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MATHEMATICS C4

Pure Mathematics

P.M. MONDAY, 12 June 2006

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$f(x) = \frac{2x^2 + 4}{(x-2)^2(x+4)},$$

(a) express $f(x)$ in partial fractions, [4]

(b) hence find the value of $f'(0)$. [3]

2. Find the equation of the normal to the curve

$$2x^3 + 6xy^2 - y^4 = 27$$

at the point (2, 1). [5]

3. Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

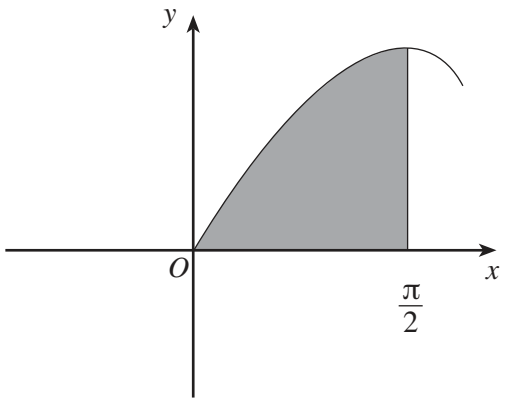
$$2 + 3\cos 2\theta = \cos\theta.$$ [6]

4. *(a)* Express $4\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Hence find the greatest value of

$$\frac{1}{4\sin x + 3\cos x + 7}.$$ [2]

5.



The diagram shows the shaded region bounded by the curve $y = \sin x$, the x-axis and the line $x = \frac{\pi}{2}$. The shaded region is rotated through four right-angles about the x-axis. Find the volume of the solid formed. [5]

6. The parametric equations of the curve C are

$$x = \frac{1}{t}, \quad y = t^2 .$$

(a) Show that the tangent to C at the point P with parameter p has equation

$$y + 2p^3x - 3p^2 = 0. \qquad [4]$$

(b) The tangent to C at the point P intersects the x-axis at A and the y-axis at B. Show that $PB = 2PA$. [5]

7. *(a)* Find $\int x \ln x \, dx$. [5]

(b) Use the substitution $u = 2\sin x + 3$ to evaluate

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{(2\sin x + 3)^2} \, dx \ . \qquad [4]$$

8. Water leaks from a hole at the bottom of a large water tank. The depth of the water at time t minutes is x metres. The rate of decrease of x is directly proportional to $\sqrt{x}$.

(a) Write down a differential equation that is satisfied by x. [1]

(b) Given that the depth of water in the tank when $t = 0$ is 9 metres, show that

$$kt = 6 - 2\sqrt{x} \ ,$$

where k is a positive constant. [4]

(c) Given that the depth of water in the tank is 4 metres when $t = 20$, find the time taken for the tank to empty. [3]

9. The position vectors of the points A and B are given by

$$\mathbf{a} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}, \ \mathbf{b} = 2\mathbf{i} + 8\mathbf{j} - 2\mathbf{k}.$$

(a) Find the vector equation of the line AB. [3]

(b) The vector equation of the line L is

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + p\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

where p is a constant. Given that AB and L intersect, find the value of p. [6]

(c) Given $\mathbf{c} = 3\mathbf{i} - \mathbf{j} - \mathbf{k}$, find $\mathbf{b}.\mathbf{c}$. What does your answer tell you about the vectors $\mathbf{b}$ and $\mathbf{c}$?[3]

TURN OVER

10. Expand $\left(1+\frac{x}{8}\right)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of x for which the expansion is valid. Hence by writing $x = 1$ in your expansion, show that $\sqrt{2} \approx \frac{256}{181}$. [5]

11. Complete the following proof by contradiction to show that $\sqrt{2}$ is irrational.

Assume that $\sqrt{2}$ *is rational. Then* $\sqrt{2}$ *may be written in the form* $\frac{a}{b}$ *, where a and b are positive integers having no common factor.*

$\therefore$ $a^2 = 2b^2$.

$\therefore$ a^2 *has a factor 2.*

$\therefore$ *a has a factor 2 so that* $a = 2k$,

where k is an integer. [4]