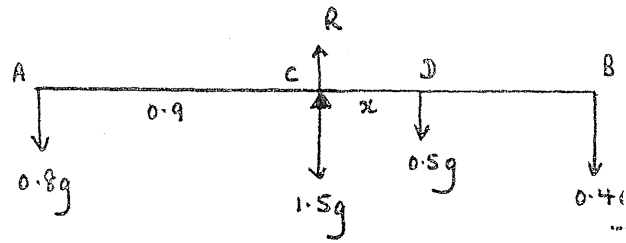
$$\text{required angle} = \tan^{-1} \left( \frac{y}{x} \right)$$
$$= \tan^{-1} \left( \frac{99}{20} \right)$$
$$= \underline{78.58^\circ}$$

MI

ft x, y

AI ✓

8.



(a)

$$\begin{aligned} R &= 0.8g + 1.5g + 0.5g + 0.4g \\ &= \underline{3.2g \text{ N}} \\ &= (31.36 \text{ N}) \end{aligned}$$

M1

(b)

Moments about C

dim. correct, all forces eq<sup>n</sup>

M1

correct moment

B1

$$0.8(g) \times 0.9 = 0.5(g) x + 0.4(g) \times 0.9$$

A1

$$x = \underline{0.72 \text{ m}}$$

cao

A1

9(a)

|        | Area          | Dist. of c. of m. from AB | Dist. of c. of m. from AD |          |
|--------|---------------|---------------------------|---------------------------|----------|
| ABCD   | $8 \times 12$ | 4                         | 6                         | BI       |
| Circle | $\pi(2)^2$    | 6                         | 3                         | BI       |
| Lamina | $96 - 4\pi$   | $x$                       | $y$                       | areas BI |

Moments about AB

$$\begin{aligned}
 (96 - 4\pi)x + (4\pi)6 &= 96 \times 4 && \text{ft c. of m.} && \text{AI} \\
 &= 3.69877 && && \text{AI} \\
 &= \underline{3.7 \text{ cm}} && \text{cao} && \text{AI}
 \end{aligned}$$

Moments about AD

$$\begin{aligned}
 (96 - 4\pi)y + (4\pi)3 &= 96 \times 6 && \text{ft c. of m.} && \text{AI} \\
 &= 6.45185 && && \text{AI} \\
 &= \underline{6.5 \text{ cm}} && \text{cao} && \text{AI}
 \end{aligned}$$

(b)

$$\begin{aligned}
 \tan \theta &= \frac{8-x}{y} && \text{MI} && \text{m} \\
 &= 0.66
 \end{aligned}$$

$$\theta = \underline{33.69^\circ} \quad \text{ft } x, y \quad \text{AI}$$

(c)

$$BP = \underline{3.7 \text{ cm}} \quad \text{ft } x \quad \text{BI}$$

New Syllabus

Mathematics M2 (June 2005)

Marks Scheme

FINAL

1(a).

$$T = 5g$$

BI

$$T = \frac{\lambda (1.3 - 0.8)}{0.8}$$

MI

$$5 \times 9.8 = \frac{\lambda \times 0.5}{0.8}$$

$$\lambda = \underline{78.4 \text{ N}}$$

AI

(b)

