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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

Tystysgrif Addysg Gyffredinol
Uwch Gyfrannol/Uwch

MARKING SCHEMES

SUMMER 2007

MATHEMATICS

WJEC
CBAC

INTRODUCTION

The marking schemes which follow were those used by the WJEC for the 2007 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

The WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

MATHEMATICS C1

1. (a) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1
 Gradient of $AB = 2$ (or equivalent) A1
- (b) Gradient of $CD = \frac{k - (-1)}{5 - 2}$ B1
 Use of gradient $CD = \text{gradient } AB$ M1
 $\frac{k - (-1)}{5 - 2} = 2 \Rightarrow k = 5$ (convincing) A1
- (c) Use of gradient of $L = \frac{-1}{\text{gradient of } CD}$ M1
 Equation of $L : y - 3 = -\frac{1}{2}[x - (-1)]$ A1
 (f.t. candidate's gradient for CD)
 Equation of $L : x + 2y - 5 = 0$ (convincing) A1
- (d) A correct method for finding the equation of CD M1
 Equation of $CD : 2x - y - 5 = 0$ A1
 (f.t. candidate's gradient for CD)
 An attempt to solve equations of L and CD simultaneously M1
 $x = 3, y = 1$ (c.a.o.) A1
2. (a) $2\sqrt{8} = 2 \times 2 \times \sqrt{2}$ B1
 $\sqrt{18} = 3\sqrt{2}$ B1
 $\frac{12}{\sqrt{2}} = 3 \times 2 \times \sqrt{2}$ B1
 $2\sqrt{8} + \sqrt{18} - \frac{12}{\sqrt{2}} = \sqrt{2}$ (c.a.o.) B1
- (b) $\frac{5 + \sqrt{15}}{5 - \sqrt{15}} = \frac{(5 + \sqrt{15})(5 + \sqrt{15})}{(5 - \sqrt{15})(5 + \sqrt{15})}$ M1
 Numerator: $25 + 15 + 5\sqrt{15} + 5\sqrt{15}$ A1
 Denominator: $25 - 15$ A1
 $\frac{5 + \sqrt{15}}{5 - \sqrt{15}} = 4 + \sqrt{15}$ (c.a.o.) A1

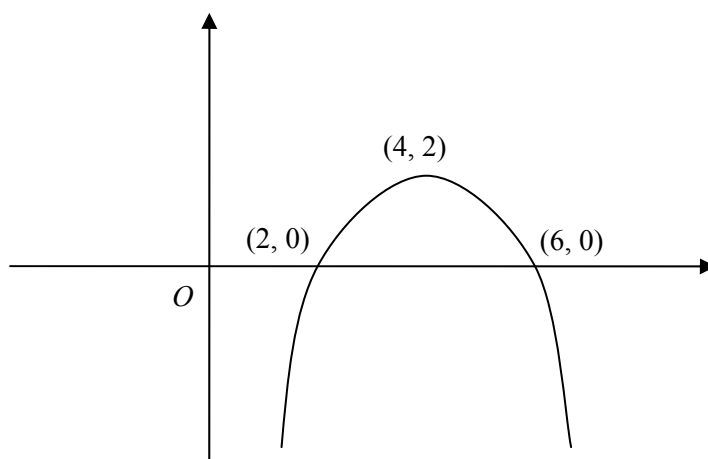
Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5 - \sqrt{15}$

3.	(a)	Either: use of $f(3) = 0$ Or: division by $(x - 3)$ A convincing argument that $p = 24$ Special case Candidates who assume $p = 24$ and verify the result using either method are awarded B1	M1 A1
	(b)	$f(x) = (x - 3)(x^2 + ax + b)$ with one of a, b correct $f(x) = (x - 3)(x^2 - 2x - 8)$ $f(x) = (x - 3)(x - 4)(x + 2)$ Roots are $x = 3, x = 4, x = -2$	M1 A1 A1 (f.t. one slip) A1 (f.t. one slip)
	(c)	Either: evaluation of $f(2)$ Or: division by $(x - 2)$ Remainder = 8	M1 A1
4.	(a)	An attempt to differentiate, at least one non-zero term correct $\frac{dy}{dx} = -16x^{-2} + 3$ An attempt to substitute $x = 4$ in expression for $\frac{dy}{dx}$ When $x = 4, \frac{dy}{dx} = -1 + 3 = 2$ Equation of tangent is $y - 18 = 2(x - 4)$ (f.t. if M1 and m1 both awarded)	M1 A1 m1 A1 A1
	(b)	$4x + 7 = x^2 + 2x + 4$ An attempt to collect terms, form and solve quadratic equation $x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = 3, x = -1$ When $x = 3, y = 19$, when $x = -1, y = 3$ The line $y = 4x + 7$ intersects the curve $y = x^2 + 2x + 4$ at the points $(-1, 3)$ and $(3, 19)$	M1 m1 A1 (both values) A1 (both values) A1 (f.t. one slip) E1 (f.t. candidate's points)
	(a)	$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ (-1 for each error) (-1 for any subsequent 'simplification') Substituting x for a and $\frac{1}{2x}$ for b in the $10a^3b^2$ term Identifying $\frac{5}{2}$ (o.e.) as the coefficient of x	B2 M1 A1
	(b)	Coefficient of $x^2 = \frac{n(n-1)}{2}$ An attempt to solve $\frac{n(n-1)}{2} = 36$ $n = 9$	B1 M1 A1 (c.a.o.)

6. $y = x^2 - 12x + 10$
 $y + \delta y = (x + \delta x)^2 - 12(x + \delta x) + 10$ B1
Subtracting y from above to find δy M1
 $\delta y = 2x\delta x + (\delta x)^2 - 12\delta x$ A1
Dividing by δx and letting $\delta x \rightarrow 0$ M1
 $\frac{dy}{dx} = 2x - 12$ (c.a.o.) A1
dx
7. (a) $a = 2$ B1
 $b = 1$ B1
 $c = 3$ B1
- (b) $\frac{1}{c+4}$ on its own or maximum value = $\frac{1}{c+4}$,
with correct explanation or no explanation B2
 $\frac{1}{c+4}$ on its own or maximum value = $\frac{1}{c+4}$,
with incorrect explanation B1
minimum value = $\frac{1}{c+4}$ with no explanation B1
minimum value = $\frac{1}{c+4}$ with incorrect explanation B0
- Special case**
Candidates who give maximum value = $\frac{1}{3}$ are awarded B1
8. (a) An expression for $b^2 - 4ac$, with $b = (2k + 1)$, and at least one of a or c correct M1
 $b^2 - 4ac = (2k + 1)^2 - 4(1)(k^2 + k + 1)$ A1
 $b^2 - 4ac = -3$ (or < 0 , convincing) A1
 $b^2 - 4ac < 0 \Rightarrow$ no real roots (f.t. one slip) A1
- (b) Finding critical points $x = -3, x = -\frac{1}{2}$ B1
 $-3 < x < -\frac{1}{2}$ or $-\frac{1}{2} > x > -3$ or $(-3, -\frac{1}{2})$ or $-3 < x$ and $x < -\frac{1}{2}$
or a correctly worded statement to the effect that x lies strictly between -3 and $-\frac{1}{2}$
(f.t. candidate's critical points) B2
Note: $-3 \leq x \leq -\frac{1}{2}$,
 $-3 < x, x < -\frac{1}{2}$
 $-3 < x \leq -\frac{1}{2}$
 $-3 \leq x < -\frac{1}{2}$
all earn B1

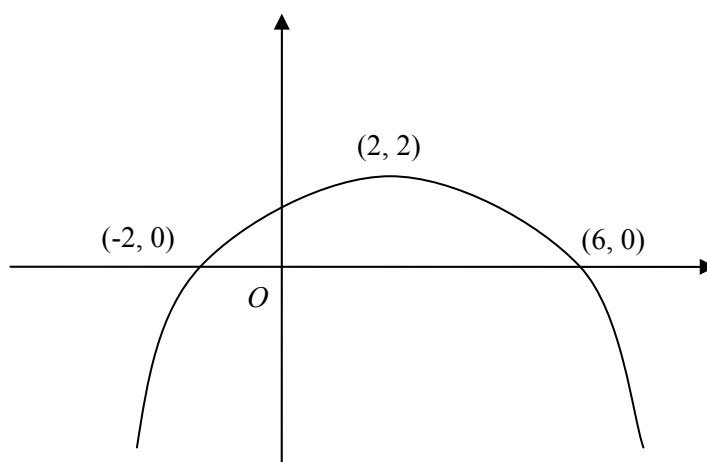
9. (a)



