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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

Tystysgrif Addysg Gyffredinol
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MARKING SCHEMES

SUMMER 2006

MATHEMATICS
C1-C4 and FP1-FP3

WJEC
CBAC

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS C1

1. (a) Gradient of $AC(BD) = \frac{\text{increase in } y}{\text{increase in } x}$ M1
 Gradient $AC = 2$ A1
 Gradient $BD = -\frac{1}{2}$ A1
 Gradient $AC \times \text{Gradient } BD = -1$ M1
 $\therefore AC$ and BD are perpendicular A1
- (b) A correct method for finding the equation of $AC(BD)$ M1
 Equation of AC : $y - 2 = 2(x - 3)$ (or equivalent) A1
 (f.t. candidate's gradient of AC)
 Equation of AC : $2x - y - 4 = 0$ (convincing) A1
 Equation of BD : $y - 3 = -\frac{1}{2}(x + 4)$ (or equivalent) A1
 (f.t. candidate's gradient of BD)
- (c) An attempt to solve equations of AC and BD simultaneously M1
 $x = 2, y = 0$ (c.a.o.) A1
- (d) A correct method for finding the length of AE M1
 $AE = \sqrt{5}$ A1
2. (a) $\frac{5 - \sqrt{3}}{\sqrt{3} + 1} = \frac{(5 - \sqrt{3})(\sqrt{3} - 1)}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$ M1
 Numerator: $5\sqrt{3} - 5 - 3 + \sqrt{3}$ A1
 Denominator: $3 - 1$ A1
 $\frac{5 - \sqrt{3}}{\sqrt{3} + 1} = 3\sqrt{3} - 4$ A1
- Special case**
 If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $a + \sqrt{3}b$
- (b) Removing brackets M1
 $\sqrt{12} = 2 \times \sqrt{3}$ B1
 $\sqrt{12} \times \sqrt{3} = 6$ B1
 $(2 + \sqrt{3})(4 - \sqrt{12}) = 2$ (c.a.o.) A1

3.	(a)	An attempt to find $\frac{dy}{dx}$	M1
		$\frac{dy}{dx} = 2x - 4$	A1
		Value of $\frac{dy}{dx}$ at $A = -2$ (f.t. candidate's $\frac{dy}{dx}$)	A1
		Equation of tangent at A: $y - 4 = -2(x - 1)$ (or equivalent) (f.t. one error)	A1
	(b)	Gradient of normal $\times$ Gradient of tangent $= -1$	M1
		Equation of normal at A: $y - 4 = \frac{1}{2}(x - 1)$ (or equivalent) (f.t. candidate's numerical value for $\frac{dy}{dx}$)	A1
4.	(a)	An expression for $b^2 - 4ac$, with $b = \pm 4$, and at least one of a or c correct	M1
		$b^2 - 4ac = 4^2 - 4k(k - 3)$	A1
		$b^2 - 4ac = 4(k - 4)(k + 1)$	A1
		Putting $b^2 - 4ac = 0$	m1
		$k = -1, 4$ (f.t. one slip)	A1
	(b)	$a = 4$	B1
5.		$b = -14$	B1
		Least value $= -14$ (f.t. candidate's b)	B1
	(a)	Use of $f(2) = -20$	M1
		$8p - 4 + 2q - 6 = -20$	A1
		Use of $f(3) = 0$	M1
		$27p - 9 + 3q - 6 = 0$	A1
		Solving simultaneous equations for p and q	M1
		$p = 2, q = -13$ (c.a.o.)	A1
		Special case assuming $p = 2$	
		Use of one of the above equations to find q	M1
		$q = -13$	A1
		Use of other equation to verify $q = -13$	A1
	(b)	Dividing $f(x)$ by $(x - 3)$ and getting coefficient of x^2 to be 2	M1
		Remaining factor $= 2x^2 + ax + b$ with one of a, b correct	A1
		$f(x) = (x - 3)(2x + 1)(x + 2)$ (c.a.o.)	A1

6. (a) $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ B1
 An attempt to substitute $3x$ for a and $\pm \frac{1}{3}$ for b in r. h. s. of
 above expansion M1
 Required expression $= 81x^4 - 36x^2 + 6 - \frac{4}{9x^2} + \frac{1}{81x^4}$
 (3 terms correct) A1
 (all terms correct) A2
 (f.t. one slip in coefficients of $(a + b)^4$)
- (b) Either: $\frac{n(n-1)}{2} \times 2^k = 40 (k=1,2)$
 Or: ${}^nC_2 \times 2^2 = 40$ M1
 $n = 5$ A1
7. (a) $y + \delta y = (x + \delta x)^2 - 3(x + \delta x) + 4$ B1
 Subtracting y from above to find δy M1
 $\delta y = 2x\delta x + (\delta x)^2 - 3\delta x$ A1
 Dividing by δx , letting $\delta x \rightarrow 0$ and referring to limiting
 value of $\frac{\delta y}{\delta x}$ M1
 $\frac{dy}{dx} = 2x - 3$ A1
- (b) Required derivative $= -4x^{-3} + \frac{7x^{-\frac{1}{2}}}{2}$ B1, B1
8. (a) An attempt to collect like terms across the inequality M1
 $x > -\frac{7}{6}$ A1
- (b) An attempt to remove brackets M1
 $x^2 + 6x + 8 < 0$ A1
 Graph crosses x -axis at $x = -4, x = -2$ B1
 Either: $-4 < x < -2$
 Or: $-4 < x$ and $x < -2$
 Or: $(-4, -2)$ B1

9.	(a)	Translation along y-axis so that stationary point is $(0, a)$, $a = 0, -8$	M1
		Correct translation and stationary point at $(0, 0)$	A1
	(b)	Translation of 2 units to left along x-axis	M1
		Stationary point is $(-2, -4)$	A1
		Points of intersection with x-axis are $(-4, 0)$ and $(0, 0)$	A1
		Special case	
		Translation of 2 units to right along x-axis with correct labelling	B1
10.		$\frac{dy}{dx} = 3x^2 - 6x - 9$	B1
		Putting derived $\frac{dy}{dx} = 0$	M1
		$x = 3, -1$ (both correct) (f.t. candidate's <u>dy</u>)	A1
		Stationary points are $(-1, 7)$ and $(3, -25)$ (both correct) (c.a.o)	A1
		A correct method for finding nature of stationary points	M1
		$(-1, 7)$ is a maximum point (f.t. candidate's derived values)	A1
		$(3, -25)$ is a minimum point (f.t. candidate's derived values)	A1

MATHEMATICS C2

1. $h = 0.1$

$$\text{Integral} \approx \frac{0.1}{2} [1 + 1.012719 + 2(1.0000500 + 1.0007997 + 1.0040418)]$$

$$\approx 0.401$$

S. Case $h = 0.08$

$$\text{Integral} \approx \frac{0.08}{2} [1 + 1.012719 + 2(1.0000205 + 1.0003276 + 1.0016575 + 1.0052292)]$$

$$\approx 0.401$$

M1 (correct formula $h = 0.1$)

B1 (3 values)

B1 (2 values)

A1 (F.T. one slip)

M1 (correct formula $h = 0.08$)

B1 (all values)

A1 (F.T. one slip)

4

2. (a) $x = 158.2^\circ, 338.2^\circ$

(b) $3x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$x = 20^\circ, 100^\circ, 140^\circ$

(c) $2(1 - \sin^2\theta) + 3 \sin\theta = 0$

$$2 \sin^2\theta - 3 \sin\theta - 1 = 0$$

$$(2 \sin\theta + 1)(\sin\theta - 2) = 0$$

$$\sin\theta = -\frac{1}{2}, 2$$

$$\theta = 210^\circ, 330^\circ$$

B1, B1

B1 (any value)

B1, B1, B1

M1 (correct use of $\cos^2\theta = 1 - \sin^2\theta$)

M1 (attempt to solve quad in $\sin\theta$
correct formula or
($a \cos\theta + b$) ($c \sin\theta + d$)
with $ac = \text{coefft. of } \sin^2\theta$
 $bd = \text{constant term}$)

A1

B1 (210°) B1 (330°)

11

3. (a) $\text{Area} = \frac{1}{2}x \times 8 \sin 150^\circ$

$$\frac{1}{2}x \times 8 \times \sin 150^\circ = 10$$

$$x = \frac{10}{4 \sin 150^\circ} = 5$$

B1

B1 (correct equation)

B1 (C.A.O.)

$$\begin{aligned}
 (b) \quad BC^2 &= 5^2 + 8^2 + 2 \cdot 5 \cdot 8 \cos 30^\circ && \text{(o.e.)} && \text{B1} \\
 &= 25 + 64 + 68 \cdot 29 && && \text{B1} \\
 BC &\approx 12 \cdot 58 && && \text{B1}
 \end{aligned}$$

6

$$\begin{aligned}
 4. \quad (a) \quad S_n &= a + a + d + \dots + a + (n-2)d + a + (n-1)d && \text{B1 (at least 3 terms one at each end)} \\
 S_n &= a + (n-1)d + a + (n-2)d + \dots + a + d + a \\
 2S_n &= 2a + (n-1)d + 2a + (n-1)d + \dots && \text{M1} \\
 &+ 2a + (n-1)d + 2a + (n-1)d \\
 &= n[2a + (n-1)d]
 \end{aligned}$$

$$\therefore S_n = \frac{n}{2} [2a + (n-1)d] \quad \text{A1 (convincing)}$$

$$(b) \quad (i) \quad \frac{20}{2} [2a + 19d] = 540 \quad \text{B1}$$

$$\frac{30}{2} [2a + 29d] = 1260 \quad \text{B1}$$

$$2a + 19d = 54 \quad (1)$$

$$2a + 29d = 84 \quad (2)$$

$$\text{Solve (1), (2), } d = 3 \quad \text{M1 (reasonable attempt to solve equations)}$$

$$a = -\frac{3}{2} \quad \text{A1 (both) C.A.O.}$$

$$(ii) \quad 50^{\text{th}} \text{ term} = -\frac{3}{2} + (n-1)3 \quad (n = 50) \quad \text{M1 (correct)}$$

$$= 145 \cdot 5 \quad \text{A1 (F.T. derived values)}$$

9

$$5. \quad (a) \quad ar = 9 ar^3 \quad \text{M1 (} ar = kar^3, k = 9, \frac{1}{9} \text{)}$$

$$\text{A1 (correct)}$$

$$1 = 9r^2 \quad \text{A1 (F.T. value of } k \text{)}$$

$$r = \pm \frac{1}{3} \quad \text{A1 (F.T. value of } k, r = \pm 3 \text{)}$$

$$(b) \quad \frac{a}{1 - \frac{1}{3}} = 12 \quad \text{M1 (use of correct formula)}$$

$$a = 8 \quad \text{A1 (F.T. derived } r)$$

$$\text{Third term} = 8 \times \left(\frac{1}{3}\right)^2 = \frac{8}{9} \quad \text{(F.T. } r) \text{ A1}$$

7

$$6. \quad 3x^{\frac{4}{3}} + \frac{3}{2}x^{-2} + 5x(+C) \quad \text{B1, B1, B1}$$

3

$$7. \quad (a) \quad 7 + 2x - x^2 = x + 1$$

M1 (equating ys)

$$x^2 - x - 6 = 0$$

M1 (correct attempt to solve quad)

$$(x - 3)(x + 2) = 0$$

$$x = 3, -2$$

A1

$$B(3, 4)$$

A1

$$(b) \quad \text{Area} = \int_0^3 (7 + 2x - x^2) dx$$

M1 (use of integration to find areas)

$$- \int_0^3 (x + 1) dx$$

m1

$$= \int_0^3 (6 + x - x^2) dx$$

B1 (simplified)

$$= \left[6x + \frac{x^2}{2} - \frac{x^3}{3} \right]_0^3$$

B3 (3 correct integrations)

$$= 18 + \frac{9}{2} - 9 - (0 + 0 - 0)$$

M1 (use of limits)

$$= \frac{27}{2}$$

A1 (C.A.O.)