$$\text{Elastic Energy} = \frac{1}{2} \times \frac{78.4 \times 0.5^2}{0.8}$$

MI

$$= \underline{12.25 \text{ J}}$$

ft λ

AI/1

$$2(a) \quad v = \int a \, dt \quad \text{M1}$$

$$= \int 4 - 6t \, dt$$

$$v = 4t - 3t^2 + c \quad \text{A1}$$

$$t=0, v=4, \quad c = 4 \quad \text{A1}$$

$$\underline{v = 4t - 3t^2 + 4}$$

$$(b) \quad s = \int v \, dt \quad \text{M1}$$

$$= \int 4t - 3t^2 + 4 \, dt$$

$$= 2t^2 - t^3 + 4t + c \quad \text{A1}$$

$$t=0, s=0, \quad c=0 \quad \text{A1}$$

$$\underline{s = 2t^2 - t^3 + 4t}$$

$$(c) \quad \text{Particle at rest} \Rightarrow v = 0 \quad \text{M1}$$

$$3t^2 - 4t - 4 = 0$$

$$(3t + 2)(t - 2) = 0$$

$$t = 2 \quad \text{A1}$$

$$\text{At } t=2, \quad s = 2(2)^2 - 2^3 + 4(2)$$

$$= \underline{8} \quad \text{A1}$$

$$(d) \quad \text{When } t=3, \quad v = 4(3) - 3(3)^2 + 4 = -11$$

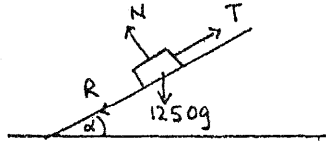
$$\text{Speed} = 11 \quad \text{B1}$$

$$a = 4 - 6(3) \quad \text{B1}$$

$$= -14$$

$a$  is negative, so velocity is decreasing, ie speed increasing B1

3.



(a)

$$T = \frac{P}{v}$$

med M1

$$= \frac{30 \times 1000}{7.5}$$

A1

$$= 4000 \text{ N}$$

N2L with  $a = 0$

dim. correct, all forces

M1

$$T - R = 1250g \sin \alpha$$

A1 A1

$$4000 - 1550 = 1250 \times 9.8 \sin \alpha$$

$$\sin \alpha = 0.2$$

$$\alpha \approx 11.5^\circ$$

A1

$$4. \quad \text{K.E at A} = \frac{1}{2} \times 240 \times 2^2 \quad \text{MI AI}$$

$$= 480 \text{ J}$$

$$\text{P.E.} = mgh \quad \text{used MI}$$

$$\text{change in P.E} = 240 \times 9.8 (30 - 22) = (18816 \text{ J}) \quad \text{AI}$$

$$\text{W.D against resistance} = 132 \times 88 \quad \text{MI AI}$$

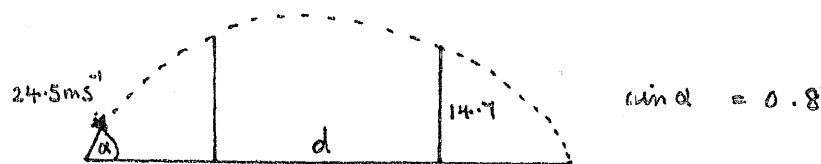
$$= 11616 \text{ J}$$

work-energy principle

$$480 + 18816 - 11616 = \frac{1}{2} \times 240 v^2 \quad \text{MI}$$

$$v = \underline{8 \text{ m.s}^{-1}} \quad \text{AI}$$

5.



$$(a)(i) \quad u_v = 24.5 \sin \alpha$$

$$= 19.6$$

BI

using  $s = ut + \frac{1}{2}at^2$  with  $s = 14.7$ ,  $u = 19.6(c)$ ,  $a = -9.8$  MI

$$14.7 = 19.6t - 4.9t^2$$

AI

$$t^2 - 4t + 3 = 0$$

MI

$$(t-1)(t-3) = 0$$

$$t = 1, 3$$

AI

$$(ii) \quad u_H = 24.5 \cos \alpha$$

BI

$$= 14.7$$

$$d = 14.7(3-1)$$

MI

$$= \underline{29.4 \text{ m}}$$

ft c's t

AI/✓

(b) Using  $v = u + at$  with  $u = 19.6$ ,  $a = -9.8$ ,  $t = 0.75$  MI

$$v = 19.6 - 9.8 \times 0.75$$

$$= 12.25$$

AI

$$\text{Speed} = \sqrt{12.25^2 + 14.7^2}$$

MI

$$= \underline{19.14 \text{ ms}^{-1}}$$

ft v

AI/✓



$$\text{Direction } \theta = \tan^{-1}\left(\frac{12.25}{14.7}\right)$$

MI

$$= \underline{39.8^\circ}$$

ft

AI/✓

6(a).  $\vec{OP} = \underline{r} = (2t-5)\underline{i} + (t-3)\underline{j} + (7-2t)\underline{k}$