Concave down curve with stationary point at $(a, 2)$, $a \neq 1$
 $a = 4$
 Points of intersection with x -axis $(2, 0)$, $(6, 0)$

B1
 B1
 B1

(b)



Concave down curve with positive intercept on y -axis
 Stationary point $(2, 2)$
 Points of intersection with x -axis $(-2, 0)$, $(6, 0)$

B1
 B1
 B1

10. $\frac{dy}{dx} = 3x^2 - 2x - 1$

B1

Putting derived $\frac{dy}{dx} = 0$

M1

$$(3x + 1)(x - 1) = 0 \Rightarrow x = -1/3, x = 1$$

(both roots required)

(f.t. candidate's $\frac{dy}{dx}$)

A1

Stationary points are $(-1/3, 59/27)$ and $(1, 1)$

(both correct) (c.a.o.)

A1

A correct method for finding nature of stationary points

M1

$(-1/3, 59/27)$ is a maximum point

(f.t. candidate's derived values)

A1

$(1, 1)$ is a minimum point

(f.t. candidate's derived values)

A1

MATHEMATICS C2

- 1.
- | | 0 | 1 | | | |
|--|--|-------------|----------------------|--|--------------------|
| | $\pi/8$ | 1.175875602 | | | |
| | $\pi/4$ | 1.306562965 | | | |
| | $3\pi/8$ | 1.387039845 | (3 values correct) | | B1 |
| | $\pi/2$ | 1.414213562 | (5 values correct) | | B1 |
| | Correct formula with $h = \pi/8$ | | | | M1 |
| | $I \approx \frac{\pi/8}{2} \times \{1 + 1.414213562 + 2(1.175875602 + 1.306562965 + 1.387039845)\}$ | | | | |
| | $I \approx 1.994$ | | | | (f.t. one slip) A1 |
| | Special case for candidates who put $h = \pi/10$ | | | | |
| | 0 | 1 | | | |
| | $\pi/10$ | 1.144122806 | | | |
| | $\pi/5$ | 1.260073511 | | | |
| | $3\pi/10$ | 1.344997024 | | | |
| | $2\pi/5$ | 1.396802247 | | | |
| | $\pi/2$ | 1.414213562 | (all values correct) | | B1 |
| | Correct formula with $h = \pi/10$ | | | | M1 |
| | $I \approx \frac{\pi/10}{2} \times \{1 + 1.414213562 + 2(1.144122806 + 1.260073511 + 1.344997024 + 1.396802247)\}$ | | | | |
| | $I \approx 1.9$ | | | | (f.t. one slip) A1 |
- 2.
- (a) $3x = 60^\circ, 240^\circ, 420^\circ, 600^\circ$ (one value) B1
 $x = 20^\circ, 80^\circ, 140^\circ$ B1, B1, B1
 Note: Subtract 1 mark for each additional root in range, ignore roots outside range.
- (b) $4 \cos^2 \theta - \cos \theta = 2(1 - \cos^2 \theta)$ (correct use of $\sin^2 \theta = 1 - \cos^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation in $\cos \theta$, either by using the quadratic formula or by getting the expression into the form $(a \cos \theta + b)(c \cos \theta + d)$, with $a \times c = \text{coefficient of } \cos^2 \theta$ and $b \times d = \text{constant}$ m1
 $6 \cos^2 \theta - \cos \theta - 2 = 0 \Rightarrow (3 \cos \theta - 2)(2 \cos \theta + 1) = 0$
 $\Rightarrow \cos \theta = \frac{2}{3}, -\frac{1}{2}$ A1
 $\theta = 48.19^\circ, 311.81^\circ, 120^\circ, 240^\circ$ (48.19°, 311.81°) B1
 (120°) B1
 (240°) B1
- Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
 $\cos \theta = +, -$, f.t. for 3 marks, $\cos \theta = -, -$, f.t. for 2 marks
 $\cos \theta = +, +$, f.t. for 1 mark

3.	(a)	$7^2 = x^2 + (3x)^2 - 2 \times x \times 3x \times \cos 60^\circ$	(correct use of cos rule)	M1
		$7^2 = x^2 + 9x^2 - 3x^2$		A1
		$7x^2 = 7^2 \Rightarrow x = \sqrt{7}$	(convincing)	A1
(b)	Either:			
		$\frac{7}{\sin 60^\circ} = \frac{\sqrt{7}}{\sin ACB}$	(correct use of sin rule)	M1
		$ACB = 19.11^\circ$		A1
	Or:			
		$(\sqrt{7})^2 = 7^2 + (3\sqrt{7})^2 - 2 \times 7 \times (3\sqrt{7}) \times \cos ACB$	(correct use of cos rule)	M1
		$ACB = 19.11^\circ$		A1
4.	(a)	$a + 2d = k(a + 5d)$	$(k = 4, \frac{1}{4})$	M1
		$a + 2d = 4(a + 5d)$		A1
		$\frac{20[2a + 19d]}{2} = 350$		B1
		An attempt to solve simultaneous equations		M1
		$d = 5$	$(a = -30)$	A1
		$a = -30$	$(d = 5)$	A1
			(c.a.o.)	A1
			(f.t. one error)	A1
	(b)	$-30 + (n - 1) \times 5 = 125$	(equation for n 'th term and an attempt to solve, f.t. candidate's values for a, d)	M1
		$n = 32$	(c.a.o.)	A1
5.	(a)	$S_n = a + ar + \dots + ar^{n-2} + ar^{n-1}$	(at least 3 terms, one at each end)	B1
		$rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n$		
		$S_n - rS_n = a - ar^n$	(multiply first line by r and subtract)	M1
		$(1 - r)S_n = a(1 - r^n)$		
		$S_n = \frac{a(1 - r^n)}{1 - r}$	(convincing)	A1
		$S_\infty = \frac{a}{1 - r}$		B1
	(b)	(i)		
		$\frac{a}{1 - r} = 10$		B1
		$\frac{a}{1 - 2r} = 15$		B1
		An attempt to eliminate a		M1
		$r = 0.25$	(c.a.o.)	A1
		(ii)		
		$a = 7.5$		B1
		$S_4 = \frac{7.5[1 - 0.25^4]}{1 - 0.25}$		
		$S_4 \approx 9.96$	(award even if sum calculated for 2 nd series) (f.t. candidate's derived values of a, r)	M1 A1

6.	(a)	$\frac{2x^{5/2}}{5/2} + \frac{9x^{-3}}{-3}$	(+ c)	B1,B1
	(b)	(i)	$x^2 + 2 = 3x$ An attempt to rewrite and solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b = 2$ $(x - 2)(x - 1) = 0 \Rightarrow x = 1, x = 2$ (both values, c.a.o.)	M1 m1 A1
			$A(1, 3), B(2, 6)$ (both values, f.t. candidate's x values)	A1
		(ii)	Either: Total area = $\int_0^1 3x \, dx + \int_1^2 (x^2 + 2) \, dx$ (use of integration) (addition of integrals)	M1 m1
			$= [(3/2)x^2]_0^1 + [(1/3)x^3 + 2x]_1^2$ (correct integration) $= [3/2 - 0] + [(8/3 + 4) - (1/3 + 2)]$ (use of candidate's 0, x_A , x_B as limits) $= 35/6$ (c.a.o.)	B3 M1 A1
			Or: Area of triangle = $3/2$ (f.t. candidate's coordinates for A)	B1
			Area under curve = $\int_1^2 (x^2 + 2) \, dx$ (use of integration)	M1
			$= [(1/3)x^3 + 2x]_1^2$ (correct integration) $= (8/3 + 4) - (1/3 + 2)$ (use of candidate's x_A , x_B as limits) $= 13/3$	B2 M1
			Finding total area by adding values Total area = $3/2 + 13/3 = 35/6$ (c.a.o.)	m1 A1