12

8.	(a) Let $\log_a x = p$ $\therefore x = a^p$ $x^n = (a^p)^n = a^{pn}$ $\log_a x^n = pn = n \log_a x$	B1 (props of logs) B1 (laws of indices) B1 (convincing)
(b)	$\ln 5^{3x+1} = \ln 6$ $(3x+1) \ln 5 = \ln 6$ $3x \ln 5 = \ln 6 - \ln 5$ $\therefore x = \frac{\ln 6 - \ln 5}{3 \ln 5}$ (o.e.) ≈ 0.0378	M1 (taking logs) A1 (correct) m1 (reasonable attempt to isolate x) A1 (C.A.O.)
9.	(a) Centre $(-1, 4)$ Radius = $\sqrt{1^2 + 4^2 - 8} = 3$	B1 B1 (use of formula or std form) B1 (answer)
(b)	 $DP^2 = 29$ (o.e.) $PT^2 = DP^2 - (\text{radius})^2$ $= 29 - 9$ $= 20$ $PT = \sqrt{20}$	B1 (F.T. coords of centre) M1 (use of Pythagoras) A1 (convincing)
(c)	Equation of circle is $(x-4)^2 + (y-6)^2 = 20$ or $x^2 + y^2 - 8x - 12y + 32 = 0$	M1 (use of $x^2 + y^2 + 2gx + 2fy + c = 0$ or $(x-4)^2 + (y-6)^2$ = any +ve no) A1 (either)
10.	(a) $x = 2 \times 4 + 4\theta = 8 + 4\theta$ $A = \frac{1}{2} \times 4^2 \theta = 8\theta$ $8 + 4\theta = 3 \times 8\theta$ $20\theta = 8, \theta = 0.4$	B1 B1 B1 (correct equation) B1 (convincing)
(b)	Area = $\frac{1}{2} \times 4^2 \times 0.4 - \frac{1}{2} \times 4^2 \times \sin 0.4$ ≈ 0.085	B1 (sector) B1 (Δ) M1 (sector - Δ) A1 (C.A.O.)

7

8

8

MATHEMATICS C3

1. $h = 0.25$ M1 ($h = 0.25$ correct formula)
- Integral $\approx \frac{0.25}{3} [0 + 0.8325546 + 4(0.4723807 + 0.7480747)$ B1 (3 values)
- $+ 2(0.6367614)]$ B1 (2 values)
- ≈ 0.582 A1 (F.T. one slip)

4

2. (a) $a = b = 45^\circ$, for example B1 (choice of values)
- $\cos(a + b) = 0$
- $\cos a + \cos b = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \approx 1.41$ B1 (for correct demonstration)
- $(\therefore \cos(a + b) \neq \cos a + \cos b)$
- [other cases, $\cos 1^\circ + \cos 2^\circ = 0.999848 + 0.999391$
 ≈ 1.999
- $\cos 3^\circ \approx 0.999$
 $\cos 2^\circ + \cos 3^\circ \approx 0.9994 + 0.9986 = 1.998$
 $\cos 5^\circ = 0.9962]$
- (b) $7 - (1 + \tan^2 \theta) = \tan^2 \theta + \tan \theta$ M1 (substitution of $\sec^2 \theta = 1 + \tan^2 \theta$)
- $2 \tan^2 \theta + \tan \theta - 6 = 0$ M1 (attempt to solve quad
 $(a \tan \theta + b)(c \tan \theta + d)$
with $ac =$ coefficient of $\tan^2 \theta$
 $bd =$ constant term,
or formula)
- $(2 \tan \theta - 3)(\tan \theta + 2) = 0$
- $\tan \theta = \frac{3}{2}, -2$ A1
- $\theta = 56.3^\circ, 236.3^\circ, 116.6^\circ, 296.6^\circ$ B1 ($56.3^\circ, 236.3^\circ$)
B1 ($116.6^\circ, 296.6^\circ$)
- Full F.T. for $\tan \theta = t$,
2 marks for $\tan \theta = -, -$
1 mark for $\tan \theta = +, +$

8

3. (a) $\frac{dy}{dx} = \frac{2 \cos 2t}{-\sin t}$ M1 (attempt to use $\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$),
 B1 ($-\sin t$)
 B1 ($k \cos 2t, k = 1, 2, -2, \frac{1}{2}$)
 A1 $\left(\frac{2 \cos 2t}{-\sin t}, \text{C.A.O.} \right)$
- (b) $4x^3 + 2x^2 \frac{dy}{dx} + 4xy + 2y \frac{dy}{dx} = 0$ B1 ($2x^2 + 4xy$)
 B1 ($2y \frac{dy}{dx}$)
 B1 ($4x^3, 0$)
 B1 (C.A.O.)
- $\frac{dy}{dx} = \frac{(4x^3 + 4xy)}{2x^2 + 2y}$

8

4. (a) (i) $\left[\frac{e^{2x}}{2} - x \right]_0^a = \frac{e^{2a}}{2} - a - \frac{1}{2}$ M1 ($ke^{2x}, k = \frac{1}{2}, 1$)
 A1 $\left(\frac{e^{2x}}{2} - x \right)$
 A1 (F.T. one slip (in k))
- (ii) $\frac{e^{2a}}{2} - a - \frac{1}{2} = \frac{1}{2}(9 - a)$
 $e^{2a} - 2a - 1 = 9 - a$
 $e^{2a} - a - 10 = 0$ B1 (convincing)
- (b)

<u>a</u>	<u>f(a)</u>	
1	-3.61	change of sign
2	42.6	indicates presence
		of root (between 1 and 2)

M1 (attempt to find values or signs)
A1 (correct values or signs and conclusion)
- $a_0 = 1.2, a_1 = 1.2079569, a_2 = 1.20831198, a_3 = 1.2083278$
 $a_4 \approx 1.20833$ (1.2083285)
 Try 1.208324, 1.208335
 B1 (a_1)
 B1 (a_4 to 5 places, C.A.O.)
- | | |
|----------|-------------|
| <u>a</u> | <u>f(a)</u> |
| 1.208325 | -0.00008 |
| 1.208335 | 0.00014 |

M1 (attempt to find values or signs)
A1 (correct values or signs)
- Change of sign indicates root is 1.20833
 (correct to 5 decimal places)
 A1

11

5. (a) (i) $\frac{1}{1+(4x)^2} \times 4 \left(= \frac{4}{1+16x^2} \right)$

(Allow M1 for $\frac{4}{1+4x^2}$)

(ii) $\frac{1}{1+x^2} \times 2x = \frac{2x}{1+x^2}$

(iii) $3x^2 e^{3x} + 2x e^{3x}$

M1 $\left(\frac{k}{1+(4x)^2} \quad k=1,4 \right)$

A1 ($k=4$)

M1 $\left(\frac{f(x)}{1+x^2}, f(x) \neq 1 \right)$

A1 ($f(x) = 2x$)

M1 ($x^2 f(x) + e^{3x} g(x)$)

A1 ($f(x) = ke^{3x}, g(x) = 2x$)

A1 (all correct)

(b) $\frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$

M1 $\left(\frac{\sin x f(x) - \cos x g(x)}{\sin^2 x} \right)$

A1 ($f(x) = -\sin x, g(x) = \cos x$)

A1 (convincing)

10

6. (a) $5 \mid x \mid = 2$
 $x = \pm \frac{2}{5}$

B1

B1 (both)

(F.T. $a \mid x \mid = b$)

(b) $7x - 5 \geq 3$
 $x \geq \frac{8}{7}$
 $7x - 5 \leq -3$
 $x \leq \frac{2}{7}$

B1

M1 ($7x - 5 \leq -3$)

A1

5

7. (a) (i) $-\frac{7}{15(5x+2)^3} \quad (+C) \quad (\text{o.e.})$

M1 $\left(\frac{k}{(5x+2)^3} \right) \quad \text{A1} \left(k = \frac{7}{15} \right)$

(ii) $\frac{1}{4} \ln \mid 8x+7 \mid \quad (+C) \quad (\text{o.e.})$

M1 ($k \ln \mid 8x+7 \mid$)

A1 ($k = \frac{1}{4} \quad (\text{o.e.})$)

$$(b) \quad \left[\frac{1}{3} \sin 3x \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \frac{1}{3} \sin \left(\frac{\pi \times 3}{3} \right) - \frac{1}{3} \sin \left(\frac{\pi}{6} \times 3 \right)$$

$$= -\frac{1}{3}$$

$$M1 \left(k \sin 3x, k = \frac{1}{3}, -\frac{1}{3}, 3 \right)$$

$$A1 \left(k = \frac{1}{3} \right)$$

M1 (use of limits, F.T. allowable k)

A1 (C.A.O.)

8

$$8. \quad (a) \quad f'(x) = 1 + \frac{1}{x^2}$$

$$f'(x) > 0 \text{ since } (1 > 0 \text{ and } \frac{1}{x^2} > 0)$$

Least value when $x = 1$ and is 0

B1

B1

B1

$$(b) \quad \text{Range of } f \text{ is } [0, \infty)$$

B1

$$(c) \quad 3\left(x - \frac{1}{x}\right)^2 + 2 = \frac{3}{x^2} + 8$$

B1 (correct composition)

M1 (writing equations and correct expansion of binomial)

$$3\left(x^2 - 2 + \frac{1}{x^2}\right) = \frac{3}{x^2} + 8$$

$$3x^2 = 12$$

$$x = \pm 2 \text{ (accept 2)}$$

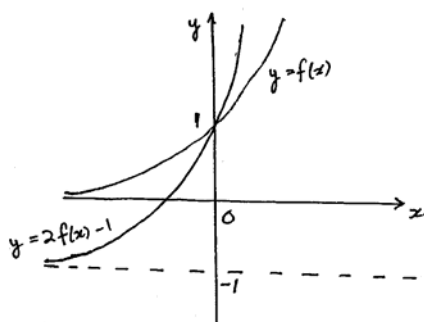
$$x = 2 \text{ since domain of } f \text{ is } x \geq 1$$

A1 (C.A.O.)

B1 (F.T. removal of -ve root)

8

9.



$$y = f(x) \text{ B1 } (0, 1)$$

B1 (correct behaviour for large +ve, -ve x)

$$y = 2f(x) - 1$$

B1 $(0, 1)$

B1 (behaviour for -ve x , must approach -ve value of y)

B1 (greater slope even only if in 1st quadrant)

5

10. (a) Let $y = \sqrt{x+1}$

$$y^2 = x + 1$$

$$x = y^2 - 1$$

$$f^{-1}(x) = x^2 - 1$$

M1 (attempt to isolate x , $y^2 = x + 1$)

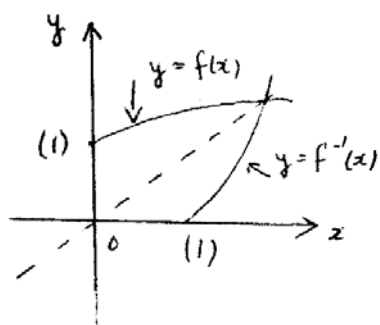
A1

A1 (F.T. one slip)

(b) domain $[1, \infty)$, range $[0, \infty)$

B1, B1

(c)



B1 (parabola $y = x^2 - 1$)

B1 (relevant part of parabola)

B1 ($y = f(x)$, F.T. graph of $y = f^{-1}(x)$)

