

973/01

MATHEMATICS C1

Pure Mathematics

A.M. MONDAY, 22 May 2006

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A, B, C, D have coordinates $(3, 2), (-4, 3), (5, 6), (4, -1)$, respectively.

(a) Show that the lines AC and BD are perpendicular. [5]

(b) Show that the line AC has equation

$$2x - y - 4 = 0$$

and find the equation of the line BD . [4]

(c) Find the coordinates of E , the point of intersection of AC and BD . [2]

(d) Find the length of AE . [2]

2. Simplify each of the following, expressing your answers in surd form.

(a) $\frac{5 - \sqrt{3}}{\sqrt{3} + 1}$, [4]

(b) $(2 + \sqrt{3})(4 - \sqrt{12})$. [4]

3. The curve C has equation $y = x^2 - 4x + 7$. The point A has coordinates $(1, 4)$.

(a) Find the equation of the tangent to C at A . [4]

(b) Find the equation of the normal to C at the point A . [2]

4. (a) Given that the equation

$$kx^2 - 4x + k - 3 = 0$$

has equal roots, find the values of k . [5]

(b) Express $x^2 + 8x + 2$ in the form $(x + a)^2 + b$. Hence write down the least value of $x^2 + 8x + 2$. [3]

5. The polynomial

$$f(x) \equiv px^3 - x^2 + qx - 6$$

has $x - 3$ as a factor. When $f(x)$ is divided by $x - 2$, the remainder is -20 .

(a) Show that $p = 2$ and find the value of q . [6]

(b) Factorise $f(x)$. [3]

6. (a) Expand $(a + b)^4$. Hence expand $\left(3x - \frac{1}{3x}\right)^4$, simplifying each term of the expansion. [4]
 (b) The coefficient of x^2 in the expansion of $(1 + 2x)^n$ is 40. Given that n is a positive integer, find the value of n . [2]

7. (a) Given that $y = x^2 - 3x + 4$, show from first principles that

$$\frac{dy}{dx} = 2x - 3. \quad [5]$$

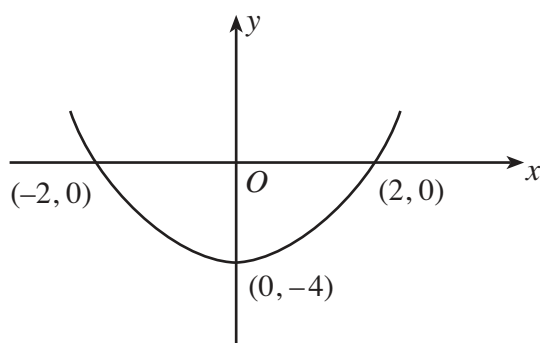
- (b) Differentiate $\frac{2}{x^2} + 7\sqrt{x}$ with respect to x . [2]

8. Solve the following inequalities.

(a) $1 - 5x < x + 8$ [2]

(b) $(x + 8)(x + 1) < 3x$ [4]

9.



The diagram shows the graph of $y = f(x)$. The curve passes through the points $(2, 0)$ and $(-2, 0)$, and has a minimum point at $(0, -4)$.

Sketch on separate diagrams the graphs of

(a) $y = f(x) + 4$, [2]

(b) $y = f(x + 2)$, [3]

indicating the coordinates of the points of intersection with the x -axis and the coordinates of the stationary points.

10. The curve C has equation

$$y = x^3 - 3x^2 - 9x + 2.$$

Find the coordinates of the stationary points of C and determine the nature of each of these stationary points. [7]