7.	(a)	(i)	Let $x = \log_a p, y = \log_a q$ Then $p = a^x, q = a^y$ (relationship between log and power) $pq = a^x \times a^y = a^{x+y}$ (the laws of indices) $\log_a pq = x + y = \log_a p + \log_a q$ (convincing)	B1 B1 B1
		(ii)	$\log_a x + \log_a (3x + 4) = \log_a x(3x + 4)$ (addition law)	B1
			$2 \log_a (3x - 4) = \log_a (3x - 4)^2$ (power law)	B1
			$x(3x + 4) = (3x - 4)^2$ (removing logs)	M1
			An attempt to rearrange and solve quadratic	m1
			$3x^2 - 14x + 8 = 0 \Rightarrow x = 2/3, x = 4$ (c.a.o.)	A1
	(b)	Either:		
		$x \log_{10} 3 = \log_{10} 11$		B1
		$x = \frac{\log_{10} 11}{\log_{10} 3} \Rightarrow x \approx 2.183$		B1
		Or:		
8.	(a)	$A(-2, 8)$		B1
		A correct method for finding the radius		M1
		Radius = $\sqrt{50}$		A1
	(b)	An attempt to substitute $(x + 2)$ for y in the equation of the circle		M1
		$x^2 - 4x - 5 = 0$ (or $2x^2 - 8x - 10 = 0$)		B1
		$x = -1, x = 5$ (correctly solving candidate's quadratic, both values)		A1
		Points of intersection are $(-1, 1), (5, 7)$ (c.a.o.)		A1
	(a)	Length of arc = 6θ		B1
		Circumference of circle = $2 \times \pi \times 6$		B1
		$2 \times \pi \times 6 = 2 \times 6\theta + 24$		
		(expression of the given information as an equation using the candidate's expressions for length of arc and circumference)		M1
		$\theta = \pi - 2$ (convincing)		A1
		Either:		
		Area of unshaded sector = $(1/2) \times 6^2 \times (\pi - 2)$ [= 20.55]		B1
9.	(b)	Area of shaded sector = $36\pi - (1/2) \times 6^2 \times (\pi - 2)$ (f.t. candidate's expression/value for area of unshaded sector)		M1
		Area of shaded sector = 92.55 cm^2 (c.a.o.)		A1
		Or:		
		Angle in shaded sector = $2\pi - (\pi - 2)$ [= 5.14]		B1
		Area of shaded sector = $(1/2) \times 6^2 \times [2\pi - (\pi - 2)]$		M1
		Area of shaded sector = 92.52 cm^2 (c.a.o.)		A1
		Accept answers in the interval [92.5, 92.6]		

MATHEMATICS C3

1.	$h = 0.1$	M1 ($h = 0.1$ correct formula)
	$\text{Integral} \approx \frac{0.1}{3} [0.5 + 0.4279957 + 4(0.4772563 + 0.4420154) + 2(0.4582276)]$ ≈ 0.184	B1 (3 values) B1 (2 values) A1 (F.T. one slip)
4		
2.	(a) $\theta = 0$ for example $\cos \theta = 1$ $1 - 2 \cos^2 \theta = -1$ ($\therefore \cos 2\theta \neq 1 - 2 \cos^2 \theta$) (b) $\operatorname{cosec}^2 \theta - 1 = 7 - 2 \operatorname{cosec} \theta$ $\operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta - 8 = 0$ $(\operatorname{cosec} \theta + 4)(\operatorname{cosec} \theta - 2) = 0$ $\operatorname{cosec} \theta = -4, 2$ $\sin \theta = -\frac{1}{4}, \frac{1}{2}$ $\theta = 194.5, 345.5, 30^\circ, 150^\circ$	B1 (choice of values and attempt to use) B1 (for correct demonstration) M1 (substitution of $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$) M1 (attempt to solve) A1 B1 (194.5), B1 (345.5) B1 (30°, 150°)
8		
3.	(a) (i) $\frac{dy}{dx} = \frac{5t^4 + 20t^2}{10t}$ (ii) $\frac{5t^4 + 20t^2}{10t} = 1$ $\frac{t^3 + 4t}{2} = 1$ $t^3 + 4t - 2 = 0$	M1 (attempt to use $\frac{dy}{dx} = \dot{y}/\dot{x}$) A1 A1 M1 (use of equation and attempt to simplify) A1 (convincing)

(b)	$\frac{t}{0}$	$\frac{t^3 + 4t - 2}{-2}$	Change of sign indicates presence of root between 0 and 1	M1 (attempt to find signs or values)
	1	3		A1 (correct values or signs and conclusion)
	$t_0 = 0.5, t_1 = 0.46875$			B1 (t_1)
	$t_2 = 0.4742508, t_3 = 0.4733336, t_4 = 0.4734880$ (0.4735)			B1 (t_4 to 4 decimal places)
	Try 0.47345, 0.47355			
	$\frac{t}{0.47345}$	$\frac{t^3 + 4t - 2}{-0.00007}$		M1 (attempt to find signs or values)
	0.47355	0.0004		M1 (correct values or signs)
	Change of sign indicates root is 0.4735 (correct to 4 decimal places)			A1

12

4.	(a)	Graph	B1
		Graph	M1 (shape) A1 ((0, 4)) A1 ((± 2, 0))
	(b)	$5x - 3 > 4$ $x > \frac{7}{5}$	B1
		or $5x - 3 < -4$ $x < -\frac{1}{5}$	M1 A1 (must have 'or' in either part) (o.e.)
		<u>Alternatively</u> $(5x - 3)^2 > 16$ $25x^2 - 30x - 7 > 0$ $(5x + 1)(5x - 7) > 0$ $-\frac{1}{5}, \frac{7}{5}$ $x < -\frac{1}{5} \text{ or } x > \frac{7}{5}$ (o.e.)	M1 (forming quadratic and attempting to solve) A1 (fixed points) A1 (F.T. fixed points)

7

5. $6y \frac{dy}{dx} + 2xy^3 + 3x^2y^2 \frac{dy}{dx} + 4x^3 - 2x = 0$

B1 $(6y \frac{dy}{dx})$

B1 $(2xy^3 + 3x^2y^2 \frac{dy}{dx})$

B1 (correct differentiation of x^4, x^2)

$\frac{dy}{dx} = -4$

B1 (F.T. one slip)

4

6. (a) (i) $x^2 \cos x + 2x \sin x$

M1 $(x^2 f(x) + \sin x g(x))$

A1 $(f(x) = \cos x, g(x) = 2x)$

(ii) $\frac{2x}{x^2 + 3}$

M1 $(\frac{f(x)}{x^2 + 3}, f(x) \neq 1), A1$

(iii) $-2e^{9-2x}$

M1 $(ke^{9-2x}) A1 (k = -2)$

(iv) $-\frac{24}{(3x+7)^3}$

M1 $(\frac{k}{(3x+7)^3}, \text{allow unsimplified})$

A1 (simplified answer)

(v) $-\frac{24}{\sqrt{1-(3x)^2}} \quad (\text{o.e.})$

M1 $(\frac{k}{\sqrt{1-(3x)^2}}) A1 (k = 3)$

(S. Case allow B1 for $\frac{3}{\sqrt{1-3x^2}})$

(b) $\frac{dy}{dx} = \frac{(1 - \tan x)(\sec^2 x) - (1 + \tan x)(-\sec^2 x)}{(1 - \tan x)^2}$

M1 $((1 - \tan x)f(x) - \frac{(1 + \tan x)g(x)}{(1 + \tan x)^2})$

$= \frac{2 \sec^2 x}{(1 - \tan x)^2}$

A1 $(f(x) = \sec^2 x, g(x) = -\sec^2 x)$

A1 (simplified)

which is positive since $\sec^2 > 0, (1 - \tan x)^2 > 0$

B1 (convincing)

14

7. (a) (i) $-\frac{1}{2} \ln |5 - 2x| \quad (+ C)$

M1 A1

$$(ii) \quad \frac{(3x+2)^{21}}{63} \quad (+ C)$$

M1 ($k(3x+2)^{21}$)

A2($k=\frac{1}{63}$) or A1 ($k=\frac{1}{21}$ or $\frac{1}{3}$)

$$(iii) \quad \frac{1}{7} e^{7x} \quad (+ C)$$

M1 (ke^{7x}) A1 ($k=\frac{1}{7}$)

$$(b) \quad \int_0^{\frac{\pi}{3}} \cos(3x + \frac{\pi}{3}) dx$$

$$= \left[\frac{1}{3} \sin(3x + \frac{\pi}{3}) \right]_0^{\frac{\pi}{3}}$$

M1 [$k \sin(3x + \frac{\pi}{3})$], allow

$k = 1, \frac{1}{3}, -\frac{1}{3}, 3]$

A1 ($k = \frac{1}{3}$)

$$= \frac{1}{3} \sin\left(\frac{4\pi}{3}\right) - \frac{1}{3} \sin \frac{\pi}{3}$$

m1 (F.T. $k(\sin \frac{4\pi}{3} - \sin \frac{\pi}{3})$)

$$= -\frac{2}{3} \sin \frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{3} \text{ or } -0.577 \text{ correct to 3 decimal places } -(0.578)$$

A1 (C.A.O.)