MATHEMATICS C4

1. (a) Let $\frac{2x^2 + 4}{(x-2)^2(x+4)} \equiv \frac{A}{x+4} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ M1 (Correct form)
- $2x^2 + 4 \equiv A(x-2)^2 + B(x-2) + C(x+4)$ M1 (correct clearing of fractions and attempt to substitute)
- $\underline{x=2} \quad 12 = C(6), \quad C = 2$
- $\underline{x=-4} \quad 36 = A(36), \quad A = 1$ A1 (two constants, C.A.O.)
- $\underline{\text{Coefft of } x^2} \quad 2 = A + B, \quad B = 1$ A1 (third constant, F.T. one slip)
- (b) $f'(x) = \frac{-1}{(x+4)^2} - \frac{1}{(x-2)^2}$ B1 (first two terms)
- $-\frac{4}{(x-2)^3}$ B1 (third term)
- $f'(0) = -\frac{1}{16} - \frac{1}{4} + \frac{4}{8} = \frac{3}{16} \quad (\text{o.e.})$ B1 (C.A.O.)
2. $6x^2 + 6y^2 + 12xy \frac{dy}{dx} - 4y^2 \frac{dy}{dx} = 0$ B1 ($4y^3 \frac{dy}{dx}$)
- B1 ($6y^2 + 12xy \frac{dy}{dx}$)
- $\frac{dy}{dx} = -\frac{3}{2}$ B1 (C.A.O.)
- Gradient of normal = $\frac{3}{2}$ M1 (F.T. candidate's $\frac{dy}{dx}$)
- Equation is $y - 1 = \frac{2}{3}(x - 2)$ A1 (F.T. candidate's gradient of normal)

7

5

3. $2 + 3(2 \cos^2 \theta - 1) = \cos \theta$ M1 (correct substitution for $\cos 2\theta$)
- $6 \cos^2 \theta - \cos \theta - 1 = 0$ M1 (correct method of solution, $(a \cos \theta + b)(\cos \theta + d)$ with $ac = \text{coefft of } \cos^2 \theta$, $bd = \text{constant term}$, or correct formula)
- $(3 \cos \theta + 1)(2 \cos \theta - 1) = 0 \quad \cos \theta = -\frac{1}{3}, \frac{1}{2}$ A1
- $\theta = 109.5^\circ, 250.5^\circ, 60^\circ, 300^\circ$ B1 (109.5°)
B1 (250.5°)
B1 ($60^\circ, 300^\circ$)

6

Full F.T. for $\cos \theta = +, -$
2 marks for $\cos \theta = -, -$
1 mark for $\cos \theta = +, +$
Subtract 1 mark for each additional value in range, for each branch.

4. (a) $R \cos \alpha = 4, R \sin \alpha = 3$
 $R = 5, \alpha = \tan^{-1} \left(\frac{3}{4} \right) = 36.87^\circ$ M1 (reasonable approach)
(or $36.9^\circ, 37.0^\circ$) A1 (α)
B1 ($R = 5$)
- (b) Write as $\frac{1}{5 \sin(x^\circ + 36.87^\circ) + 7}$ M1 (attempt to use $\sin(x + \alpha) = \pm 1$)
- Greatest value = $\frac{1}{-5 + 7} = \frac{1}{2}$ A1 (F.T. one slip)

5

5. Volume = $\pi \int_0^{\frac{\pi}{2}} \sin^2 x dx$ B1
- = $(\pi) \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx$ M1 ($a + b \cos 2x$)
- = $(\pi) \left[\frac{x}{2} - \frac{\sin 2x}{4} \right]$ A1 ($a = \frac{1}{2}, b = \frac{1}{2}$)
(F.T. a, b)
- = $\frac{\pi^2}{4}$ (≈ 2.467 , accept 2.47, 3 sig. figures) A1 (C.A.O.)

5

6.	(a)	$\frac{dy}{dx} = \frac{2t}{-\frac{1}{t^2}} - 2t^3$	M1 $\left(\frac{\dot{y}}{\dot{x}}\right)$ A1 (simplified)
		Equation of tangent is	
		$y - p^2 = -2p^3 \left(x - \frac{1}{p}\right)$	M1 $(y - y_1 = m(x - x_1))$ o.e.
		$y - p^2 = -2p^3x + 2p^2$	
		$2p^3x + y - 3p^2 = 0$	A1 (convincing)
	(b)	$y = 0, x = \frac{3}{2p} \quad (\text{o.e.})$	B1
		$x = 0, y = 3p^2$	B1
		$PA^2 = \left(\frac{3}{2p} - \frac{1}{p}\right)^2 + (p^2 - 0)^2 = \frac{1}{4p^2} + p^4 \quad (\text{o.e.})$	M1 (correct use of distance formula in context) A1 (one correct simplified distance C.A.O.) A1 (convincing)
		$PB^2 = 4PA^2, PB = 2PA$	
			9
7.	(a)	$\int x \ln x dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$	M1 $(f(x) \ln x - \int g(x) \cdot \frac{1}{x} dx)$ A1 $(f(x)=g(x))$ A1 $(f(x) = g(x) = \frac{x^2}{2})$
		$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$	A1
		$= \frac{x^2}{2} \ln x - \frac{x^2}{4} (+C)$	A1 (C.A.O.)
	(b)	$u = 2 \sin x + 3, dx = 2 \cos x dx$ (limits are 3, 4)	
		$\int_3^4 \frac{1}{2} \frac{du}{u^2}$	M1 $\left(\int \frac{a}{u^2} du \text{ with } a = \pm \frac{1}{2}, 1, 2\right)$ A1 $(a = \frac{1}{2})$
		$= \left[-\frac{1}{2u}\right]_3^4$	A1 $(-\frac{a}{u}, \text{allowable } a)$
		$= \frac{1}{24} \quad (\text{o.e.})$	A1 (F.T. allowable a <u>or</u> one slip)

8.	(a) $\frac{dx}{dt} = -k\sqrt{x}$	B1
	(b) $\int \frac{dx}{\sqrt{x}} = \int -k dt$	M1 (attempt to separate variables, allow similar work)
	$2x^{\frac{1}{2}} = -kt + C$	A1 (unsimplified version, allow absence of C)
	$t = 0, x = 9,$	M1 (correct attempt to find C)
	$2\sqrt{9} = C$	A1 (convincing)
	$C = 6$	
	$kt = 6 - 2\sqrt{x}$	
	(c) $t = 20, x = 4$ gives $k = \frac{1}{10}$ (o.e.)	M1 (attempt to find k)
	Tank is empty when $6 = \frac{1}{10}t, t = 60$ (mins.)	A1
		8
9.	(a) $\mathbf{OP} = \mathbf{OA} + \lambda \mathbf{AB}$	M1 (correct formula for r.h.s. and method for finding AB)
	$= \mathbf{i} + 3\mathbf{j} + \mathbf{k} + \lambda (\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	B1 (AB)
	$\mathbf{r} = \mathbf{i} + 3\mathbf{j} + \mathbf{k} + \lambda (\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$	A1 (must contain r , F.T. AB)
	(b) (Point of intersection is on both lines) Equate coefficients of i and j	
	$1 + \lambda = 2 + \mu$	M1 (attempt to write equations using candidate's equations, one correct)
	$3 + 5\lambda = -1 + 2\mu$	A1 (two correct, using candidate's equations)
	$\lambda = -2, \mu = -3$	M1 (attempt to solve equations) A1 (C.A.O.)
	(Consider coefficients in k)	
	$p - \mu = 1 - 3\lambda$	M1 (use of equation in k to find <i>p</i>)
	$p = \mu + 1 - 3\lambda = 4$	A1 (F.T. candidate's λ, μ)
	(c) $\mathbf{b} \cdot \mathbf{c} = (2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}) \cdot (3\mathbf{i} - \mathbf{j} - \mathbf{k})$	M1 (correct method)
	$= 6 - 8 + 2 = 0$	A1 (correct)
	b and c are $\perp$ vectors	A1 (C.A.O.)

$$\begin{aligned}
 10. \quad \left(1 + \frac{x}{8}\right)^{\frac{1}{2}} &= 1 + \frac{1}{2}\left(\frac{x}{8}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1.2}\left(\frac{x}{8}\right)^2 + \dots \\
 &= 1 + \frac{x}{16} - \frac{x^2}{512} + \dots
 \end{aligned}$$

$$\text{B1 } \left(1 + \frac{x}{16}\right)$$

$$\text{B1 } \left(-\frac{x^2}{512}\right)$$

Expansion is valid for $|x| < 8$

B1 (only for conditions on $|x|$)

$$\left(1 + \frac{1}{8}\right)^{\frac{1}{2}} = 1 + \frac{1}{6} - \frac{1}{512}$$

$$\frac{3}{2\sqrt{2}} = \frac{543}{512} \quad (\text{o.e.})$$

B1 (expression must involve $\sqrt{2}$)

$$\sqrt{2} = \frac{3}{2} \times \frac{512}{543} = \frac{256}{181}$$

B1 (convincing)

5

$$11. \quad 4k^2 = 2b^2$$

B1

$$\left. \begin{array}{l} b^2 = 2k^2 \\ b^2 \text{ has a factor 2} \end{array} \right\} \begin{array}{l} \text{one or other of} \\ \text{these statements} \end{array}$$

B1

$\therefore b$ has a factor 2

B1 (if and only if previous B gained)

a and b have a common factor
Contradiction

B1 (depends upon previous B being gained)

($\therefore \sqrt{2}$ is irrational)

4

MATHEMATICS FP1

$$\begin{aligned}
 1. \quad \text{Sum} &= \sum_{r=1}^n r(r+1)(r+5) = \sum_{r=1}^n r^3 + 6 \sum_{r=1}^n r^2 + 5 \sum_{r=1}^n r & \text{M1A1} \\
 &= \frac{n^2(n+1)^2}{4} + \frac{6n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2} & \text{A1} \\
 &= \frac{n(n+1)}{4} (n^2 + n + 8n + 4 + 10) & \text{A1} \\
 &= \frac{n(n+1)}{4} (n^2 + 9n + 14) & \text{A1} \\
 &= \frac{n(n+1)(n+2)(n+7)}{4} & \text{A1}
 \end{aligned}$$

[Award 4/6 for a correct unfactorised answer]

$$\begin{aligned}
 2. \quad f(x+h) - f(x) &= \frac{1}{2(x+h)-3} - \frac{1}{2x-3} & \text{M1A1} \\
 &= \frac{2x-3-2(x+h)+3}{[2(x+h)-3](2x-3)} & \text{A1} \\
 &= \frac{-2h}{[2(x+h)-3](2x-3)} & \text{A1} \\
 f'(x) &= \lim_{h \rightarrow 0} \frac{-2h}{h[2(x+h)-3](2x-3)} & \text{A1} \\
 &= \frac{-2}{(2x-3)^2} & \text{A1}
 \end{aligned}$$

3. Cross multiplying,

$$\begin{aligned}
 z &= (z+1)(2+3i) & \text{M1} \\
 &= 2z + 3iz + 2 + 3i & \text{A1} \\
 z(1+3i) &= -(2+3i) & \text{A1} \\
 z &= -\frac{2+3i}{1+3i} & \text{A1} \\
 &= \frac{-(2+3i)(1-3i)}{(1+3i)(1-3i)} & \text{M1} \\
 &= -\frac{11}{10} + \frac{3}{10}i & \text{A1}
 \end{aligned}$$

Alternative solutions:

Candidates who replace z by $x + iy$ and multiply the LHS by $x + 1 - iy$ to obtain

$$\frac{x(x+1) + y^2}{(x+1)^2 + y^2} = 2; \quad \frac{y}{(x+1)^2 + y^2} = 3 \quad \text{Award M1A1}$$

No further progress is possible at this level.

Candidates who cross multiply and then replace z by $x + iy$ to obtain

$$\begin{aligned}x + iy &= 2x + 2 - 3y + i(2y + 3x + 3) \\x - 3y &= -2, 3x + y = -3\end{aligned}$$

Award M1A1
Award M1A1

Solving these equations

Award M1A1

4. (a) Let the roots be a , $2a$ and $4a$.
Then $14a^2 = 56$
 $a = 2$
The roots are 2, 4 and 8.

