

984/01

**MATHEMATICS S2**

**Statistics**

A.M. FRIDAY, 24 June 2005

(1  $\frac{1}{2}$  hours)

<p><b>NEW SPECIFICATION</b></p>
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**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Ann records the times taken (in minutes) to drive to work on twelve consecutive days with the following results.

73.3 74.7 71.2 75.8 70.0 74.0 80.5 70.5 72.6 73.4 69.1 76.9

Assume that these observations form a random sample from a normal distribution with standard deviation 4.0.

- (a) Calculate a 95% confidence interval for the mean time taken by Ann to drive to work. [5]
- (b) Ann believed that the mean time was 75 minutes. State, with a reason, whether or not your result supports this belief. [1]
2. The number of batteries sold per week by a garage may be assumed to have a Poisson distribution with a mean 5.
- (a) Find the probability that
- (i) exactly 6 are sold in a randomly chosen week, [2]
  - (ii) exactly 6 are sold in each of 3 randomly chosen weeks, [2]
  - (iii) exactly 18 are sold in a randomly chosen 3-week period. [3]
- (b) Find, approximately, the probability that more than 240 are sold in a randomly chosen 52-week period. [5]
3. The weights,  $X$  kg, of male students in a hall of residence are normally distributed with mean 70 kg and standard deviation 6 kg.
- (a) Find the probability that the weight of a randomly chosen male student lies between 67 kg and 79 kg. [5]
- The weights,  $Y$  kg, of female students in the hall of residence are normally distributed with mean 50 kg and standard deviation 5 kg.
- (b) Find the mean and variance of the random variable  $2Y - X$ . Hence find the probability that the weight of a randomly chosen male student is more than twice the weight of a randomly chosen female student. [6]
- (c) The hall of residence has a lift installed with a maximum recommended load of 500 kg. On one occasion, there are 3 male students and 6 female students in the lift. Find the probability that their combined weight exceeds the recommended maximum. [6]
4. The number of machine breakdowns per day in a certain factory may be assumed to have a Poisson distribution with mean  $\mu$ . The value of  $\mu$  is known, from past experience, to be 1.5. In an attempt to reduce the value of  $\mu$ , all the machines are fitted with new control units. To investigate whether or not this succeeds in reducing the value of  $\mu$ , the number of breakdowns,  $x$ , during a 30-day period following the fitting of these new units is recorded.
- (a) State suitable hypotheses. [1]
- (b) It is decided to conclude that the value of  $\mu$  has been reduced if  $x \leq 35$ .
- (i) What name is given to the region  $x \leq 35$ ?
  - (ii) Define the term 'significance level' and find its approximate value for this procedure. [7]

5. A garden centre claims that, on average, 45% of seeds of a certain type will germinate. A gardener believes that the percentage germinating is actually less than 45%.
- (a) To test his belief, the gardener sows 50 of these seeds and he counts the number which germinate.
- State appropriate hypotheses.
  - He finds that 18 of the seeds germinate. Find the  $p$ -value of this result.
  - Determine the maximum number of germinating seeds which would lead to the rejection of the claim at the 1% significance level. [6]
- (b) The gardener now sows 500 of these seeds. He finds that 202 of the seeds germinate.
- Use a distributional approximation to find the  $p$ -value of this result.
  - Interpret your  $p$ -value in context using a 5% significance level. [7]
6. There are 8 girls and 6 boys in a class. The teacher gives them all a problem to solve and she records the time taken by each child to complete the solution. She produces the following table.

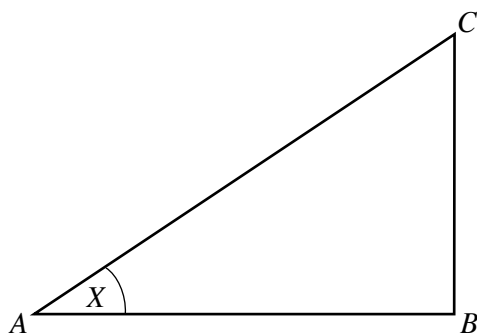
Time taken by girls (minutes)	12.4	11.2	14.3	12.7	15.1	13.6	11.9	13.9
Time taken by boys (minutes)	14.2	15.8	13.8	14.9	12.9	15.1		

She now wishes to test whether or not there is a difference between the mean times taken by boys and girls to solve this problem. You may assume that the times taken are independent random samples from normal distributions with a common standard deviation of 1.5.

- State suitable hypotheses. [1]
- Calculate the  $p$ -value of these results. [7]
- State your conclusion in context. [1]

**TURN OVER**

7.



The above diagram shows a right-angled triangle  $ABC$  in which the length of the hypotenuse  $AC$  is 2 cm and  $\widehat{BAC}$  equals  $X$  radians, where  $X$  is a uniformly distributed random variable defined on the interval  $\left[0, \frac{\pi}{3}\right]$ .

(a) Write down, in full, the probability density function of  $X$ . [2]

(b) The length of  $BC$  is denoted by  $H$  cm. Calculate

(i)  $P(H \leq \sqrt{2})$ ,

(ii)  $E(H)$ . [8]