11

8. (a) Range of $f[1, \infty)$, Range of $g[1, \infty)$

B1, B1

$$(b) \quad \begin{aligned} gf(x) &= (e^x)^2 + 1 \\ &= e^{2x} + 1 \end{aligned}$$

M1 (correct order of composition)

A1

(c) domain of gf $[0, \infty)$

B1

range of gf $[2, \infty)$

B1

(d) Graph $y = e^x$
 $y = e^{2x} + 1$

B1 (full curve with (0, 1)

B1 (truncated)

B1 (0, 2)

B1 (steeper curve)

B1 (truncated)

11

9. (a) Let $y = \frac{8}{x+2}$

$$(x+2)y = 8$$

M1 (attempt to isolate x)

$$x+2 = \frac{8}{y}$$

$$x = \frac{8}{y} - 2$$

A1

$$f^{-1}(x) = \frac{8}{x} - 2$$

A1 (F.T.)

(b) domain $(0, 4]$

B1

4

MATHEMATICS C4

1. (a) Let $\frac{x+3}{x^2(x-1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$ M1 (correct form)
- $\therefore x+3 \equiv Ax(x-1) + B(x-1) + Cx^2$ M1 (clearing fractions and attempt to solve)
- $\underline{x=1}$ $4 = C, \quad C = 4$ A1 (2 constants)
- $\underline{x=0}$ $3 = -B, \quad B = -3$ A1 (other constant)
- Equate coefficients of x^2 $0 = A + C, \quad A = -4$ (F.T. if 2 Ms scored)
- No need for display
- (b) $\int -\frac{4}{x} dx - \int \frac{3}{x^2} dx + \int \frac{4}{x-1} dx$
- $= -4 \ln(x) + \frac{3}{x} + 4 \ln|x-1|$ (o.e.) B1, B1 (two logs)
- (+ C)
2. $5x^4 + y^2 + 2xy \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$ B1 ($y^2 + 2xy \frac{dy}{dx}$)
- B1 ($3y^2 \frac{dy}{dx}$)
- $\frac{dy}{dx} = -\frac{2}{3}$ (o.e.) B1 (C.A.O.)
- Equation is $y-3 = -\frac{2}{3}(x+1)$ B1 (F.T. candidate's $\frac{dy}{dx}$)
3. $4 \cos x + 2 \sin x = R \cos(x-\alpha)$ M1 (for $R \cos(x \pm \alpha)$)
- $R \cos \alpha = 4, R \sin \alpha = 2$ B1 ($\sqrt{20}$)
- $R = \sqrt{20}, \alpha = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$ A1 (correct α for given presentation)
- $\cos(x-26.6^\circ) = \frac{3}{\sqrt{20}}$ M1 ($\cos(x \pm 26.6^\circ) = \frac{3}{\sqrt{20}}$)
- $x-26.6^\circ = 47.9^\circ, 312.1^\circ$ A1 (for one value) (C.A.O.)
- $x = 74.4^\circ, 338.7^\circ$ (C.A.O.) (accept 74, 75, 338°, 339°)
- A1, A1

6

4

7

4. $(1+4x)^{\frac{1}{2}} - \frac{1}{1+3x}$

$= 1 + \left(\frac{1}{2}\right)(4x) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{1}{2!}(4x)^2 + \dots$

$\quad - (1-3x + \frac{(-1)(-2)}{2}(3x)^2 + \dots)$

$= 1 + 2x - 2x^2 + \dots$

$\quad - 1 + 3x - 9x^2 + \dots$

$= 5x - 11x^2 + \dots$

Expansion valid for $|x| < \frac{1}{4}$

(o.e.)

B2 (-1) each error

B2 (-1) each error

(correct expansion of $(1+3x)^{-n}$)

B2

(-1 each error)

B1

7

5. Volume = $\pi \int_0^1 (e^{2x} + 1) dx$

$= \pi \left[\frac{e^{2x}}{2} + x \right]_0^1$

$= \pi \left[\frac{e^{2x}}{2} + 1 - \frac{1}{2} \right]$

≈ 13.177

B1 (with or without limits,

after squaring $\sqrt{\quad}$)

B1 (correct integration)

M1 (correct use of limits after

attempted integration)

A1 (C.A.O.)

4

6. (a) $\frac{dy}{dx} = \frac{2t}{2} = t$ M1 (correct attempt to find gradient)

Gradient of normal = $-\frac{1}{t}$ A1

Equation is

$y - p^2 = -\frac{1}{p}(x - 2p)$ M1 ($y - y_1 = m(x - x_1)$) o.e.

$py - p^3 = -x + 2p$

$x + py = p^3 + 2p$ A1 (convincing)

(b) A $y = 0, x = p^3 + 2p$ B1 (must be correct for A and B)

B $x = 0, y = p^2 + 2$ B1

$p^3 + 2p = 2(p^2 + 2)$

$p(p^2 + 2) = 2(p^2 + 2)$ M1 (candidate's OA = $k \times$ candidate's OB, $k = \frac{1}{2}$ or 2)

$p = 2$

A1 (C.A.O.)

8

7. (a) $\int x^2 \ln x dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$ M1 (Parts and correct choice of u, v)

$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$ A1

M1 (division)

$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} (+ C)$

A1 (C.A.O.)

(b) $dx = 2 \cos \theta d\theta$

When $x = 0, \theta = 0$

$x = \sqrt{2}, \sin \theta = \frac{1}{\sqrt{2}}, \theta = \frac{\pi}{4}$ B1

$\int_0^{\frac{\pi}{4}} \frac{4 \sin^2 \theta}{\sqrt{4 - 4 \sin^2 \theta}} \cdot 2 \cos \theta d\theta$ M1 (substitution for dx and x)

A1 (any limits)

$= \int_0^{\frac{\pi}{4}} \frac{4 \sin^2 \theta}{2\sqrt{1 - \sin^2 \theta}} \cdot 2 \cos \theta d\theta$

$$= \int_0^{\frac{\pi}{4}} \frac{4 \sin^2 \theta \cdot 2 \cos \theta d\theta}{2 \cos \theta}$$

$$= \int_0^{\frac{\pi}{4}} 4 \sin^2 \theta d\theta$$

$$\int = \int_0^{\frac{\pi}{4}} 2(1 - \cos 2\theta) d\theta$$

$$= [2\theta - \sin 2\theta]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{2} - 1 (\approx 0.571)$$

A1 (convincing, proof of $k = 4$,
any limits)

M1 ($a + b \cos 2\theta$)

A1 ($a = \frac{k}{2}$, $b = -\frac{k}{2}$)

A1 (correct integration of two terms)

A1 (either answer, C.A.O.)