M1
m1
A1
A1

- (b) $7a = -p$ gives $p = -14$
 $8a^3 = -q$ gives $q = -64$

B1
B1

5. (a) $\mathbf{A} + \lambda \mathbf{I} = \begin{bmatrix} -4 + \lambda & -4 & 4 \\ -1 & \lambda & 1 \\ -7 & -6 & 7 + \lambda \end{bmatrix}$

M1A1

(b) $\text{Det} = (\lambda - 4)(\lambda(7 + \lambda) + 6) + 4(-7 - \lambda + 7) + 4(6 + 7\lambda)$
 $= \lambda^3 + 3\lambda^2 + 2\lambda$

M1A1
A1

The matrix is singular when this equals zero,

ie, when $\lambda = 0$ or

$$\lambda^2 + 3\lambda + 2 = 0$$

M1
A1
m1

giving $\lambda = -1, -2$.

A1

6. (a) $\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

B1

$$\mathbf{T}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B1

$$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

M1

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

A1

- (b) Fixed points satisfy

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

M1

giving $y + 1 = x$ and $x - 1 = y$

The required equation is $y = x - 1$.

A1A1
A1

7. The statement is true for $n = 1$ since $9 - 5 = 4$ is divisible by 4. B1
 Let $T_n = 9^n - 5^n$ and assume that T_k is divisible by 4. M1
 Consider $T_{k+1} = 9^{k+1} - 5^{k+1}$ M1
 $= 9^k \cdot 9 - 5^k \cdot 5$ m1
 $= 9^k \cdot 8 - 5^k \cdot 4 + (9^k - 5^k)$ A1
 It follows that T_{k+1} is divisible by 4. A1
 It follows by induction that $9^n - 5^n$ is divisible by 4 for all positive integers n .
8. Using reduction to echelon form,

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 3 \\ 0 & 7 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 19 \\ 35 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 3 \\ 0 & 0 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 19 \\ 42 \end{bmatrix}$$
 M1A1A1
 It follows that $z = 3, y = 2$ and $x = 1$. A1
 B1B1B1
9. (a) $\ln y = -x \ln(x)$ M1
 $\frac{1}{y} \frac{dy}{dx} = -\ln(x) - 1$ m1A1
 At the stationary point,
 $\ln(x) = -1$ M1
 so $x = e^{-1}$ (0.368) and $y = (e^{-1})^{-e^{-1}}$ (1.44) A1A1
- (b) Differentiating again,
 $\frac{1}{y} \frac{d^2 y}{dx^2} - \frac{1}{y^2} \left(\frac{dy}{dx} \right)^2 = -\frac{1}{x}$ M1A1A1
 whence the printed result. AG
 At the stationary point, $\frac{dy}{dx} = 0$ so M1
 $\frac{d^2 y}{dx^2} = -\frac{y}{x} < 0$ A1
 It is therefore a maximum. A1
10. $u + iv = (x + iy)^2 = x^2 - y^2 + 2xyi$ M1
 $u = x^2 - y^2; v = 2xy$ A1A1
 Substituting $y = x - 1$, M1
 $u = x^2 - (x - 1)^2 = 2x - 1$ A1
 $v = 2x(x - 1)$ A1
 Substituting from first into second, M1
 $v = \frac{(u^2 - 1)}{2}$ A1

MATHEMATICS FP2

1. (i) f is continuous because $f(x)$ passes through zero from both sides around $x = 0$. B1B1
- (ii) For $x \geq 0$, $f'(x) = \cos x = 1$ when $x = 0$ and for $x < 0$, $f'(x) = 1$. M1A1
So f' is continuous. A1

2. Converting to trigonometric form,

$$i = \cos(\pi/2) + i\sin(\pi/2) \quad \text{B2}$$

$$\text{Cube roots} = \cos(\pi/6 + 2n\pi/3) + i\sin(\pi/6 + 2n\pi/3) \quad (\text{si}) \quad \text{M1A1}$$

$$n = 0 \text{ gives } \cos(\pi/6) + i\sin(\pi/6) = \frac{\sqrt{3}}{2} + \frac{1}{2}i \quad \text{M1A1}$$

$$n = 1 \text{ gives } \cos(5\pi/6) + i\sin(5\pi/6) \quad \text{M1}$$

$$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i \quad \text{A1}$$

$$n = 2 \text{ gives } \cos(3\pi/2) + i\sin(3\pi/2) = -i \quad \text{A1}$$

[FT on candidate's first line but award a max of 4 marks for the cube roots of 1]

3. (a) $f'(x) = -\frac{1}{x^2(1+x^2)^2} \times (3x^2 + 1) < 0 \text{ for all } x > 0 \quad (\text{cso}) \quad \text{M1A1}$
- (b) f is odd because $f(-x) = -f(x) \quad \text{B2} \quad \text{A1}$
- (c) The asymptotes are $x = 0$ and $y = 0$. B1B1
- (d) Graph G2

4. (a) Completing the square,
 $2\{(x-1)^2 - 1\} - (y+2)^2 + 4 = 4$ M1A1
 $\frac{(x-1)^2}{1} - \frac{(y+2)^2}{2} = 1$ A1
The centre is (1,-2) A1
- (b) In the usual notation, $a = 1$, $b = \sqrt{2}$. M1
 $2 = 1(e^2 - 1)$ A1
 $e = \sqrt{3}$ A1
Foci are $(1 \pm \sqrt{3}, -2)$, Directrices are $x = 1 \pm \frac{1}{\sqrt{3}}$ (FT) B1B1
5. Putting $t = \tan(\theta/2)$ and substituting, M1
 $\frac{3(1-t^2)}{1+t^2} + \frac{4.2t}{1+t^2} = 3 - t$ A1
 $3 - 3t^2 + 8t = 3 + 3t^2 - t - t^3$ A1
 $t^3 - 6t^2 + 9t = 0$ A1
Either $t = 0$ B1
giving $\theta/2 = n\pi$ or $\theta = 2n\pi$ B1
Or $t = 3$ B1
giving $\theta/2 = 1.25 + n\pi$ B1
 $\theta = 2.50 + 2n\pi$ (Accept degrees) B1
6. (a) Putting $n = 1$,
LHS = $\cos\theta + i\sin\theta = \text{RHS}$ so true for $n = 1$. B1
Let the result be true for $n = k$, ie
 $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ M1
Consider
 $(\cos\theta + i\sin\theta)^{k+1} = (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ M1
 $= \cos k\theta \cos\theta - \sin k\theta \sin\theta + i(\sin k\theta \cos\theta + \cos k\theta \sin\theta)$ A1
 $= \cos(k+1)\theta + i\sin(k+1)\theta$ A1
Thus, true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$,
proof by induction follows. A1
- (b) $\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$ M1
 $= \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$
 $+ 10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$ A1
 $= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta$
 $- 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$ A1

Equating imaginary parts,

$$\begin{aligned} \sin 5\theta &= 5\cos^4 \theta \sin \theta - 10\cos^2 \theta \sin^3 \theta + \sin^5 \theta & \text{M1} \\ &= 5\sin \theta (1 - \sin^2 \theta)^2 - 10\sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta & \text{A1} \\ &= 5\sin \theta - 10\sin^3 \theta + 5\sin^5 \theta - 10\sin^3 \theta + 10\sin^5 \theta + \sin^5 \theta & \text{A1} \\ &= 16\sin^5 \theta - 20\sin^3 \theta + 5\sin \theta & \text{A1} \end{aligned}$$

7. (a) Let $\frac{x}{(x+2)(x^2+4)} \equiv \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$ M1

$$x \equiv A(x^2+4) + (x+2)(Bx+C)$$

$$x = -2 \text{ gives } A = -1/4 \quad \text{A1}$$

$$\text{Coeff of } x^2 \text{ gives } B = 1/4 \quad \text{A1}$$

$$\text{Constant term gives } C = 1/2 \quad \text{A1}$$

(b) Integral = $\int_2^3 \left(-\frac{1}{4(x+2)} + \frac{x}{4(x^2+4)} + \frac{1}{2(x^2+4)} \right) dx$ M1

$$= \left[-\frac{1}{4} \ln(x+2) + \frac{1}{8} \ln(x^2+4) + \frac{1}{4} \arctan\left(\frac{x}{2}\right) \right]_2^3 \quad \text{A1A1A1}$$

$$= -\frac{1}{4} \ln 5 + \frac{1}{8} \ln 13 + \frac{1}{4} \arctan\left(\frac{3}{2}\right) + \frac{1}{4} \ln 4 - \frac{1}{8} \ln 8 - \frac{1}{4} \arctan 1 \quad \text{A1}$$

$$= 0.054 \quad \text{cao} \quad \text{A1}$$

8. (a) The line meets the circle where $x^2 + m^2(x-2)^2 = 1$ M1

$$(1+m^2)x^2 - 4m^2x + 4m^2 - 1 = 0 \quad \text{A1}$$

$$x \text{ coordinate of M} = \frac{\text{Sum of roots}}{2} \quad \text{M1}$$

$$= \frac{2m^2}{1+m^2} \quad \text{AG}$$

Substitute in the equation of the line.

$$y = m \left(\frac{2m^2}{1+m^2} - 2 \right) \quad \text{M1A1}$$

$$= -\frac{2m}{1+m^2} \quad \text{AG}$$

(b) Dividing, $\frac{x}{y} = -m$ or $m = -\frac{x}{y}$ M1A1

Substituting,

$$y = \frac{2x/y}{1+x^2/y^2} \quad \text{M1A1}$$

$$= \frac{2xy}{x^2+y^2} \quad \text{A1}$$

$$\text{whence } x^2 + y^2 - 2x = 0 \quad \text{A1}$$

[Accept alternative forms, e.g.

$$y = \sqrt{x(2-x)} \quad]$$

MATHEMATICS FP3

1. (a) $2 \sinh^2 x + 1 = 2 \left(\frac{e^x - e^{-x}}{2} \right)^2 + 1$ M1

$$= \frac{(e^{2x} - 2 + e^{-2x} + 2)}{2}$$
 A1

$$= \frac{(e^{2x} + e^{-2x})}{2} = \cosh 2x$$
 A1
- (b) Substitute to obtain the quadratic equation M1
 $2 \sinh^2 x - 3 \sinh x + 1 = 0$ A1
 $\sinh x = 1, 1/2$ M1A1
 $x = 0.881, 0.481$ cao A1A1
2. $t = \tan\left(\frac{x}{2}\right) \Rightarrow dx = \frac{2dt}{1+t^2}$ B1
 $(0, \pi/2) \rightarrow (0, 1)$ B1

$$I = \int_0^1 \frac{2dt/(1+t^2)}{1+3(1-t^2)/(1+t^2)}$$
 M1A1

$$= \int_0^1 \frac{dt}{2-t^2}$$
 A1

$$= \frac{1}{2\sqrt{2}} \left[\ln \left(\frac{\sqrt{2}+t}{\sqrt{2}-t} \right) \right]_0^1 \quad \text{or} \quad \left[\frac{1}{\sqrt{2}} \tanh^{-1} \left(\frac{t}{\sqrt{2}} \right) \right]_0^1$$
 M1A1

$$= \frac{1}{2\sqrt{2}} \ln \left(\frac{\sqrt{2}+1}{\sqrt{2}-1} \right) \quad \text{or} \quad \frac{1}{\sqrt{2}} \tanh^{-1} \left(\frac{1}{\sqrt{2}} \right) (0.623) \quad \text{cao}$$
 A1
3. (a) $f(0) = 0$ si B1
 $f'(x) = \frac{1}{\sec x} \times \sec x \tan x = \tan x, \quad f'(0) = 0$ si B1B1
 $f''(x) = \sec^2 x, \quad f''(0) = 1$ si B1
 $f'''(x) = 2 \sec^2 x \tan x, \quad f'''(0) = 0$ si B1B1
 $f^{(4)}(x) = 4 \sec^2 x \tan^2 x + 2 \sec^4 x, \quad f^{(4)}(0) = 2$ si B1B1
- [This expression has several similar looking forms, eg $6 \sec^2 x \tan^2 x + 2 \sec^2 x$]
 The series is

$$f(x) = \frac{x^2}{2} + \frac{x^4}{12} + \dots$$
 B1
- (b) Substituting the series gives
 $x^4 + 126x^2 - 12 = 0$ M1
 Solving,
 $x^2 = 0.0952,$ M1A1
 $x = 0.3085$ A1