$$OP^2 = (2t-5)^2 + (t-3)^2 + (7-2t)^2 \quad M1$$

$$= 4t^2 - 20t + 25$$

$$+ t^2 - 6t + 9$$

$$+ 4t^2 - 28t + 49$$

$$= 9t^2 - 54t + 83 \quad \text{combining} \quad A1$$

$P$  is closest to  $O$  when  $OP^2$  is minimum M1

$$\frac{d}{dt}(OP^2) = 0 \quad \text{+ differentiation} \quad M1$$

$$18t - 54 = 0$$

$$t = \underline{3} \quad A1$$

(b)  $\underline{r} = (-5\underline{i} - 3\underline{j} + 7\underline{k}) + (2\underline{i} + \underline{j} - 2\underline{k})t$  any method M1

$$\therefore \underline{v} = 2\underline{i} + \underline{j} - 2\underline{k} \quad \text{is constant} \quad A1$$

$$|\underline{v}| = \sqrt{2^2 + 1^2 + 2^2} = 3 \quad A1$$

(c) When  $P$  is closest to  $O$ ,

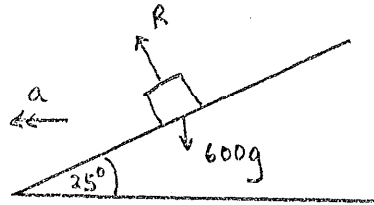
$$\vec{OP} = \underline{i} + \underline{k} \quad B1$$

$$\vec{OP} \cdot \underline{v} = 2 \times 1 - 2 \times 1 \quad M1$$

$$= 0$$

$\therefore$  Direction of velocity of  $P$  is  $\perp$  to  $OP$  A1

7.



(a) Resolve vertically

M1

$$R \cos 25^\circ = 600g$$

A1

$$R = \frac{600 \times 9.8}{\cos 25^\circ}$$

$$= \underline{6487.86 \text{ N}}$$

A1

(b) Now horizontally

M1

$$R \sin 25^\circ = 600a$$

A1

$$a = \frac{42^2}{r}$$

B1

$$\frac{600 \times 9.8}{\cos 25^\circ} \times \sin 25^\circ = 600 \times \frac{42^2}{r}$$

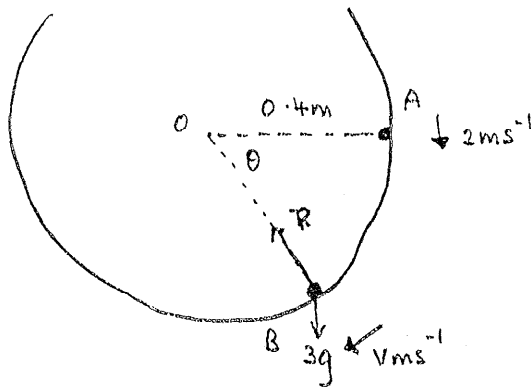
$$r = \frac{42^2 \cos 25^\circ}{9.8 \sin 25^\circ}$$

$$= \underline{386.01 \text{ m}}$$

cao

A1

8.



(a) Conservation of energy

M1

$$\frac{1}{2} m \cdot 2^2 = \frac{1}{2} m v^2 - mg \cdot 0.4 \sin \theta$$

A1 A1

$$v^2 = 4 + 7.84 \sin \theta$$

A1

$$(b) \quad R - mg \sin \theta = m \frac{v^2}{0.4}$$

M1 A1 A1

$$R = \frac{m}{0.4} (4 + 7.84 \sin \theta) + 9.8 m \sin \theta$$

$$= 30 + 58.8 \sin \theta + 29.4 \sin \theta$$

$$R = 30 + 88.2 \sin \theta$$

A1

$$(c) \quad R = 0 \quad \text{when} \quad \theta = \sin^{-1} \left( -\frac{30}{88.2} \right)$$

