New Syllabus

Mathematics M2 (June 2005)

Marks Scheme

FINAL

1(a).

$T = 5g$ B1

$T = \frac{\lambda(1.3 - 0.8)}{0.8}$ M1

$5 \times 9.8 = \frac{\lambda \times 0.5}{0.8}$

$\lambda = \underline{78.4 \text{ N}}$ A1

(b) Elastic Energy $= \frac{1}{2} \times \frac{78.4 \times 0.5^2}{0.8}$ M1

$= \underline{12.25 \text{ J}}$ ft λ A1

2(a) $v = \int a \, dt$ M1

$= \int 4 - 6t \, dt$

$v = 4t - 3t^2 + c$ A1

$t = 0$, $v = 4$, $c = 4$ A1

$\underline{v = 4t - 3t^2 + 4}$

(b) $s = \int v \, dt$ M1

$= \int 4t - 3t^2 + 4 \, dt$

$= 2t^2 - t^3 + 4t + c$ A1

$t = 0$, $s = 0$, $c = 0$ A1

$\underline{s = 2t^2 - t^3 + 4t}$

(c) Particle at rest $\Rightarrow$ $v = 0$ M1

$3t^2 - 4t - 4 = 0$

$(3t + 2)(t - 2) = 0$

$t = 2$ A1

At $t = 2$, $s = 2(2)^2 - 2^3 + 4(2)$

$= \underline{8}$ A1

(d) When $t = 3$, $v = 4(3) - 3(3)^2 + 4 = -11$

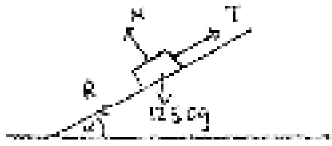
Speed $= +11$ B1

$a = 4 - 6(3)$ B1

$= -14$

a is negative, so velocity is decreasing, ie speed increasing B1

3.



(a) $T = \frac{P}{v}$ used M1

$= \frac{30 \times 1000}{7.5}$ B1

$= 4000$ N

N2L with $a = 0$ dim correct, all forces M1

$T - R = 1250g \sin\alpha$ A1 A1

$4000 - 1550 = 1250 \times 9.8 \sin\alpha$

$\sin\alpha = 0.2$

$\alpha = 11.5°$ A1

4. K.E at A $= \frac{1}{2} \times 240 \times 2^2$ M1 A1

$= 480J$

P.E. $= mgh$ used M1

change in P.E $= 240 \times 9.8(30-22) = (18816J)$ B1

W.D against resistance $= 132 \times 88$ M1 A1

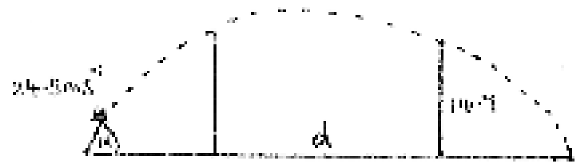
$= 11616\ J$

Work-energy principle

$480 + 18816 - 11616 = \frac{1}{2} \times 240\ v^2$ M1

$v = \underline{8\ ms^{-1}}$ A1

5.



$\sin\alpha = 0.8$

(a)(i) $u_v = 24.5 \sin\alpha$ B1

$= 19.6$

Using $s = ut + \frac{1}{2}at^2$ with $s = 14.7$, $u = 19.6$ (c), $a = \pm 9.8$ M1

$14.7 = 19.6t - 4.9t^2$ A1

$t^2 - 4t + 3 = 0$ m1

$(t-1)(t-3) = 0$

$t = 1, 3$ A1

(ii) $u_H = 24.5\cos\alpha$ B1

$= 14.7$

$d = 14.7(3-1)$ m1

$= \underline{29.4 \text{ m}}$ ft t A1$\checkmark$

(b) Using $v = u + at$ with $u = 19.6$, $a = \pm 9.8$, $t = 0.75$ M1

$v = 19.6 - 9.8 \times 0.75$

$= 12.25$ A1

Speed $= \sqrt{12.25^2 + 14.7^2}$ M1

$= \underline{19.14 \text{ ms}^{-1}}$ ft v A1$\checkmark$

Direction $\theta = \tan^{-1}\left(\frac{12.25}{14.7}\right)$ M1

$= \underline{39.8^\circ}$ ft A1$\checkmark$

6(a). $\vec{OP} = \underline{r} = (2t-5)\underline{i} + (t-3)\underline{j} + (7-2t)\underline{k}$