4. (a) $\frac{dx}{d\theta} = 1 + \cos \theta, \frac{dy}{d\theta} = -\sin \theta$ B1B1
- $$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 1 + 2\cos \theta + \cos^2 \theta + \sin^2 \theta$$
- M1A1
- $$= 2(1 + \cos$$
- A1
- $$= 4\cos^2\left(\frac{\theta}{2}\right)$$
- AG
- (b) Arc length $= \int_0^\pi 2\cos\left(\frac{\theta}{2}\right)d\theta$ M1A1
- $$= 4\left[\sin\left(\frac{\theta}{2}\right)\right]_0^\pi$$
- A1
- $$= 4$$
- A1
- (c) CSA $= 2\pi \int_0^\pi (1 + \cos \theta) \cdot 2\cos\left(\frac{\theta}{2}\right)d\theta$ M1A1
- $$= 2\pi \int_0^\pi 2\cos^2\left(\frac{\theta}{2}\right) \cdot 2\cos\left(\frac{\theta}{2}\right)d\theta \quad \text{or} \quad 4\pi \int_0^\pi \cos\left(\frac{\theta}{2}\right)d\theta + 4\pi \int_0^\pi \cos \theta \cos\left(\frac{\theta}{2}\right)d\theta$$
- A1
- $$= 8\pi \int_0^\pi \cos^3\left(\frac{\theta}{2}\right)d\theta \quad \text{or} \quad 4\pi \int_0^\pi \cos\left(\frac{\theta}{2}\right) + 2\pi \int_0^\pi \left(\cos\left(\frac{3\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right)\right)d\theta$$
- A1
- $$= 16\pi \int_0^\pi \left\{1 - \sin^2\left(\frac{\theta}{2}\right)\right\} d\sin\left(\frac{\theta}{2}\right) \quad \text{or} \quad 6\pi \int_0^\pi \cos\left(\frac{\theta}{2}\right)d\theta + 2\pi \int_0^\pi \cos\left(\frac{3\theta}{2}\right)d\theta$$
- M1A1
- $$= 16\pi \left[\sin\left(\frac{\theta}{2}\right) - \frac{1}{3}\sin^3\left(\frac{\theta}{2}\right)\right]_0^\pi \quad \text{or} \quad 12\pi \left[\sin\left(\frac{\theta}{2}\right)d\theta\right]_0^\pi + \frac{4\pi}{3} \left[\sin\left(\frac{3\theta}{2}\right)\right]_0^\pi$$
- A1
- $$= \frac{32}{3}\pi \quad (33.5)$$
- A1
5. (a) $I_n = -\int_0^\pi \theta^n d\cos \theta$ M1
- $$= \left[-\theta^n \cos \theta\right]_0^\pi + \int_0^\pi \cos \theta \cdot n\theta^{n-1}d\theta$$
- A1A1
- $$= \pi^n + n \int_0^\pi \theta^{n-1} \cos \theta d\theta$$
- A1
- $$= \pi^n + n \int_0^\pi \theta^{n-1} d\sin \theta$$
- M1
- $$= \pi^n + n \left[\theta^{n-1} \sin \theta\right]_0^\pi - n(n-1) \int_0^\pi \theta^{n-2} \sin \theta d\theta$$
- A1A1
- $$= \pi^n - n(n-1)I_{n-2}$$
- A1

(b)	$I_4 = \pi^4 - 12I_2$	B1
	$= \pi^4 - 12(\pi^2 - 2I_0)$	B1
	$= \pi^4 - 12\pi^2 + 24 \int_0^\pi \sin \theta d\theta$	M1
	$= \pi^4 - 12\pi^2 + 24[-\cos \theta]_0^\pi$	A1
	$= \pi^4 - 12\pi^2 + 48$	A1

6.	(a)	$\text{Area} = \frac{1}{2} \int_0^{\pi/2} \sinh^2 \theta d\theta$ $= \frac{1}{4} \int_0^{\pi/2} (\cosh 2\theta - 1) d\theta$ $= \frac{1}{4} \left[\frac{\sinh 2\theta}{2} - \theta \right]_0^{\pi/2}$ $= \frac{1}{4} \left(\frac{\sinh \pi}{2} - \frac{\pi}{2} \right) \quad (1.05)$	M1A1 A1 A1
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(b)	(i)	Consider	
		$x = r \cos \theta = \sinh \theta \cos \theta$	M1A1
		$\frac{dx}{d\theta} = \cosh \theta \cos \theta - \sinh \theta \sin \theta$	M1A1

At P,		
	$\cosh \theta \cos \theta = \sinh \theta \sin \theta$	M1
	so $\tanh \theta = \cot \theta$	A1

(ii)	The Newton-Raphson iteration is	
	$x \rightarrow x - \frac{(\tanh \theta - \cot \theta)}{(\sec^2 \theta + \operatorname{cosec}^2 \theta)}$	M1A1
	Starting with $x_0 = 1$, we obtain	M1
	$x_1 = 0.9348$	A1

MS3
£3.00

WELSH JOINT EDUCATION COMMITTEE
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MARKING SCHEMES

SUMMER 2006

MATHEMATICS
M1-M3 and S1-S3

WJEC
CBAC

INTRODUCTION

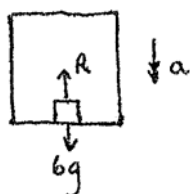
The marking schemes which follow were those used by the WJEC for the 2006 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS M1

1.



N2L applied to object
 $6g - R = 6a$

o.e.

M1
 A1

Accelerating $a = 3$

$$R = 6g - 6 \times 3$$

$$= \underline{40.8\text{N}}$$

c.a.o.

A1

Constant speed $a = 0$

$$R = 6g$$

$$= \underline{58.8\text{N}}$$

B1

Decelerating $a = -2$

$$R = 6 + 6 \times 2$$

$$= \underline{70.8\text{N}}$$

c.a.o.

A1

2.

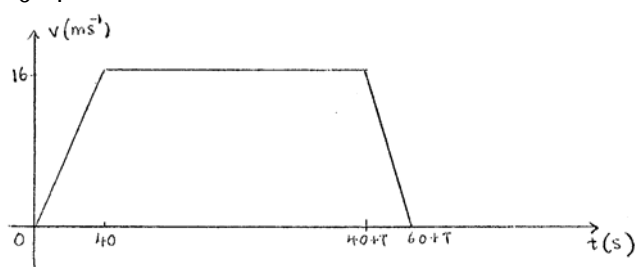
(a) Use of $v = u + at$ with $u = 0$, $a = 0.4$, $v = 1.5$

M1

$$t = \frac{16}{0.4} = \underline{40\text{s}}$$

A1

(b)



B1
 B1
 B1
 B1

(c) $\frac{1}{2} \times 40 \times 16 + 16T + \frac{1}{2} \times 20 \times 16 = 2400$

M1, B1

$$16T + 4800 = 2400$$

f.t.(a)

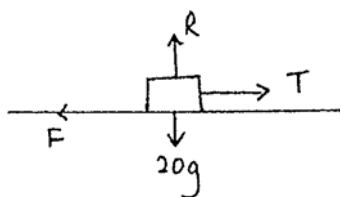
A1

$$T = \underline{120\text{s}}$$

c.a.o.

A1

3.



(a) $R = 20g$

B1

$$F = \text{limiting friction} = \mu R$$

$$= 0.3 \times 20g$$

$$= 58.8\text{N}$$

used

M1

N2L

dim. correct

M1 A1

$$T - F = 20a$$

$$a = \frac{65 - 58.8}{20}$$

$$= \underline{0.31 \text{ ms}^{-2}}$$

c.a.o.

A1

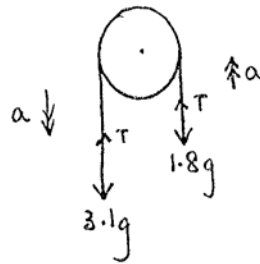
(b) $T < \text{limiting friction}$

$$\therefore F = T$$

$$= 45\text{N}$$

B1

4.



N2L applied to A and B

$$3.1g - T = 3.1a$$

M1
B1

N2L applied to B

$$T - 1.8g = 1.8a$$

A1

Adding

$$3.1g = 1.8g = 4.9a$$

$$a = \underline{2.6\text{ms}^{-1}}$$

c.a.o.

m1
A1

$$T = 1.8(2.6 + 9.8)$$

$$= \underline{22.32\text{N}}$$

c.a.o.

A1

5.

(a) Using $s = ut + \frac{1}{2}at^2$ with $s = 0$, $u = 22.05$, $a = (\pm) 9.8$

o.e.

M1

$$0 = 22.05t - \frac{1}{2} \times 9.8 t^2$$

A1

$$t = \underline{4.5\text{s}}$$

A1

$$v = \underline{22.05\text{ms}^{-1}}$$

B1

(b) Using $v^2 = u^2 + 2as$ with $v = 0$, $u = 22.05$, $a = (\pm) 9.8$

o.e.

M1

$$0 = 22.05^2 - 2 \times 9.8s$$

A1

$$s = \underline{24.8 (0625)\text{m}}$$

c.a.o.

A1

(c) Using $v = u + at$ with $u = 22.05$, $a = (\pm) 9.8$, $t = 3$

o.e.

M1

$$v = 22.05 - 9.8 \times 3$$

f.t. (a) if used

A1

$$v = -7.35$$

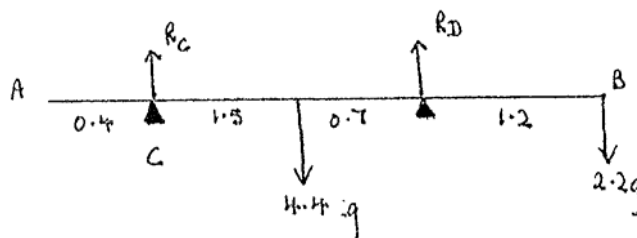
$$\text{Speed} = \underline{7.35\text{ms}^{-1}}$$

A1

Direction is downwards

B1

6.



Moments about C

all forces dim. correct

M1

$$4.4g \times 1.5 + 2.2g \times 3.4 = R_D \times 2.2$$

$$R_D = \underline{62.72\text{N}} \quad (6.4g)$$

c.a.o.

A2 B1
A1

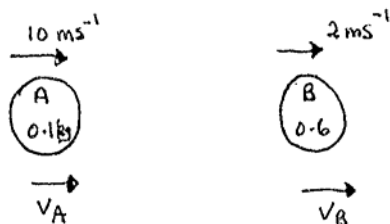
$$\text{Resolve } \uparrow R_C + R_D = 4.4g + 2.2g$$

$$R_C = 0.2g = \underline{1.96\text{N}}$$

f.t. R_D

M1
A1

7. (a)



$$e = \frac{3}{4}$$

Conservation of momentum

used

M1

$$0.1 \times 10 + 0.6 \times 2 = 0.1 V_A + 0.6 V_B$$

$$V_A + 6V_B = 22$$

A1

Restitution

used

M1

$$V_B - V_A = -\frac{3}{4} (2 - 10)$$

$$-V_A + V_B = 6$$

A1

Adding $7V_B = 28$

dep. on both Ms

M1

$$V_B = 4 \text{ ms}^{-1}$$

c.a.o.

A1

$$V_A = -2 \text{ ms}^{-1}$$

c.a.o.

A1

(b) After B collide with wall, it is moving with speed V_B' towards A

$$V_B' = \frac{1}{4} \times 4 = 1 \text{ ms}^{-1}$$

f.t.