M1 A1

$$= 199.89^\circ$$

$$\therefore \text{Greatest } \theta \text{ is } 199.89^\circ$$

A1

Marble leaves the surface of the bowl at  $19.89^\circ$  above the horizontal and moves under the action of gravity like a projectile

B1

**Mark Scheme for S1 (New Syllabus) June 2005**

1. (a) Prob =  $\frac{5}{11} \times \frac{4}{10} \times \frac{6}{9} \times 3 = \frac{4}{11}$  (0.364) (or  $\frac{\binom{5}{2} \times \binom{6}{1}}{\binom{11}{3}}$ ) M1A1A1
- [M1 mult probs, A1 for 3, A1 ans OR M1 ratio of combs, A1 correct, A1 ans]
- (b) P(3 boys) =  $\frac{5}{11} \times \frac{4}{10} \times \frac{3}{9} = \frac{\binom{2}{33}}{\binom{5}{3}} \left( \text{or } \frac{\binom{5}{3}}{\binom{11}{3}} \right)$  B1
- P(3 girls) =  $\frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} = \frac{\binom{4}{33}}{\binom{6}{3}} \left( \text{or } \frac{\binom{6}{3}}{\binom{11}{3}} \right)$  B1
- P(3 same gender) = Sum =  $\frac{2}{11}$  (0.182) M1A1
2. (a)(i)  $P(A) = \frac{20+30+30}{150} = \frac{8}{15}$  M1A1
- (ii) EITHER  $P(B|A) = \frac{20}{20+30+30} = \frac{1}{4}$  M1A1
- OR  $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{20/150}{8/15} = \frac{1}{4}$
- [M0 if A,B assumed independent in numerator]
- (iii) EITHER  $P(A \cup B) = \frac{15+20+30+30}{150} = \frac{19}{30}$  M1A1
- OR  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $= \frac{80+35-20}{150} = \frac{19}{30}$
- (iv) EITHER  $P(B) = \frac{35}{150} \neq P(B|A)$  M1A1
- OR  $P(A \cap B)(2/15) \neq P(A)P(B)(28/225)$   
 so not independent. A1
3. (a)  $\frac{1}{2}$ . B1
- (b)  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$  M1A1
- (c)  $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}$  M1A1
- (d) Prob =  $\frac{1/2}{1-1/4} = \frac{2}{3}$  M1A1
4. (a)(i)  $P(X=10) = e^{-15} \cdot \frac{15^{10}}{10!} = 0.049$  (0.1185 – 0.0699 or 0.9301 – 0.8815) M1A1
- (ii)  $P(X < 12) = 0.1848$  M1A1
- (b) We require  $P(X \geq 21) = 0.083$  M1A1
- (c) Reqd number = 25. [Award B1 for 24 or 26] B2