$OP^2 = (2t-5)^2 + (t-3)^2 + (7-2t)^2$ M1

$= 4t^2 - 20t + 25$

$+ t^2 - 6t + 9$

$+ 4t^2 - 28t + 49$

$= 9t^2 - 54t + 83$ combining A1

P is closest to O when OP^2 is minimum M1

$\frac{d}{dt}(OP^2) = 0$ ← differentiation M1

$18t - 54 = 0$

$t = \underline{3}$ A1

(ii) $\underline{r} = (-5\underline{i} - 3\underline{j} + 7\underline{k}) + (2\underline{i} + \underline{j} - 2\underline{k})t$ any method M1

$\therefore \underline{v} = 2\underline{i} + \underline{j} - 2\underline{k}$ is constant A1

$|\underline{v}| = \sqrt{2^2+1^2+2^2} = 3$ A1

(c) When P is closest to O,

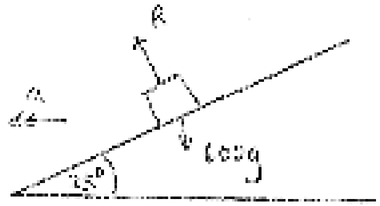
$\vec{OP} = \underline{i} + \underline{k}$ A1

$\vec{OP} \cdot \underline{v} = 2\times1 - 2\times1$ M1

$= 0$

$\therefore$ Direction of velocity of P is $\perp$ to OP A1

7.



(a) Resolve vertically M1

$R \cos 25^\circ = 600g$ A1

$R = \dfrac{600 \times 9.8}{\cos 25^\circ}$

$= \underline{6487.86\ N}$ A1

(b) N2L horizontally M1

$R \sin 25^\circ = 600a$ B1

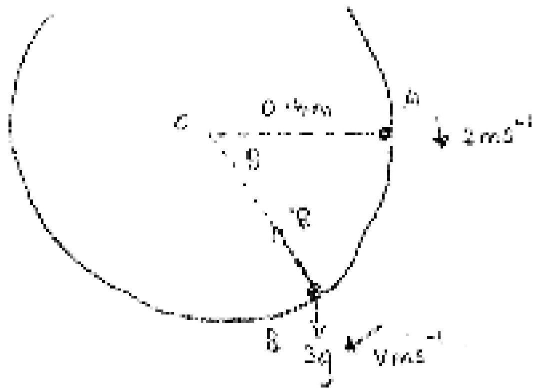
$a = \dfrac{42^2}{r}$ B1

$\dfrac{600 \times 9.8}{\cos 25^\circ} \times \sin 25^\circ = 600 \times \dfrac{42^2}{r}$

$r = \dfrac{42^2 \cos 25^\circ}{9.8 \sin 25^\circ}$

$= \underline{386.01\ m}$ cao A1

8.



(a) Conservation of energy M1

$$\tfrac{1}{2}m \cdot 2^2 = \tfrac{1}{2}mv^2 - mg \cdot 0{\cdot}4\sin\theta$$ A1 B1

$$v^2 = \underline{4 + 7{\cdot}84\sin\theta}$$ A1

(b) $$R - mg\sin\theta = m\frac{v^2}{0{\cdot}4}$$ M1 A1 A1

$$R = \frac{m}{0{\cdot}4}\left(4 + 7{\cdot}84\sin\theta\right) + 9{\cdot}8\,m\sin\theta$$

$$= 20 + 58{\cdot}8\sin\theta + 29{\cdot}4\sin\theta$$

$$R = \underline{30 + 88{\cdot}2\sin\theta}$$ A1

(c) $R = 0$ when $\theta = \sin^{-1}\left(-\frac{30}{88{\cdot}2}\right)$ M1 A1

$$= 199{\cdot}89^{\circ}$$

∴ Greatest θ is $\underline{199{\cdot}89^{\circ}}$ A1

Marble leaves the surface of the bowl at 19·89° above the horizontal and moves under the action of gravity like a projectile. B1