B1

Since $|V_B'| = 1$ and $|V_A'| = 2$, B will not catch up with A

B1

(c) $I = 0.6 (4 + 1)$
 $= 3 \text{ NS}$

f.t. V_B, V_B'

M1

A1

8. (a)

	Area	dist. from AB	dist. from AE	
ABDE	60	3	5	B1
BCD	18	4	12	B1 B1
Lamina	78	x	y	B1 (areas)

Moments about AB

$$60 \times 3 + 18 \times 4 = 78x$$

f.t. cand's values

M1 A1

$$x = \frac{43}{13}$$

c.a.o.

A1

Moments about AE

$$60 \times 5 + 18 \times 12 = 78y$$

f.t. cand's values

M1 A1

$$y = \frac{86}{13}$$

c.a.o.

A1

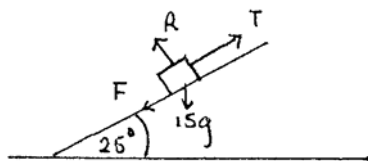
(b) $AX = \frac{43}{13} \text{ cm}$

f.t. x

B1

9.

$$\mu = 0.4$$



$\perp$ to plane	$R = 15g \cos 25^\circ$		B1
------------------	-------------------------	--	----

Limiting friction	$F = \mu R$	used si	M1
	$= 0.4 \times 15g \cos 25^\circ$		M1
	$= 53.2909 \text{ N}$		

Max T when body on point of moving up plane

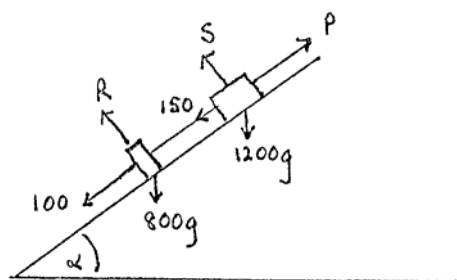
$T = 15g \sin \alpha + F$	f.t. F	M1 A1
$T = 115.42 \text{ N}$	c.a.o.	A1

Min T when body on point of moving down plane

$T = 15g \sin \alpha - F$	f.t. F	M1 A1
$T = 8.83 \text{ N}$	c.a.o.	A1

MATHEMATICS M2

1.



$$\sin \alpha = \frac{1}{28}$$

(a) $P = \frac{45 \times 1000}{v}$ M1 A1

N2L to whole system M1

$$\frac{45000}{25} - 2000g \sin \alpha - 150 - 100 = 2000 a$$
 A2

$$1800 - 700 - 250 = 2000a$$

$$a = \underline{0.425 \text{ ms}^{-2}}$$
 A1

(b) N2L applied to trailer

$$T - 800 \times 9.8 \times \frac{1}{28} - 100 = 800 \times 0.425$$
 A2

$$T = \underline{720\text{N}}$$
 A1

2. (a) $\mathbf{r}_A = (\mathbf{i} - 10\mathbf{k}) + t(-2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k})$ B1

$$= (1 - 2t)\mathbf{i} + (-2t)\mathbf{j} + (-10 - 5t)\mathbf{k}$$

$$\mathbf{r}_B = (7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}) + t(\mathbf{i} - 8\mathbf{j} - 5\mathbf{k})$$
 B1

$$= (7 + t)\mathbf{i} + (9 - 8t)\mathbf{j} + (-6 - 5t)\mathbf{k}$$

(b) When $t = 2$

$$\mathbf{r}_A - \mathbf{r}_B = (-3 - 9)\mathbf{i} + (-4 + 7)\mathbf{j} + (-20 + 16)\mathbf{k}$$
 M1

$$= -12\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$$

Distance between A and B $= \sqrt{12^2 + 3^2 + 4^2}$ m1

$$= 13$$
 A1

3.	(a)	$s = \int (12t - 3t^2) dt$		M1
		$= 6t^2 - t^3 (+C) \text{ m}$		A1

When $t = 1, s = 0$

$\therefore 0 = 6 - 1 + C$	o.e.	M1
$C = -5$		

$\therefore s = 6t^2 - t^3 - 5 \text{ m}$	A1
---	----

(b)	$a = \frac{dv}{dt}$	attempted	M1
	$a = \underline{12 - 6t}$		

(c)	Power	= F.v.used	M1
	F	= ma	
		= $3(12 - 9)$	
		= 9	B1

Power	= $9 \times (12 \times 1.5 - 3 \times 1.5^2)$	
	= 9×11.25	
	= <u>101.25W</u>	f.t. F A1

4.	Initial energy	= PE = mgh	used	M1
		= $3 \times 9.8 \times 1.2$		
		= 35.28J	si	A1

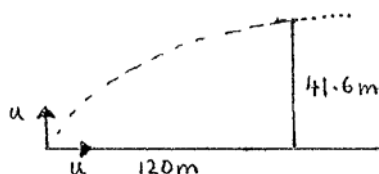
Final EE	= $\frac{1}{2} \times \frac{\lambda(1.2 - 0.8)^2}{0.8}$	M1
	= $\frac{1}{2} \times \frac{35.4 \times 0.4^2}{0.8}$	
	= 3.54 J	A1

Conservation of energy

$1.5v^2 + 3.54 = 35.28$	M1 A1
-------------------------	-------

$v = \underline{4.6 \text{ ms}^{-1}}$	A1
---------------------------------------	----

5. (a)



Let u be initial horizontal and vertical velocities ($v \sin/\cos 45$) o.e.

B1

t be time to hit target.

Horizontal motion $ut = 120$

M1 A1

Vertical motion

Using $s = ut + \frac{1}{2}at^2$ with $s = 41.6$, $u = u$ (c), $a = (\pm) 9.8$

M1

$$41.6 = 120 + \frac{1}{2} \times (-9.8) \left(\frac{120}{u} \right)^2$$

m1 (subst.)

$$u = 30 \text{ ms}^{-1}$$

A1

$$\begin{aligned} \text{Speed of projection} &= \sqrt{2u^2} \\ &= 30\sqrt{2} = \underline{42.4264 \text{ ms}^{-1}} \\ t &= \frac{120}{u} \\ &= \underline{4s} \end{aligned}$$

B1

convincing

A1

(b) Using $v = u + at$ with $u = 30$ (c), $a = (\pm) 9.8$, $t = 4$

M1

$$\begin{aligned} v &= 30 - 9.8 \times 4 \\ &= -9.2 \end{aligned}$$

f.t. u

A1

f.t. u

A1

$$\text{Resultant speed} = \sqrt{(-9.2)^2 + 30^2} \text{ (c)}$$

M1

$$= \underline{31.38 \text{ ms}^{-1}}$$

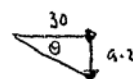
f.t. u

A1

$$\text{Direction of motion} = \tan^{-1} \left(\frac{9.2}{30(c)} \right)$$

M1

$$= \underline{17.05^\circ}$$



[No if +9.2 used]
i.e. angle above horizontal

A1

6. (a) $\mathbf{r} = \cos 3t \mathbf{i} + \sin 3t \mathbf{j}$

$$\mathbf{v} = \frac{d}{dt}(\mathbf{r}) \quad \text{used} \quad \text{M1}$$

$$= -3 \sin 3t \mathbf{i} + 3 \cos 3t \mathbf{j} \quad \text{A2}$$

(b) Consider $\mathbf{v} \cdot \mathbf{r} = (-3 \sin 3t \mathbf{i} + 3 \cos 3t \mathbf{j}) \cdot (\cos 3t \mathbf{i} + \sin 3t \mathbf{j})$ M1

$$= -3 \sin 3t \cos 3t + 3 \sin 3t \cos 3t \quad \text{dot product} \quad \text{B1}$$

$$= 0$$

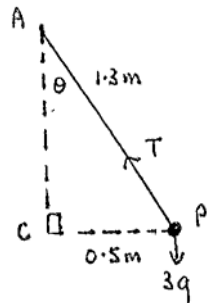
$\therefore \mathbf{v}$ is perpendicular to $\mathbf{r}$ for all values of t A1

(c) Speed of P = $|\mathbf{v}|$ si M1

$$= \sqrt{(-3 \sin 3t)^2 + (3 \cos 3t)^2} \quad \text{M1}$$

$$= \sqrt{9(\sin^2 3t + \cos^2 3t)}$$

$$= \underline{3} \quad \text{c.a.o.} \quad \text{A1}$$

7. (a)  Resolve $\uparrow$ M1

$$T \cos \theta = mg \quad \text{A1}$$

$$T = 3 \times 9.8 \times \frac{1.3}{1.2}$$

$$= \underline{31.85\text{N}} \quad \text{c.a.o.} \quad \text{A1}$$

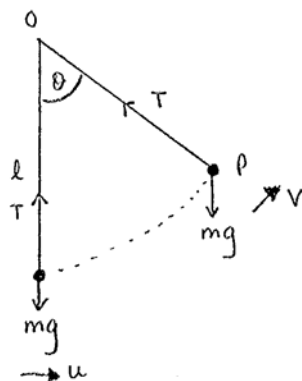
(b) N2L towards C

$$T \sin \theta = m r \omega^2 \quad \text{M1}$$

$$\omega^2 = 31.85 \times \frac{0.5}{1.3} \times \frac{1}{3 \times 0.5} \quad \text{A1 B1 } (a = r \omega^2)$$

$$\omega = \underline{2.86} \text{ rads}^{-1} \quad \text{f.t. T} \quad \text{A1}$$

8.



(a) At lowest point

$$T - mg = \frac{mv^2}{r}$$

M1 (ma) A1

$$T = 2mg, v = u, r = l \quad mg = \frac{mu^2}{l}$$

m1

$$u^2 = gl$$

$$u = \sqrt{gl}$$

convincing

A1

(b) Conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 = mgl(1 - \cos \theta)$$

M1

A1 (KE)

A1 (all correct)

$$v^2 = gl - 2gl(1 - \cos \theta)$$

$$= 2gl \cos \theta - gl$$

(c) At max θ , $v^2 = 0$

used

M1

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi^c}{3} = 60^\circ$$

c.a.o.

A1

(d) At general θ

$$T - mg \cos \theta = \frac{mv^2}{l}$$

M1(ma)

$$M1 \left(\frac{mv^2}{r} \right)$$

$$\text{Subst. for } v^2 \text{ and } r = l \quad T = \frac{m}{l} 2gl \cos \theta + mg \cos \theta - \frac{m}{l} gl$$

m1

$$= 3mg \cos \theta - mg$$

$$T = mg \quad \cos = \frac{2}{3}$$

$$\theta = \underline{48.2^\circ, 0.84^\circ}$$

A1

MATHEMATICS M3

1.	(a)	$a = v \frac{dv}{dx}$	used	M1
		$= \left(\frac{B}{x+A} \right) \left[-B(x+A)^{-2} \right]$		A1
		$= \frac{-B^2}{(x+A)^3}$		A1
	(b)	$t = 0, v = 12, x = 0$		M1
		$\therefore B = 12A$		
		$t = 0, a = -16, x = 0$		
		$-B^2 = -16A^3$		A1
		$144A^2 = 16A^3$		
		$A = 9$		
		$B = 108$	as required	
			convincing	A1
	(c)	$v = \frac{dx}{dt} = \frac{108}{x+9}$		M1
		$\int (x+9)dx = 108 \int dt$		
		$\frac{x^2}{2} + 9x = 108t + C$		A1
		$t = 0, x = 0 \Rightarrow C = 0$	f.t. minor error	A1
		$\therefore 216t = x^2 + 18x$		
		$t = \frac{1}{216}x(x+18)$		A1
2.	Auxiliary equation $m^2 + 2m + 10 = 0$			B1
		$m = \frac{-2 \pm \sqrt{4-40}}{2}$		
		$= -1 \pm 3i$		B1
		$\therefore$ C.F. - is $x = e^{-t} (A \sin 3t + B \cos 3t)$		B1
	For P.I. try $x = at + b$			M1
		$\frac{dx}{dt} = a$		
		$\therefore 2a + 10(at + b) = 5t - 14$		A1
		$10a = 5$	comp. coeff.	M1
		$a = \frac{1}{2}$		

$$2a + 10b = -14$$

$$b = -\frac{3}{2}$$

both c.a.o. A1

$$\therefore \text{General solution is } x = e^{-t} (A \sin 3t + B \cos 3t) + \frac{1}{2}t - \frac{3}{2}$$