5. (a)(i)  $P(+) = 0.01 \times 0.9 + 0.99 \times 0.05 = 0.0585$  M1A1A1  
 [M1 Use of Law of Total Prob, A1 all correct, A1 ans]  
 (ii)  $P(\text{Not D} | +) = \frac{0.99 \times 0.05}{0.0585} = 0.846$  (cao) B1B1B1  
 (b) The probability of not having the disease given a + result is too high. B1
6. (a)(i) Binomial B1  
 (ii) Mean = 10/6, Variance = 50/36 B1B1  
 (iii)  $P(X \leq 2) = \left(\frac{5}{6}\right)^{10} + \binom{10}{1}\left(\frac{5}{6}\right)^9\left(\frac{1}{6}\right) + \binom{10}{2}\left(\frac{5}{6}\right)^8\left(\frac{1}{6}\right)^2$  M1A1  
 $= 0.775$  A1  
 (b)  $Y$  is  $B(81, 1/36) \approx \text{Po}(81/36 \text{ or } 2.25)$  B1B1  
 $P(Y = 4) = e^{-2.25} \times \frac{2.25^4}{4!}$  M1  
 $= 0.113$  A1
7. (a)  $k(2 + 3 + 4 + 5 + 6) = 1$  giving  $k = 1/20$ . M1A1  
 [Accept verification]  
 (b) Mean =  $\frac{1}{20}(1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6) = 3.5$  M1A1  
 $E(X^2) = \frac{1}{20}(1 \times 2 + 4 \times 3 + 9 \times 4 + 16 \times 5 + 25 \times 6) = 14$  M1A1  
 Var =  $14 - 3.5^2 = 1.75$  A1  
 (c) Possibilities are 2 and 2, 1 and 3 (si) B1  
 Prob =  $\frac{1}{400}(3 \times 3 + 2 \times 2 \times 4)$  M1A1  
 $= 0.0625$  A1  
 (d)  $E(Y) = 2 \times 3.5 + 3 = 10$  M1A1  
 Var( $Y$ ) =  $4 \times 1.75 = 7$  M1A1
8. (a)  $F(0.8) = 0.8192$  B1  
 $F(0.2) = 0.0272$  B1  
 Prob =  $0.8192 - 0.0272 = 0.792$  (FT on 1 slip) B1  
 (b)  $F(0.45) = 0.241$ ,  $F(0.46) = 0.255$  B1B1  
 so the root of  $F(q) = 0.25$  is between 0.45 and 0.46. B1  
 (c)  $f(x) = \text{Deriv of } 4x^3 - 3x^4$  M1  
 $= 12x^2(1 - x)$  A1  
 (d)  $E(X) = 12 \int_0^1 x \cdot x^2(1 - x) dx$  M1A1  
 $= 12 \left[ \frac{x^4}{4} - \frac{x^5}{5} \right]_0^1$  A1  
 $= 0.6$  A1

**Mark Scheme for A/AS level Mathematics - S2 (New Syllabus) – June 2005**

1. (a)  $\bar{x} = \frac{882}{12} = 73.5$  B1  
 95% confidence limits are  
 $73.5 \pm 1.96 \sqrt{\frac{16}{12}}$  B1B1B1  
 giving [71.2, 75.8]. B1  
 (b) Her belief is supported because 75 is inside the interval. B1
2. (a)(i)  $\text{Prob} = e^{-5} \cdot \frac{5^6}{6!} = 0.146$  M1A1  
 [Or using tables 0.7622 – 0.6160 or 0.3840 – 0.2378]  
 (ii)  $\text{Prob} = 0.146^3 = 0.0031$  M1A1  
 (iii) T is Poi(15) M1  
 $\text{Prob} = e^{-15} \cdot \frac{15^{18}}{18!} = 0.0706$  m1A1  
 [Or using tables 0.8195 – 0.7489 or 0.2511 – 0.1805]  
 (b) T is Poi(260)  $\approx$  N(260, 260) B1  
 $z = \frac{240.5 - 260}{\sqrt{260}} = -1.21$  M1A1A1  
 Prob = 0.8869 A1
3. (a)  $z_1 = \frac{79 - 70}{6} = 1.5, z_2 = \frac{67 - 70}{6} = -0.5$  M1A1A1  
 Prob = 0.93319 – (1 – 0.69146) or 1 – 0.3085 – 0.0668 M1  
 = 0.625 A1  
 (b) Mean =  $2 \times 50 - 70 = 30$  B1  
 Var =  $4 \times 25 + 36 = 136$  B1  
 Prob =  $P(X > 2Y) = P(2Y - X) < 0$  M1  
 $z = \frac{0 - 30}{\sqrt{136}} = -2.57$  A1A1  
 Prob = 0.00508 A1  
 (c) E(Total) =  $3 \times 70 + 6 \times 50 = 510$  B1  
 Var(Tot) =  $3 \times 36 + 6 \times 25 = 258$  B1  
 Total is N(510, 258) M1  
 $z = \frac{500 - 510}{\sqrt{258}} = -0.62$  A1A1  
 Prob = 0.7324 A1

