984/01

MATHEMATICS S2

STATISTICS 2

A.M. THURSDAY, 8 June 2006

$(1\frac{1}{2}$ hours$)$

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Bill is an athlete specialising in the long-jump. He does 10 jumps at a training session with the following results (in metres).

$$6{\cdot}21 \quad 6{\cdot}33 \quad 6{\cdot}02 \quad 6{\cdot}11 \quad 6{\cdot}13 \quad 6{\cdot}40 \quad 6{\cdot}51 \quad 6{\cdot}29 \quad 6{\cdot}16 \quad 6{\cdot}44$$

You may assume that the above distances form a random sample from a normal distribution with unknown mean μ metres and standard deviation 0·1 metres.

Calculate a 95% confidence interval for μ.

Bill had stated beforehand that the mean length of his jumps was 6·3 metres.

Giving a reason, state whether or not your interval supports this claim. [6]

2. The random variable X is uniformly distributed on the interval $[a, b]$. Given that the variance of X is 3, show that

$$b - a = 6.$$

Given further that the mean of X is 10, find the values of a and b. [7]

3. The weights of adult dogs of a certain breed may be assumed to be normally distributed. For male dogs, the mean weight is 30 kg and the standard deviation is 2 kg. For female dogs, the mean weight is 25 kg and the standard deviation is 1·8 kg.

   (a) (i) Find the probability that a randomly chosen male dog weighs between 28 kg and 34 kg.

       (ii) Determine the weight that is exceeded by 1% of female dogs. [8]

   (b) Find the probability that the weight of a randomly chosen male dog exceeds the weight of a randomly chosen female dog. [5]

4. There are 5 computers in an office working continuously. You may assume that, for each computer independently of the others, the number of 'crashes' occurring during a week follows a Poisson distribution with mean 0·8.
   During a randomly chosen week, find the probability that

   (a) each computer crashes exactly once, [4]

   (b) the total number of crashes on all the computers is five. [4]

5. On any weekday, the number of passengers, X, using an early morning bus service may be assumed to follow a Poisson distribution. In the past, the mean value of X has been 2·4. The local council wishes to increase this mean so they decide to offer this service free of charge.

   (a) During the 5-day week following this offer, a total of 18 passengers used this service. Clearly stating your hypotheses, calculate the p-value of this result. Interpret your value in context. [5]

   (b) During the 100 weekdays following this offer, a total of 280 passengers used this service. Determine, at the 1% significance level, whether or not the offer has resulted in an increase in the mean number of passengers using this early morning bus service. [7]

6. Mr. Jones, a candidate in an election, believes that 40% of the electors intend to vote for him. His agent, however, believes that the support for him is less than this. To resolve this difference in opinion, they set up the hypotheses

$$H_0 : p = 0{\cdot}4 \text{ versus } H_1 : p < 0{\cdot}4$$

where p denotes the proportion of the electorate intending to vote for Mr. Jones.

(a) They decide to question a random sample of 50 electors. They define the critical region to be $X \leqslant 14$, where X denotes the number in the sample intending to vote for Mr. Jones.

   (i)   Determine the significance level of this procedure.

   (ii)   Given that p is actually $0{\cdot}3$, find the probability that they draw the wrong conclusion.

                                             [5]

(b) They now decide to carry out a larger survey in which they question 500 randomly chosen electors. They find that 185 of these intend to vote for Mr. Jones. Find the *p*-value of this result and interpret it in context. [7]

7. A Motoring Organisation wished to determine whether or not the fuel consumptions of two car models, A and B, are the same. To do this, six cars of each model were given 10 litres of petrol and driven around a track until they ran out of petrol. The distances travelled (in miles) by the cars were as follows.

Model A	83·1	84·1	84·6	85·1	81·2	82·9
Model B	81·0	82·2	79·9	83·4	81·8	80·7

You may assume that these are random samples from normal distributions with common standard deviation 1·5 miles.

(a) State suitable hypotheses. [1]

(b) Calculate the *p*-value and state your conclusion when the significance level is

   (i)   1%,

   (ii)   5%. [10]

8. *(a)* State the Central Limit Theorem. [1]

(b) When a cubical die is thrown, the score obtained has a mean of $\dfrac{7}{2}$ and a variance of $\dfrac{35}{12}$. Such a die is thrown 50 times. Find, approximately, the probability that the mean of the 50 scores obtained exceeds 3. [5]