B1

$$\text{When } t = 0, x = 4\frac{1}{2}, \frac{dx}{dt} = 3\frac{1}{2}$$

used M1

$$4\frac{1}{2} = B - \frac{3}{2}$$

$$B = 6$$

f.t. a.b. A1

$$\frac{dx}{dt} = -e^{-t} (A \sin 3t + B \cos 3t) + e^{-t} (3A \cos 3t - 3B \sin 3t) + \frac{1}{2}$$

B1

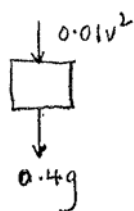
$$3\frac{1}{2} = -B + 3A + \frac{1}{2}$$

$$A = 3$$

c.a.o. A1

$$\therefore x = 3e^{-t} (\sin 3t + 2 \cos 3t) + \frac{1}{2}t - \frac{3}{2}$$

3.



$$(a) \quad \text{N2L} \quad -0.01v^2 - 0.4g = 0.4a$$

M1

$$0.4 v \frac{dv}{dx} = -3.92 - 0.01 v^2$$

$$a = v \frac{dv}{dx} \quad \text{A1}$$

$$\times 100 \quad 40 v \frac{dv}{dx} = -(392 + v^2)$$

convincing A1

$$(b) \quad 40 \int \frac{v dv}{(392 + v^2)} = - \int dx$$

sep. var. M1

$$20 \ln (392 + v^2) = -x + C$$

A1 A1

$$t = 0, v = 17, x = 0$$

m1

$$\therefore 20 \ln (392 + 17^2) = C$$

$$C = 20 \ln (681)$$

f.t. minor error A1

$$x = 20 \ln (681) - 20 \ln (392 + v^2)$$

$$= 20 \ln \left(\frac{681}{392 + v^2} \right)$$

$$\text{At greatest height, } v = 0$$

m1

$$\therefore x = 20 \ln \left(\frac{681}{392} \right)$$

$$= \underline{11.05 \text{ m}}$$

c.a.o. A1

$$(c) \quad \text{Speed of ball when it returns to } O \text{ is less than } 17 \text{ ms}^{-1}$$

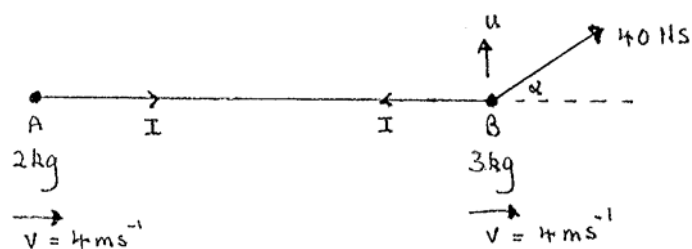
B1

because energy used (lost) in overcoming air resistance.

B1

4.	(a)	Period = $\frac{2\pi}{\omega} = 4$		M1
		$\omega = \frac{\pi}{2}$		A1
		Using $v_{\text{MAX}} = a\omega$ with $v_{\text{MAX}} = 3\pi$, $\omega = \frac{\pi}{2}$		M1
		$3\pi = a \times \frac{\pi}{2}$		
		$a = \underline{6 \text{ m}}$	c.a.o.	A1
	(b)	Using $v^2 = \omega^2 (a^2 - x^2)$ with $\omega = \frac{\pi}{2}$ (c), $a = 6$ (c), $x = 4.8$		M1
		$v^2 = \frac{\pi^2}{4} (36 - 4.8^2)$	f.t. a, ω	A1
		$v = \underline{1.8\pi}$		
		$= \underline{5.65 \text{ ms}^{-1}}$	f.t. a, ω	A1
	(c)	Let x = distance from O , $y = 0$, p is at O		
		$x = 6 \sin\left(\frac{\pi}{2}t\right)$	f.t. a, ω	B1
		$4.8 = 6 \sin\left(\frac{\pi}{2}t\right)$		M1
		$\sin\left(\frac{\pi}{2}t\right) = \frac{4.8}{6} = 0.8$		
		$t = \frac{2}{\pi} \sin^{-1}(0.8)$		
		$= \underline{0.59\text{s}}$	f.t. a, ω	A1
	(d)	Max acceleration when $x = a$		M1
		Max acceleration = $\omega^2 a$		M1
		$= \frac{\pi^2}{4} \times 6$		
		$= \frac{3\pi^2}{2}$		
		$= \underline{14.8 \text{ ms}^{-1}}$	f.t. a, ω	A1
	(e)	Distance travelled = $\frac{12}{4}$ oscillations		
		$= 3 \times (4a)$		M1
		$= \underline{72\text{m}}$	f.t. a	A1

5. (a)



Impulse = change in momentum

Apply to A $I = 2v$ M1
 $= 2 \times 4$ A1

Apply to B $-I = -40 \cos \alpha + 3v$ M1
 $= -40 \cos \alpha + 3 \times 4$ A1
 $\therefore -8 = -40 \cos \alpha + 12$ M1

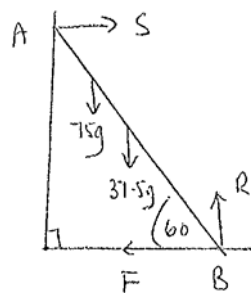
$40 \cos \alpha = 20$
 $\cos \alpha = \frac{1}{2}$
 $\alpha = \underline{60^\circ}$ A1

(b) $40 \sin \alpha = 3u$
 $u = 40 \times \frac{\sqrt{3}}{2} \times \frac{1}{3}$
 $= \frac{20\sqrt{3}}{3}$ A1

Speed of b $= \sqrt{\left(\frac{20\sqrt{3}}{3}\right)^2 + 4^2}$ M1
 $= \underline{12.22 \text{ ms}^{-1}}$ A1

$\theta = \tan^{-1}\left(\frac{20\sqrt{3}}{3 \times 4}\right)$ M1
 $= \underline{70.89^\circ}$ A1

6. (a)



Moments about B

dim correct, attempted equation M1

$$37.5g \times 4 \cos 60^\circ + 75g \times x \cos 60^\circ = S \times 8 \sin 60^\circ$$

B1 A2

$$\begin{aligned} \text{Resolve } \uparrow \quad R &= 37.5g + 75g \\ &= 112.5g \end{aligned}$$

M1

$$\begin{aligned} \text{Resolved } \rightarrow S &= F \\ F &= \mu R \\ \therefore S &= \mu 112.5g \end{aligned}$$

M1

M1

Substitute S into moment equation and $\mu = 0.25$

m1

$$\begin{aligned} x(75g \cos 60^\circ) &= 0.25 \times 112.5g \times 8 \sin 60^\circ - 37.5g \times 4 \cos 60^\circ \\ x &= \frac{112.5\sqrt{3} - 75}{37.5} \end{aligned}$$

A1

$$= \underline{3.196}$$

c.a.o.

A1

(b) Substitute $x = 8$ and $s = \mu 112.5g$ (c) into moment equation

M2

$$\mu 112.5g \times 8 \sin 60^\circ = 37.5g \times 4 \cos 60^\circ + 75g \times 8 \cos 60^\circ$$

A1

$$\mu = \frac{300 + 75}{450\sqrt{3}}$$

$$= \underline{0.481}$$

c.a.o.

A1

(c) Person modelled as particle/
Ladder modelled as rod

B1

MATHEMATICS S1

1. (a) $P(\text{A eats red sweet}) = \frac{1}{3}$ B1
- (b) $P(\text{B eats red sweet}) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ M1A1
- [Method must be shown, award M1 for either $\frac{2}{3} \times \frac{1}{2}$ or $\frac{2}{3} \times \frac{1}{3}$]
- (c) $P(\text{C eats red sweet}) = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{1}$ or $1 - \frac{2}{3}$ M1
- $= \frac{1}{3}$ (no working required) A1
- [FT on answers to (a) and (b) - Special case – award 3/5 for writing down the correct answers with no working]
2. (a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
 $= 0.05$ M1A1
- $P(A), P(B) = 0.12$ or $P(A|B) = 1/12$ or $P(B|A) = 1/4$ B1
 A and B are not independent B1
- (b) $P(\text{exactly one of A,B}) = P(A \cap B') + P(A' \cap B)$ M1
- [Award M0 if independence assumed having stated not independent in (a).
 If independence stated in (a), FT partially bearing in mind that the problem is now easier]
- $= 0.2 - 0.05 + 0.6 - 0.05$ A1A1
 $= 0.7$ A1
- OR
- $P(\text{exactly one of A,B}) = P(A \cup B) - P(A \cap B)$ M1A1
 $= 0.75 - 0.5$ A1
 $= 0.25$ A1
3. (a) $\text{Var}(X) = 4$ B1
 $E(Y) = 2 \times 4 + 8 = 16$ M1A1
 $\text{Var}(Y) = 4 \times 4 = 16$ M1A1
- (b) Because Y can only take the values 8,10,12 etc B1

4.	(a)	(i)	$P(X > 10) = 0.6528$ (or $1 - 0.3472$)	M1A1
		(ii)	$P(X = 15) = 0.8444 - 0.7720$ or $0.2280 - 0.1556$ $= 0.0724$ (cao)	B1B1 B1
	(b)	(i)	$P(Y = 5) = e^{-6.3} \cdot \frac{6 \cdot 3^5}{5!} = 0.152$	M1A1
		(ii)	$P(Y < 3) = e^{-6.3} \left(1 + 6 \cdot 3 + \frac{6 \cdot 3^2}{2} \right)$ $= 0.0498$ (cao)	M1A1 A1
5.	(a)		$P(DY) = 0.4 \times 0.04 + 0.35 \times 0.05 + 0.25 \times 0.06$ $= 0.0485$	M1A1 A1
	(b)	(i)	$P(A DY) = \frac{0.4 \times 0.04}{0.0485}$ $= 0.330$	B1B1 B1
		(ii)	$P(B DY) = \frac{0.35 \times 0.05}{0.0485}$ $= 0.361$	B1
			$P(C DY) = 1 - 0.691 = 0.309$ (si) So most likely to have come from Farm B.	B1 B1
6.	(a)	(i)	$B(50, 0.2)$	B1
		(ii)	Mean $= 50 \times 0.2 = 10$ Var $= 50 \times 0.2 \times 0.8$ SD $= 2.83$ ($2\sqrt{2}$)	M1A1 M1 A1
		(iii)	$P(8 \leq X \leq 12) = 0.8139 - 0.1904$ or $0.8096 - 0.1861$ $= 0.6235$ (cao)	B1B1 B1
			[For candidates summing probs award M1A1 for an expression involving binomial probs and A1 for the answer]	
7.	(b)		X is approx Po(10). $P(X < 10) = 0.4579$ (or $1 - 0.5421$)	M1A1 M1A1
		(a)	Sum of probabilities $= 15k$ $15k = 1$ so $k = \frac{1}{15}$	B1 B1
	(b)		$E(X) = 1 \times \frac{1}{15} + 2 \times \frac{2}{15} + \dots + 5 \times \frac{5}{15}$ $= \frac{11}{3}$	M1 A1
			$E(X^2) = 1 \times \frac{1}{15} + 4 \times \frac{2}{15} + \dots + 25 \times \frac{5}{15}$ (= 15)	M1A1
			$\text{Var}(X) = 15 - \left(\frac{11}{3} \right)^2$ $= 1.56$	M1 A1

(c) The possibilities are (1,5), (2,4),(3,3).

$$P(Y=6) = 2 \times \frac{1}{15} \times \frac{5}{15} + 2 \times \frac{2}{15} \times \frac{4}{15} + \left(\frac{3}{15}\right)^2$$