12

8. (a) $\frac{dP}{dt} = kP$

B1

(b) $\int \frac{dP}{P} = \int k dt$

M1 (Separation of variables and $\int$)

$$\ln P = kt + C$$

$$t = 0, P = 50$$

$$\ln 50 = C$$

$$\ln P - \ln 50 = kt$$

$$\ln \frac{P}{50} = kt$$

$$\frac{P}{50} = e^{kt}$$

$$P = 50e^{kt}$$

A1

F.T. right hand side)

M1 (attempt to find C)

M1 (combination of logs and

attempt to exponentiate)

A1

(c) $65 = 50 e^{7k}$

$$\ln \frac{65}{50} = 7k$$

$$k = 0.03748$$

M1 (taking logs correctly)

A1

After sixteen years, $P = 50 \exp (0.03748 \times 16)$
 $\approx \text{£}91$ (nearest pound)

M1 (use of formula)

A1 (C.A.O.)

10

9.	(a)	(i)	$\mathbf{AB} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} - (\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$	M1 ($\mathbf{b} - \mathbf{a}$)
			$= 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$	A1
		(ii)	Equation of AB is	M1 ($\mathbf{a} + \lambda (\mathbf{b} - \mathbf{a})$)
			$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda (2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$	(o.e.) A1 (must involve $\mathbf{r}$ or $\mathbf{OP} = \dots$ F.T. $\mathbf{AB}$)
		(iii)	(The point lies on both lines)	
			$\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda (2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})$	
			$= 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu (\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	
			$1 + 2\lambda = 2 + \mu$	M1 (attempt to equate components, one correct)
			$3 + 3\lambda = 3 + \mu$	A1 (other correct) M1 (correct attempt to solve)
			$\lambda = -1, \mu = -3$	A1 (F.T. candidate's equations)
(b)			Position vector of point of intersection is $-\mathbf{i} - 5\mathbf{k}$	A1 (C.A.O.)
			$\cos \theta = \frac{(\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})}{ \mathbf{i} + 2\mathbf{j} - \mathbf{k} 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} }$	M1 (attempt to use correct formula)
			$= \frac{1 \times 3 - 2 \times 1 - 1 \times 2}{\sqrt{1^2 + 2^2 + (-1)^2} \sqrt{3^2 + 1^2 + 2^2}}$	M1 (correct attempt to find scalar product)
			$= \frac{-1}{\sqrt{6}\sqrt{14}}$	A1 (scalar product) B1 (one correct modulus)
				B1 (F.T. arithmetic slip in scalar product)
			$0 = 96.3^\circ$ (accept nearest degree)	B1 (C.A.O.)

15

10.	$3n + 2n^3 = 3(2k) + 2(2k)^3$	
	$= 6k + 16k^3$	
	$= 2(3k + 8k^3)$	B1 (either $2x$ () or even + even = even)
	which is even Contradiction (Thus n is odd)	B1

2

MATHEMATICS FP1

1. $f(x+h) - f(x) = (x+h)^4 - x^4$ B1
 $= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4$ M1A1
 $= 4x^3h + 6x^2h^2 + 4xh^3 + h^4$ A1
 $\frac{f(x+h) - f(x)}{h} = 4x^3 + 6x^2h + \dots$ M1
 $f'(x) = \lim_{h \rightarrow 0} 4x^3 + 6x^2h + \dots$ A1
 $= 4x^3$ AK
2. $\frac{(1+7i)}{(3+i)} = \frac{(1+7i)(3-i)}{(3+i)(3-i)} = 1+2i$ M1A1
 $2z + \bar{z} = 2(x+iy) + x-iy = 3x+iy$ M1A1
 Equating real and imaginary parts, M1
 $x = 1/3, y = 2$ A1
3. (a) One of the other roots is $2-3i$. B1
 Let the 3rd root be r .
 Then, $(2+3i)(2-3i)r = 13r = -26$, M1A1
 $r = -2$ A1
 (b) $p = -\text{sum of roots} = -2$ M1A1
 $q = (2+3i)(2-3i) - 2(2+3i+2-3i) = 5$ M1A1
4. (a) $T_n = 3n^2 + 2n - 3(n-1)^2 - 2(n-1)$ M1
 $= 3n^2 + 2n - 3n^2 + 6n - 3 - 2n + 2$ A1
 $= 6n - 1$ AG
 [Do not accept a solution which 'goes backwards']
 (b) $\text{Sum} = \sum_{r=1}^n (6r-1)^2 = 36 \sum_{r=1}^n r^2 - 12 \sum_{r=1}^n r + \sum_{r=1}^n 1$ M1A1
 $= \frac{36n(n+1)(2n+1)}{6} - \frac{12n(n+1)}{2} + n$ A1
 $= 12n^3 + 18n^2 + 6n - 6n^2 - 6n + n$ A1
 $= 12n^3 + 12n^2 + n$ A1

5. The statement is true for $n = 1$ since the formula gives the correct answer $1/2$. B1
 Let the statement be true for $n = k$, ie
- $$\sum_{r=1}^k r \times \left(\frac{1}{2}\right)^r = 2 - (k+2)\left(\frac{1}{2}\right)^k \quad \text{M1}$$
- Consider $\sum_{r=1}^{k+1} r \times \left(\frac{1}{2}\right)^r = 2 - (k+2)\left(\frac{1}{2}\right)^k + (k+1) \times \left(\frac{1}{2}\right)^{k+1}$ M1A1
- $$= 2 - \left(\frac{1}{2}\right)^{k+1} (2k+4-k-1)$$
- A1
- $$= 2 - (k+1+2)\left(\frac{1}{2}\right)^{k+1}$$
- A1
- Thus, true for $n = k \Rightarrow$ True for $n = k+1$. A1
 Since the statement is true for $n = 1$ and true for k implies true for $k+1$,
 the statement is proved to be true by mathematical induction. A1
6. $\ln y = x \ln x$ M1
 $\frac{1}{y} \frac{dy}{dx} = \ln x + 1$ m1A1
 $\frac{dy}{dx} = x^x (\ln x + 1)$ A1
 $\frac{d^2 y}{dx^2} = x^x (\ln x + 1)(\ln x + 1) + x^x \cdot \frac{1}{x}$ M1A1A1
 whence the printed result. AG
7. (a) $\text{Det} = 2(-20 - 8) + 1(1 + 15) + 2(24 - 4)$ M1A1
 $= 0$ therefore singular A1
- (b) (i) Using reduction to echelon form, M1
 $2x + y + 2z = 3$ A1
 $5y - 4z = -7$ A1
 $15y - 12z = 2k - 3$ A1
 For consistency, we require M1
 $2k - 3 = -21$ A1
 $k = -9$
- (ii) Put $z = \alpha$. M1
 $y = \frac{4\alpha - 7}{5}$ A1
 $x = \frac{11 - 7\alpha}{5}$ M1A1

8. (a) Under T_1 , $(1, 0) \rightarrow (a, c)$ and $(0, 1) \rightarrow (b, d)$. B1B1
 We are now given that $(1, 0) \rightarrow (0, -1)$ and $(0, 1) \rightarrow (-1, 0)$ M1
 So $a = 0$, $c = -1$, $b = -1$, $d = 0$. A1
- (b) (i) $T_2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ B1
- $T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ M1
- $= \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ A1
- (ii) Fixed points satisfy
- $\begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ M1
- giving
- $-y + 2 = x$
- $-x + 2 = y$ A1
- These equations are consistent confirming that fixed points lie on the line A1
 $x + y = 2$ A1
- (iii) A reflection in the line $x + y = 2$. B1
9. $u + iv = (x + iy)^2 = x^2 - y^2 + 2xyi$ M1
 $u = x^2 - y^2$; $v = 2xy$ A1A1
 Substituting for y , M1
 $u = x^2 - (2x^2 - 1) = 1 - x^2$ A1
 $v^2 = 4x^2(2x^2 - 1)$ A1
 Eliminating x , M1
 $v^2 = 4(1 - u)(2 - 2u - 1)$ A1
 $= 4(1 - u)(1 - 2u)$ A1

