

981/01

MATHEMATICS M2

Mechanics 2

P.M. TUESDAY, 6 June 2006

(1 $\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9.8 ms^{-2} .

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A car of mass 1200 kg is towing a trailer of mass 800 kg up a slope inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{28}$. The resistance to motion acting on the car is 150 N and that acting on the trailer is 100 N. The car's engine is working at 45 kW.

- (a) Calculate the acceleration of the car and trailer when the speed is 25 ms^{-1} . [6]
- (b) Determine the tension in the rigid tow-bar connecting the car and the trailer when the speed is 25 ms^{-1} . [4]

2. Particle A is moving with constant velocity $-2\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$, and at time $t = 0$ s it has position vector $\mathbf{i} - 10\mathbf{k}$. Particle B is moving with constant velocity $\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}$, and at time $t = 0$ s it has position vector $7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$.

- (a) Write down the position vectors of A and B at time t s. [2]
- (b) Find the distance between A and B when $t = 2$ s. [3]

3. A particle P , of mass 3 kg, moves along the horizontal x -axis under the action of a resultant force F N. Its velocity $v \text{ ms}^{-1}$ at time t seconds is given by

$$v = 12t - 3t^2.$$

- (a) Given that the particle is at the origin O when $t = 1$, find an expression for the displacement of the particle from O at time t s. [4]
- (b) Find the acceleration of the particle at time t s. [2]
- (c) Find the power of the force F when $t = 1.5$. [3]

4. A light elastic string, of natural length 0.8 m and modulus of elasticity 35.4 N, has one end A attached to a fixed point and the other end B attached to a particle P of mass 3 kg. Initially P is held at rest at A . It is then released and allowed to fall. Calculate the speed of P when the length of the string is 1.2 m. [7]

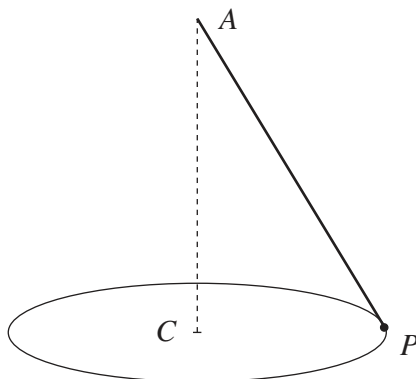
5. A stone is projected in a direction which makes an angle of 45° above the horizontal. It strikes a small target whose horizontal and vertical distances from the point of projection are 120 m and 41.6 m respectively. The target is above the level of the point of projection.

- (a) Find the speed of projection and show that the time taken for the stone to reach the target is 4 s. [8]
- (b) Determine, correct to two decimal places, the speed and direction of motion of the stone as it hits the target. [7]

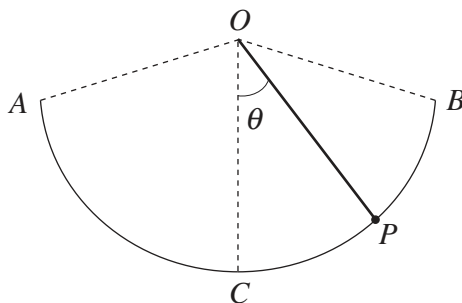
6. A particle P moves such that its position vector \mathbf{r} with respect to the origin O at time t is given by

$$\mathbf{r} = \cos 3t \mathbf{i} + \sin 3t \mathbf{j}.$$

- (a) Find an expression for \mathbf{v} , the velocity of P at time t . [3]
- (b) Show that the direction of \mathbf{v} is perpendicular to that of \mathbf{r} for all values of t . [3]
- (c) Find the speed of P . [3]
7. The diagram shows a small body P , of mass 3 kg, attached by means of a light inextensible string, of length 1.3 m, to a fixed point A . The point C is vertically below A , and P describes a horizontal circle, with centre C and radius 0.5 m, with a uniform angular speed of ω radians per second about C .



- (a) Find the tension in the string. [3]
- (b) Calculate, correct to two decimal places, the value of ω . [4]
8. One end of a light rod of length l m is attached to a fixed point O and the other end is attached to a particle P of mass m kg. The particle P is set in motion so that it moves back and forth along the minor arc AB of a vertical circle with centre O and radius l m, as shown in the diagram.



When P is at its lowest point C , its speed is $u \text{ ms}^{-1}$ and the tension in the rod is $2mg \text{ N}$.

- (a) Show that $u = \sqrt{gl}$. [4]
- (b) The speed of P when OP makes an angle θ with the vertical is denoted by $v \text{ ms}^{-1}$. Show that $v^2 = gl(2\cos\theta - 1)$. [3]
- (c) Find the greatest value of θ . [2]
- (d) Find the value of θ when the tension in the rod is $mg \text{ N}$. [4]