B1B1B1

$$= \frac{7}{45}$$

B1

8. (a) (i) $P(0.25 \leq X \leq 0.5) = F(0.5) - F(0.25)$

M1

$$= \frac{1}{2}(0.5^2 + 0.5) - \frac{1}{2}(0.25^2 + 0.25)$$

A1

$$= \frac{7}{32} \quad (0.219)$$

A1

(ii) The median satisfies

$$\frac{1}{2}(m^2 + m) = \frac{1}{2}$$

M1

$$m^2 + m - 1 = 0$$

A1

$$m = \frac{-1 + \sqrt{5}}{2}$$

M1

$$= 0.618$$

A1

[Special case for candidates integrating $F(x)$:-

For obtaining $2m^3 + 3m^2 - 6 = 0$

M1A1]

(b) (i) $f(x) = F'(x)$

M1

$$= \frac{1}{2}(2x+1)$$

A1

(ii) $E(X) = \frac{1}{2} \int_0^1 x(2x+1)dx$

M1A1

[FT on candidate's $f(x)$ from (i). If candidates use $F(x)$ instead of $f(x)$, not having obtained an answer in (i), award M1]

$$= \frac{1}{2} \left[\frac{2x^3}{3} + \frac{x^2}{2} \right]_0^1$$

A1

$$= \frac{7}{12} \quad (0.583)$$

A1

MATHEMATICS S2

1. $\bar{x} = \frac{62 \cdot 6}{10} (= 6 \cdot 26)$ B1
- SE of $\bar{x} = \frac{0 \cdot 1}{\sqrt{10}} (= 0 \cdot 0316)$ B1
- 95% conf limits are
 $6 \cdot 26 \pm 1 \cdot 96 \times 0 \cdot 0316$ M1A1
- [M1 correct form, A1 1·96]
giving [6·20, 6·32] A1
Yes because 6·3 is within the interval. B1
2. Variance = $\frac{(b-a)^2}{12} = 3$ M1A1
- $(b-a)^2 = 36$ A1
 $b-a = 6$ AG
- Mean = $\frac{a+b}{2} = 10$ M1
 $b+a = 20$ A1
- Solving, $a = 7, b = 13$. M1A1
3. (a) (i) $z_1 = \frac{34-30}{2} = 2; z_2 = \frac{28-30}{2} = -1$ M1A1
- Prob = $0 \cdot 97725 - 0 \cdot 15866$ or $0 \cdot 8413 - 0 \cdot 02275$ B1B1
 $= 0 \cdot 819$ (cao) B1
- (ii) Reqd weight = $25 + 2 \cdot 326 \times 1 \cdot 8$ M1A1
 $= 29 \cdot 2$ A1
[M1 for $25 \pm z\sigma$]
- (b) $X - Y$ is $N(5, 7 \cdot 24)$ B1B1
We require $P(X - Y > 0)$
- $z = \frac{5}{\sqrt{7 \cdot 24}} = (\pm) 1 \cdot 86$ M1A1
- Prob = $0 \cdot 969$ (cao) A1
4. (a) Prob of 1 crash on a computer = $0 \cdot 8 \times e^{-0 \cdot 8}$ M1
 $= 0 \cdot 3595$ A1
Prob of 1 crash on each of 5 computers = $0 \cdot 3595^5$ M1
 $= 0 \cdot 006$ A1
- (b) Distribution of Total is $Po(4)$. B1
- $P(\text{Total} = 5) = e^{-4} \cdot \frac{4^5}{5!}$ M1A1
 $= 0 \cdot 156$ (cao) A1

5.	(a)	$H_0 : \mu = 2 \cdot 4$ versus $H_1 : \mu > 2 \cdot 4$ (Accept $\mu = 12$)	B1
		In 5 days, number of passengers Y is Poi(12) under H_0 . [M1A0 for normal approx]	si B1
		$p\text{-value} = P(Y \geq 18) = 0.0630$ We cannot conclude that the mean has increased.	M1 A1 B1
	(b)	Under H_0 the number of passengers in 100 days is $Po(240) \approx N(240, 240)$	B1B1
		$z = \frac{279 \cdot 5 - 240}{\sqrt{240}}$ $= 2.55$	M1A1 A1
		Either $p\text{-value} = 0.00539$ or $CV = 2.326$ [No cc gives $z = 2.58$, $p = 0.00494$, wrong cc gives $z = 2.61$, $p = 0.00453$] We conclude at the 1% level that the mean has increased.	A1 B1
6.	(a)	(i) X is $B(50, p)$ (si) Sig level = $P(X \leq 14 p = 0.4)$ $= 0.0540$ (cao)	B1 M1 A1
		(ii) We require $P(X \geq 15 p = 0.3) = 0.5532$	M1A1
	(b)	Under H_0 , X is now $B(500, 0.4) \approx N(200, 120)$	B1B1
		$z = \frac{185 \cdot 5 - 200}{\sqrt{120}}$ $= -1.32$ $p\text{-value} = 0.0934$	M1A1 A1 A1
		[No cc gives $z = -1.37$, $p = 0.0853$, wrong cc gives $z = -1.41$, $p = 0.0793$] Insufficient evidence to support the agent's belief. (oe).	B1
7.	(a)	$H_0 : \mu_A = \mu_B$ versus $H_1 : \mu_A \neq \mu_B$	B1
	(b)	$\bar{x}_A = \frac{501}{6} = 83.5$ $\bar{x}_B = \frac{489}{6} = 81.5$ The appropriate test statistic is	B1 B1
		$TS = \frac{\bar{x} - \bar{y}}{\sigma \sqrt{\frac{1}{m} + \frac{1}{n}}}$ $= \frac{83.5 - 81.5}{1.5 \sqrt{\frac{1}{6} + \frac{1}{6}}}$ $= 2.31 \text{ (cao)}$	M1 1A1 A1
		Prob from tables = 0.01044 $p\text{-value} = 0.021$	A1 B1
		(i) Accept H_0 (or the fuel consumptions are the same) at 1% SL	B1
		(ii) Accept H_1 (or the fuel consumptions are not the same) at 5% SL	B1

8. (a) The **mean** of a **large** (random) sample from any distribution is (approximately) **normally** distributed. B1
- (b) $E(\bar{X}) = 3.5, \text{Var}(\bar{X}) = \frac{35}{600}$ (si) B1B1
- $$z = \frac{3 - 3.5}{\sqrt{35/600}} = -2.07$$
- M1A1
- Prob = 0.981 A1

MATHEMATICS S3

1. (a) The possible combinations are given in the following table.

Combination
1 1 2
1 1 3
1 1 4
1 2 3
1 2 4
1 3 4
2 3 4

[Accept a table with 123, 124 and 134 repeated]

M1A1

The sampling distribution of the mean is

Mean	4/3	5/3	2	7/3	8/3	3
Prob	0.1	0.1	0.3	0.2	0.2	0.1

(correct means)
(correct probs)

B1
B2

[Award B1 for 4 correct probs]

The sampling distribution of the median is

Median	1	2	3
Prob	0.3	0.4	0.3

(correct medians)
(correct probs)

B1
B2

[For each table award B1 for correct probs with replacement]

2. (a) $\hat{p} = \frac{498}{1200} = 0.415$

B1

(b) $SE \approx \sqrt{\frac{0.415 \times 0.585}{1200}}$
 $= 0.0142...$

M1

A1

(c) Approx 90% confidence limits are
 $0.415 \pm 1.645 \times 0.0142$
giving [0.392, 0.438].

M1A1
A1

(d) We are using a normal approximation to a binomial situation.
The standard error and/or p are estimated and are not exact.

B1
B1

3.	$\bar{x}_A = 1.034; \bar{x}_B = 1.016$ $s_A^2 = 3.474747... \times 10^{-4}$ $s_B^2 = 1.449664... \times 10^{-4}$	B1B1 M1A1 A1
	[Accept division by n giving 3.44.. and 1.44..] $SE = \sqrt{\frac{3.474747... \times 10^{-4}}{100} + \frac{1.449664... \times 10^{-4}}{150}} (= 0.002107)$ 95% conf lims are $1.034 - 1.016 \pm 1.96 \times 0.002107$ giving [0.014, 0.022] (cao)	B1 M1A1 A1
4.	(a) UE of $\mu = 8.54$ $\sum x^2 = 734 \cdot 2,$	B1 B1
	UE of $\sigma^2 = \frac{734 \cdot 2}{9} - \frac{85 \cdot 4^2}{9 \times 10}$ $= 0.542(666..)$	M1 A1
	(b) $H_0 : \mu = 9$ versus $H_1 : \mu \neq 9$ Test statistic $= \frac{8.54 - 9}{\sqrt{0.542666.. / 10}}$ $= -1.97$ DF = 9 Crit value = 2.26	B1 M1A1 A1 B1 B1
	Insufficient evidence to reject the farmer's claim at the 5% level. [Award the final B1 only if t used]	B1
5.	(a) $\Sigma x = 75, \Sigma y = 89.2, \Sigma xy = 1270.5, \Sigma x^2 = 1375$ [B1 1 error]	B2
	(b) $b = \frac{6 \times 1270.5 - 75 \times 89.2}{6 \times 1375 - 75^2}$ $= 0.355$ $a = \frac{89.2 - 75 \times 0.355}{6}$ $= 10.4$	M1A1 A1 M1A1 A1
	[Note $S_{xx} = 437.5, S_{xy} = 155.5$]	
	(c) (i) Est resist $= 10.4 + 0.355 \times 20 = 17.5$ $SE = 0.4 \sqrt{\frac{1}{6} + \frac{(20 - 12.5)^2}{1375 - 75^2 / 6}}$ $= 0.217(343...)$	M1 A1 M1 A1
	(ii) 95% confidence limits are $17.5 \pm 1.96 \times 0.217$ giving [17.1, 18.0] [Accept 17.9 as the upper limit]	M1 A1

$$\begin{aligned}
 (d) \quad \text{Test stat} &= \frac{b - \beta}{\sigma / \sqrt{(\sum x^2 - (\sum x)^2 / n)}} & \text{M1} \\
 &= \frac{0.355... - 0.4}{0.4 / \sqrt{(1375 - 75^2 / 6)}} & \text{A1A1} \\
 &= -2.33 \text{ (Accept 2.34, 2.35)} & \text{A1}
 \end{aligned}$$

EITHER $p\text{-value} = 2 \times 0.0099 = 0.0198$ OR Crit value = 2.576
 The value 0.4 is consistent with his prediction at the 1% level. A1
B1

6. (a) $E(U) = a(\mu + 2 \times 2\mu)$ M1A1
 $= \mu$ if $a = \frac{1}{5}$ A1
 $E(V) = b(2 \times \mu + 2\mu) = \mu$ M1
 if $b = \frac{1}{4}$ A1

(b) $\text{Var}(U) = \frac{1}{25}(\sigma^2 + 4 \times 3\sigma^2) = \frac{13}{25}\sigma^2$ M1A1
 $\text{Var}(V) = \frac{1}{16}(4 \times \sigma^2 + 3\sigma^2) = \frac{7}{16}\sigma^2$ M1A1
 V is the better estimator (since it has the smaller variance). B1

(c) (i) $E(W) = \frac{\mu + k \times 2\mu}{1 + 2k} = \mu$ M1A1
 [AG so must be convincing]
 (ii) $\text{Var}(W) = \frac{\sigma^2 + k^2 \times 3\sigma^2}{(1 + 2k)^2} = \frac{(1 + 3k^2)}{(1 + 2k)^2} \sigma^2$ M1A1
 (iii) $\frac{d}{dk}(\text{Var}(W)) = \frac{6k(1 + 2k)^2 - 4(1 + 2k)(1 + 3k^2)}{(1 + 2k)^4}$ M1A1

For minimum variance,

$$\begin{aligned}
 6k(1 + 2k)^2 &= 4(1 + 2k)(1 + 3k^2) & \text{M1} \\
 3k(1 + 2k) &= 2(1 + 3k^2) & \text{A1} \\
 k &= \frac{2}{3} & \text{A1}
 \end{aligned}$$

[Award full marks if correct answer given with no/partial working]

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