MATHEMATICS FP2

1. $x = y^2 \Rightarrow dx = 2ydy$ and $[1, 4] \rightarrow [1, 2]$ B1B1
- $$I = \int_1^2 \frac{2ydy}{\sqrt{y^2(9-y^2)}} \quad \text{M1}$$
- $$= 2 \int_1^2 \frac{dy}{\sqrt{9-y^2}} \quad \text{A1}$$
- $$= 2 \left[\sin^{-1} \left(\frac{y}{3} \right) \right]_1^2 \quad \text{A1}$$
- $$= 0.78 \quad \text{A1}$$
-
2. EITHER $1 + \sqrt{3}i = 2(\cos\pi/3 + i\sin\pi/3)$ M1A1
 Square root $= \sqrt{2}(\cos\pi/6 + i\sin\pi/6)$ M1A1
 $= 1.22 + 0.71i$ A1
- Other root $= -1.22 - 0.71i$. $(\pm \frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i)$ A1
- OR Let $\sqrt{1 + \sqrt{3}i} = x + iy$ M1
 $x^2 - y^2 = 1; 2xy = 3$ A1
 $x^4 - x^2 - 3/4 = 0$ M1
 $x^2 = 3/2$ A1
 $x = \pm\sqrt{3/2}$ A1
 $y = \pm 1/\sqrt{2}$ A1
-
3. (a) Let $\frac{(x+1)(x+2)}{(x-1)(x^2+1)} \equiv \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$ M1
 $(x+1)(x+2) \equiv A(x^2+1) + (x-1)(Bx+C)$ A1
 $x = 1$ gives $A = 3$ A1
 Coeff of x^2 gives $B = -2$ A1
 Constant term gives $C = 1$ A1
- (b) $\int f(x)dx = \int \left[\frac{3}{x-1} - \frac{2x}{x^2+1} + \frac{1}{x^2+1} \right] dx$ M1
 $= 3\ln(x-1) - \ln(x^2+1) + \tan^{-1} x (+C)$ A1A1A1

4. $2\sin 3\theta \cos \theta = \cos \theta$ M1A1
 $\cos \theta (2\sin 3\theta - 1) = 0$ M1A1
 Either $\cos \theta = 0$ whence $\theta = (2n+1)\frac{\pi}{2}$ oe M1A1
 Or $\sin 3\theta = \frac{1}{2}$ whence $3\theta = \frac{\pi}{6} + 2n\pi$ or $\frac{5\pi}{6} + 2n\pi$ M1A1
 So $\theta = \frac{\pi}{18} + \frac{2n\pi}{3}$ or $\frac{5\pi}{18} + \frac{2n\pi}{3}$ oe A1
 [Accept degrees and other alternative forms]

5. (a) The equation in standard form is
 $\frac{x^2}{25} + \frac{y^2}{16} = 1$ M1
 so $a = 5$ and $b = 4$. A1
 The coordinates of the foci are $(\pm\sqrt{a^2 - b^2}, 0)$ ie $(\pm 3, 0)$ M1A1
- (b) $\frac{(5\cos\theta)^2}{25} + \frac{(4\sin\theta)^2}{16} = 1$ so the point lies on the ellipse. B1
- (c) (i) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{4\cos\theta}{5\sin\theta}$ M1A1
 Gradient of normal = $\frac{5\sin\theta}{4\cos\theta}$ A1
 Equation of normal is
 $y - 4\sin\theta = \frac{5\sin\theta}{4\cos\theta}(x - 5\cos\theta)$ M1
 $4y\cos\theta - 5x\sin\theta + 25\sin\theta\cos\theta - 16\sin\theta\cos\theta$ A1
 Whence printed result.
- (ii) Putting $y = 0$, cords of Q are $\left(\frac{9}{5}\cos\theta, 0\right)$ B1
 Putting $x = 0$, cords of R are $\left(0, -\frac{9}{4}\sin\theta\right)$ B1
 Coords of M are $(x, y) = \left(\frac{9}{10}\cos\theta, -\frac{9}{8}\sin\theta\right)$ B1
 Eliminating θ ,
 $\frac{x^2}{(9/10)^2} + \frac{y^2}{(9/8)^2} = 1$, ie an ellipse M1A1
 [These 2 marks can be gained by noting that the above are the parametric equations of an ellipse]

6. (a) $f'(x) = \frac{2x \cdot x - (x^2 + 4)}{x^2} = \frac{x^2 - 4}{x^2}$ M1A1
- The stationary points are $x = \pm 2, y = \pm 4$ A1A1
- (b) The asymptotes are $x = 0, y = x$. B1B1
- (c) Graph G2
- (d) $f(1) = 5, f(5) = 5.8$ M1
 so upper end of $f(A) = 5.8$ A1
 Since there is a minimum at $(2, 4)$, M1
 Lower end of $f(A) = 4$ A1
 ie $f(A) = [4, 5.8]$
7. (a) $z^n = \cos n\theta + i \sin n\theta$ B1
 $\frac{1}{z^n} = \frac{1}{\cos n\theta + i \sin n\theta} = \cos n\theta - i \sin n\theta$ M1A1
 [Accept $z^{-n} = \cos(-n\theta) + i \sin(-n\theta) = \cos(n\theta) - i \sin(n\theta)$]
 Adding,
 $z^n + \frac{1}{z^n} = 2 \cos n\theta$ AG
- (b) $\left(z + \frac{1}{z}\right)^5 = z^5 + 5z^4 \cdot \left(\frac{1}{z}\right) + 10z^3 \cdot \left(\frac{1}{z}\right)^2 + 10z^2 \cdot \left(\frac{1}{z}\right)^3 + 5z \cdot \left(\frac{1}{z}\right)^4 + \left(\frac{1}{z}\right)^5$ M1A1
 $= z^5 + \frac{1}{z} + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$ A1
 $(2 \cos \theta)^5 = 2 \cos 5\theta + 10 \cos 3\theta + 20 \cos \theta$ M1
 $\cos^5 \theta = \frac{1}{16} \cos 5\theta + \frac{5}{16} \cos 3\theta + \frac{5}{8} \cos \theta$ A1

8. (a) For $x = 1$, the values given by the two forms of $f(x)$ are
 $4 \cdot 1^2 = 4$ and $(1+1)^2 = 4$ M1
 These are equal so the function is continuous. A1
- (b) For $0 < x < 1$,
 $f'(x) = 8x > 0$ for all x B1
 For $1 \leq x < 2$,
 $f'(x) = 2(x+1) > 0$ for all x B1
- (c) For $0 < x < 1$ and therefore $0 < y < 4$, put
 $y = x^2$ so $x = \sqrt{y}$ M1A1
 So for $0 < x < 4$,
 $f^{-1}(x) = \sqrt{x/4}$ A1
 [correct domain required]
 For $1 \leq x < 2$ and therefore $4 \leq y < 9$, put
 $y = (x+1)^2$ so $x = \sqrt{y} - 1$ M1A1
 So for $4 \leq x < 9$,
 $f^{-1}(x) = \sqrt{x} - 1$ A1
 [correct domain required]

MATHEMATICS FP3

1. $dx = 2\cosh\theta d\theta$; $[0,1] \rightarrow [\sinh^{-1}(1/2), \sinh^{-1} 1]$ B1B1
- $$I = \int_{\sinh^{-1} 0.5}^{\sinh^{-1} 1} \frac{2\cosh\theta d\theta}{\sqrt{4\sinh^2\theta - 4\sinh\theta + 1 + 4\sinh\theta - 2 + 5}}$$
- M1A1
- $$= \int_{\sinh^{-1} 0.5}^{\sinh^{-1} 1} \frac{2\cosh\theta d\theta}{2\cosh\theta}$$
- A1
- $$= [\theta]_{\sinh^{-1} 0.5}^{\sinh^{-1} 1}$$
- A1
- $$= \sinh^{-1} 1 - \sinh^{-1} 0.5$$
- A1
- $$= 0.400$$
- A1
2. (a) $f'(x) = 3x^2 + 6x + 6$ B1
- $$= 3[(x+1)^2 + 1]$$
- M1
- $$> 0 \text{ for all } x.$$
- A1
- The equation has 1 real root. B1
- (b)(i) $f(0) = -5, f(1) = 5$, sign change means root in $(0,1)$ B1
- (ii) The Newton-Raphson iteration is
- $$x \rightarrow x - \frac{x^3 + 3x^2 + 6x - 5}{3x^2 + 6x + 6}$$
- M1A1
- Taking $x_0 = 0.5$, we obtain the following values M1
- 0.6153846154 A1
- 0.6097099622
- 0.6096954941 A1
- Required value = 0.6097 A1
- 3 (a) $2x + 2y \frac{dy}{dx} = 0$ M1
- $$\frac{dy}{dx} = -\frac{x}{y}$$
- A1
- $$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{y^2}$$
- M1
- $$= \frac{x^2 + y^2}{y^2} = \frac{a^2}{y^2}$$
- A1
- (b) $CSA = 2\pi \int_{-a}^a y \cdot \sqrt{\frac{a^2}{y^2}} dx$ M1A1
- $$= 2\pi a [x]_{-a}^a$$
- A1
- $$= 2\pi a \cdot 2a$$
- A1
- $$= 4\pi a^2$$
- AG