|   |        |                                                                                   |      |
|---|--------|-----------------------------------------------------------------------------------|------|
| 4 | (a)    | $H_0 : \mu = 1.5$ versus $H_1 : \mu < 1.5$                                        | B1   |
|   | (b)(i) | The critical region.                                                              | B1   |
|   | (ii)   | $SL = P(\text{Accept } H_1   H_0)$                                                | B1   |
|   |        | Under $H_0$ , $X$ is $P(45) \approx N(45, 45)$ .                                  | M1A1 |
|   |        | $z = \frac{35.5 - 45}{\sqrt{45}} = -1.42$                                         | A1A1 |
|   |        | $SL = 0.0778$                                                                     | A1   |
| 5 | (a)(i) | $H_0 : p = 0.45$ versus $H_1 : p < 0.45$                                          | B1   |
|   | (ii)   | Under $H_0$ , $X$ (no. germinating) is $B(50, 0.45)$ .                            | M1   |
|   |        | $p\text{-value} = P(X \leq 18   p = 0.45)$                                        | m1   |
|   |        | $= 0.1273$                                                                        | A1   |
|   | (iii)  | We find that, when $p = 0.45$ ,                                                   |      |
|   |        | $P(X \leq 14) = 0.0104$ and $P(X \leq 13) = 0.0045$                               | M1   |
|   |        | So Max $X = 13$                                                                   | A1   |
|   | (b)    | Under $H_0$ , $X$ is now $B(500, 0.45) \approx N(225, 123.75)$                    | M1A1 |
|   |        | $z = \frac{202.5 - 225}{\sqrt{123.75}}$                                           | A1A1 |
|   |        | $= -2.02$                                                                         | A1   |
|   |        | $p\text{-value} = 0.0217$                                                         | A1   |
|   |        | We conclude at the 5% level that the proportion germinating is less than 45%.     | B1   |
| 6 | (a)    | $H_0 : \mu_G = \mu_B$ versus $H_1 : \mu_G \neq \mu_B$                             | B1   |
|   | (b)    | $\Sigma g = 105.1$ , $\Sigma b = 86.7$ or $\bar{g} = 13.1375$ , $\bar{b} = 14.45$ | B1   |
|   |        | The appropriate test statistic is                                                 |      |
|   |        | $TS = \frac{\bar{g} - \bar{b}}{\sigma \sqrt{\frac{1}{m} + \frac{1}{n}}}$          | M1   |
|   |        | $= \frac{105.1/8 - 86.7/6}{1.5 \sqrt{\frac{1}{8} + \frac{1}{6}}}$                 | A1A1 |
|   |        | $= -1.62$                                                                         | A1   |
|   |        | tabular value = 0.0526                                                            | A1   |
|   |        | p-value = 0.1052                                                                  | B1   |
|   | (c)    | Insufficient evidence to conclude that the means are different.                   | B1   |

|   |      |                                                           |      |
|---|------|-----------------------------------------------------------|------|
| 7 | (a)  | The density function is                                   |      |
|   |      | $f(\theta) = 3/\pi$ for $0 < \theta < \pi/3$              | B1   |
|   |      | $= 0$ otherwise                                           | B1   |
|   | (b)  | $H = 2\sin\theta$                                         | B1   |
|   | (i)  | $P(H \leq 1) = P(\sin\theta \leq 1/\sqrt{2})$             | M1   |
|   |      | $= P(\theta \leq \pi/4)$                                  | A1   |
|   |      | $= \frac{\pi/4}{\pi/3} = 0.75$                            | A1   |
|   | (ii) | $E(H) = \frac{3}{\pi} \int_0^{\pi/3} 2\sin\theta d\theta$ | M1A1 |
|   |      | $= \frac{3}{\pi} [-2\cos\theta]_0^{\pi/3}$                | A1   |
|   |      | $= \frac{3}{\pi}$                                         | A1   |

Welsh Joint Education Committee  
245 Western Avenue  
Cardiff. CF5 2YX  
Tel. No. 029 2026 5000  
Fax. 029 2057 5994  
E-mail: [exams@wjec.co.uk](mailto:exams@wjec.co.uk)  
website: [www.wjec.co.uk/exams.html](http://www.wjec.co.uk/exams.html)

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