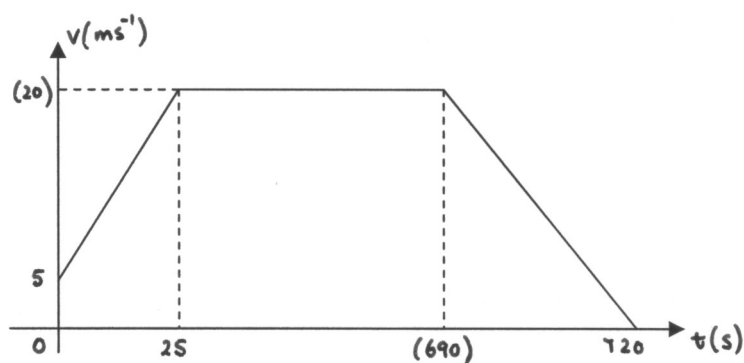
4. (a) At the point of intersection,
 $e^{\theta} = 2e^{-\theta}$ M1
 $e^{2\theta} = 2$ A1
 $\theta = \frac{1}{2} \ln 2$ A1
 $r = \sqrt{2}$ A1
- (b) C2 Area = $\frac{1}{2} \int_0^{0.5 \ln 2} 4e^{-2\theta} d\theta = \left[-e^{-2\theta} \right]_0^{0.5 \ln 2}$ (1/2) M1A1
C1 area = $\frac{1}{2} \int_0^{0.5 \ln 2} e^{2\theta} d\theta = \frac{1}{4} \left[e^{2\theta} \right]_0^{0.5 \ln 2}$ (1/4) M1A1
Reqd area = difference = 1/4 A1
5. (a) $a \cosh x + b \sinh x = r \cosh x \cosh \alpha + r \sinh x \sinh \alpha$ M1
so $r \cosh \alpha = a$ and $r \sinh \alpha = b$ A1
Dividing,
 $\tanh \alpha = \frac{b}{a}$ m1
 $\alpha = \tanh^{-1}(b/a)$ A1
 $= \frac{1}{2} \ln \left(\frac{1+b/a}{1-b/a} \right)$ A1
= printed result
Squaring and subtracting,
 $r^2 (\cosh^2 \alpha - \sinh^2 \alpha) = a^2 - b^2$ M1A1
 $r = \sqrt{a^2 - b^2}$ A1
- (b) In this case, $\alpha = \ln 2$ and $r = 4$ B1
so $4 \cosh(x + \ln 2) = 4$ B1
 $x + \ln 2 = 0$ so $x = -\ln 2$ M1A1
- [Accept solution using first principles giving M1A1 for reaching $4e^{2x} - 4e^x + 1 = 0$]
6. (a) $f'(x) = \frac{1}{\tan(\pi/4 + x)} \cdot \sec^2(\pi/4 + x)$ M1A1
 $= \frac{1}{\sin(\pi/4 + x) \cos(\pi/4 + x)}$ A1
 $= 2 \operatorname{cosec}(\pi/2 + 2x)$ A1
 $= 2 \sec 2x$ AG

(b)	$f(0) = 0$	B1
	$f'(0) = 2$	B1
	$f''(x) = 4 \sec 2x \tan 2x, (f''(0) = 0)$	B1
	$f'''(x) = 8 \sec^3 2x + 8 \sec 2x \tan^2 2x$	B1
	$f'''(0) = 8$	B1
	The series expansion is	
	$f(x) = 2x + \frac{4}{3}x^3 + \dots$	M1A1
	[FT on values of derivatives]	
(c)	$2x + \frac{4}{3}x^3 = 10x^3$	M1
	$x^2 = \frac{3}{13}$	A1
	$x = 0.48$	A1
7.	(a)	
	$I_n = -\int_0^1 x^n d\left(-\frac{2}{5}(1-x)^{5/2}\right)$	M1
	$= \left[-\frac{2}{5}x^n(1-x)^{5/2}\right]_0^1 + \int_0^1 \frac{2n}{5}x^{n-1}(1-x)^{5/2} dx$	A1A1
	$= \frac{2n}{5} \int_0^1 x^{n-1}(1-x)(1-x)^{3/2} dx$	A1A1
	$= \frac{2n}{5}I_{n-1} - \frac{2n}{5}I_n$	A1
	$\left(1 + \frac{2n}{5}\right)I_n = \frac{2n}{5}I_{n-1}$	A1
	whence the printed result	
	(b)	
	$I_0 = \int_0^1 (1-x)^{3/2} dx$	M1
	$= \left[-\frac{2}{5}(1-x)^{5/2}\right]_0^1$	A1
	$= \frac{2}{5}$	A1
	$I_2 = \frac{4}{9}I_1$	B1
	$= \frac{4}{9} \cdot \frac{2}{7}I_0$	B1
	$= \frac{16}{315}(0.05)$	B1

MATHEMATICS M1

1. (a) Using $v = u + at$ with $u = 5$, $a = 0.6$, $t = 25$ M1
 $v = 5 + 0.6 \times 25$ A1
 $v = \underline{20 \text{ ms}^{-1}}$ A1

(b)



M1 A1 A1

- (c) Using $v = u + at$ with $u = 20$, $v = 0$, $t = 30$ M1
 $a = -\frac{20}{30}$

magnification of deceleration $= -\frac{2}{3}$ ft(a) A1

- (d) Distance = Area under graph used M1
Distance $= 0.5(5 + 20)25 + 20(690 - 25) + 0.5(20)(30)$ B1(ft)
Distance $= \underline{13912.5\text{m}}$ A1(ft)

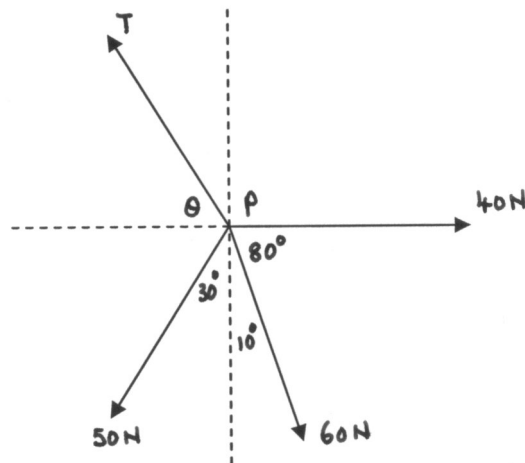
2. (a) Using $s = ut + 0.5at^2$ with $s = (-)1.75$, $a = (-)9.8$, $t = 2.5$ M1
 $-1.75 = 25u - 4.9 \times 2.5^2$ A1
 $u = \underline{11.55\text{ms}^{-1}}$

- (b) Using $v^2 = u^2 + 2as$ with $v = 0$, $u = 11.55$ (c), $a = (-)9.8$ M1
 $0 = 11.55^2 - 2 \times 9.8s$ A1
 $s = 6.80625$
Therefore greatest height above ground $= 6.80625 + 1.75$
 $= \underline{8.55625 \text{ m}}$ cao A1

- (c) Using $v = u + at$ with $u = 11.55$ (c), $a = (-)9.8$, $t = 2.5$ M1
 $v = 11.55 - 9.8 \times 2.5$ A1
 $v = \underline{-12.95 \text{ ms}^{-1}}$
Therefore speed $= \underline{12.95 \text{ ms}^{-1}}$ cao A1

- (d) Speed after bounce $= 0.8 \times 12.95$ M1
 $= \underline{10.36 \text{ ms}^{-1}}$ ft(c) A1

3.



Resolve in any direction.

$$T \cos \theta + 50 \sin 30^\circ = 40 + 60 \cos 80^\circ$$

$$T \cos \theta = 40 - 25 + 60 \cos 80^\circ$$

$$= 15 + 60 \cos 80^\circ$$

Resolve in a direction to obtain independent equation

$$T \sin \theta = 50 \cos 30^\circ + 60 \cos 10^\circ$$

sensible attempt to eliminate variable.

$$\tan \theta = \frac{50 \cos 30^\circ + 60 \cos 10^\circ}{15 + 60 \cos 80^\circ}$$

$$\theta = 76.06^\circ$$

$$T = \sqrt{(15 + 60 \cos 80^\circ)^2 + (50 \cos 30^\circ + 60 \cos 10^\circ)^2}$$

$$T = 105.5 \text{ N}$$

M1

B1 A1

M1

A1 A1

M1

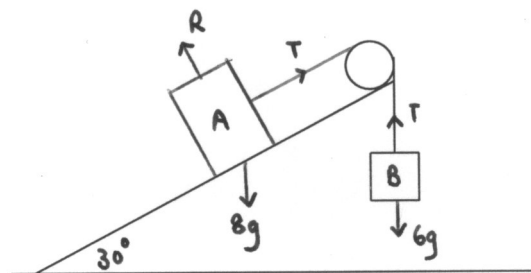
cao

A1

cao

A1

4.



(a) Apply N2L to B

$$6g - T = 6a$$

Apply N2L to A

$$T - 8g \sin 25^\circ = 8a$$

$$\text{Adding } 6g - 8g \sin 25^\circ = 14a$$

$$a = 1.4 \text{ ms}^{-2}$$

$$T = 6(g - a)$$

$$T = 50.4 \text{ N}$$

M1

A1

M1

A1

m1

cao

A1

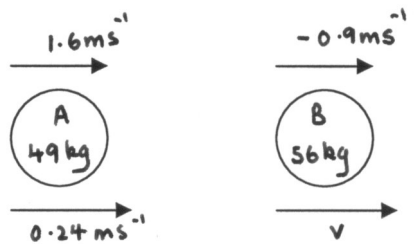
cao

A1

(b) Magnitude of acceleration of A and B are equal.

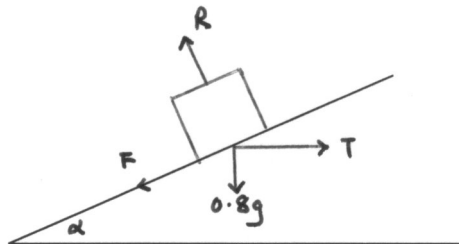
B1

5.



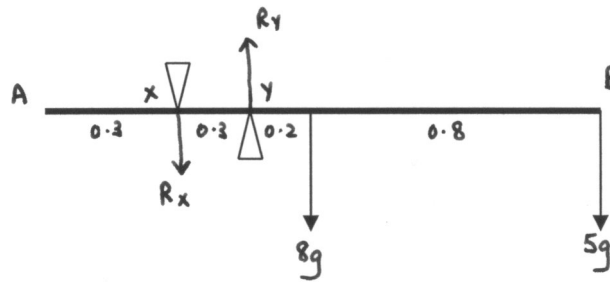
- (a) Conservation of momentum M1
 $49 \times 1.6 - 56 \times 0.9 = 49 \times 0.24 + 56v$
 $v = \underline{0.29}$ convincing A1
- (b) Restitution M1
 $0.29 - 0.24 = -e(-0.9 - 1.6)$ A1
 $e = \frac{0.05}{2.5} = 0.02$ cao A1
- (c) $|I| = 56(0.29 + 0.9)$ M1
 $= \underline{66.64 \text{ Ns}}$ A1 B1
- (d) Objects are modelled as particles. B1

6.



- Resolve vertically all forces, dim. correct M1
 $R \cos \alpha = 0.8g + F \sin \alpha$ A1
- $F = 0R$ M1
 $0.8R = 0.8g + 0.4R \times 0.6$
 $R = 14 \text{ N}$ substitution of F A1
- Resolve horizontally all forces, dim. Correct M1
 $R \sin \alpha + F \cos \alpha = T$ A1
- $14 \times 0.6 + 0.4 \times 14 \times 0.8 = T$ elimination of variable m1
 $T = \underline{12.88 \text{ N}}$ cao A1

7.



Moments about Y

$$0.3 R_X = 0.2 \times 8g + 1 \times 5g$$

$$R_X = 22g$$

$$R_X = \underline{215.6 \text{ N}}$$

all forces

M1

A1 B1

cao

A1

Resolve vertically

$$R_Y = R_X + 8g + 5g$$

$$R_Y = 35g = \underline{343 \text{ N}}$$

ft R_X only

M1

A1

A1

8. (a) Particle mass from AC from AB

P 2m 7.5 0

Q 3m 2.4 4.2

R 5m 0 3.5

B1

B1

B1

(i) Moments about AC

equation

M1

$$10 \bar{x} = 7.5 \times 2 + 2.4 \times 3 + 0 \times 5$$

ft

A1

$$10 \bar{x} = 22.2$$

$$\bar{x} = \underline{2.22 \text{ cm}}$$

cao

A1

(ii) Moments about AB

M1

$$10 \bar{y} = 0 \times 2 + 4.2 \times 3 + 3.5 \times 5$$

ft

A1

$$10 \bar{y} = 30.1$$

$$\bar{y} = \underline{3.01 \text{ cm}}$$

cao

A1

(b) $\theta = \tan^{-1} \left(\frac{3.01}{8 - 2.22} \right)$

ft

M1 A1

$$\theta = \underline{27.51^\circ}$$

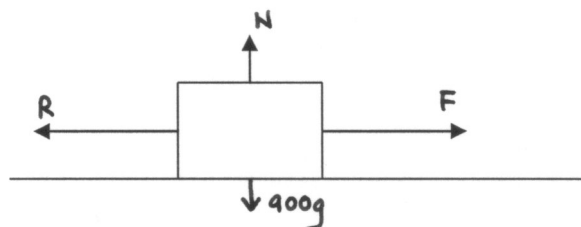
ft

A1

MATHEMATICS M2

1. (a) When P is at rest, $v = 0$. used M1
 $3t^2 - 24t + 45 = 0$
 $3(t-3)(t-5) = 0$
 Therefore P first comes to rest when $t = 3$. A1
- (b) $a = \frac{dv}{dt}$ used M1
 $a = \underline{6t - 24}$ A1
- (c) Displacement $s = \int v \, dt$ used M1
 $s = t^3 - 12t^2 + 45t (+ C)$ A1
 When $t = 0, s = 0$, therefore $C = 0$
 $s = t^3 - 12t^2 + 45t$ A1
- (d) Distance travelled in the first 3 s $= 3^3 - 12(3)^2 + 45(3)$ M1
 $= 27 - 108 + 135$
 $= \underline{54 \text{ m}}$ ft A1
- (e) Displacement after 4s $= 4^3 - 12(4)^2 + 45(4)$
 $= \underline{52 \text{ m}}$ B1
 Distance travelled in 4 s $= 54 + (54 - 52)$
 $= \underline{56 \text{ m}}$ B1

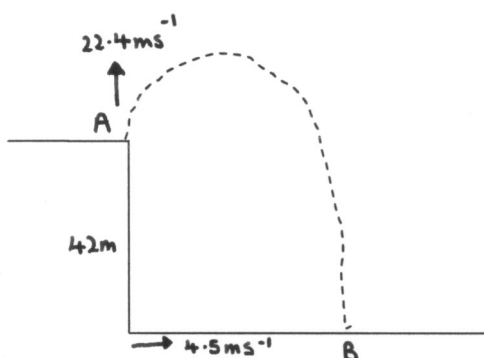
2.



- (a) At maximum speed $F = R$ used M1
 $F = \frac{P}{V}$ used M1
 Therefore $1800 = \frac{45 \times 1000}{V}$
 $V = \underline{25 \text{ ms}^{-1}}$ cao A1
- (b) N2L all forces, dim. cor. M1
 $F - R - mg \sin 4^\circ = ma$ A1
 $F = \frac{45000}{15} (= 3000)$ B1
 $3000 - 1800 - 900 \times 9.8 \sin 4^\circ = 900 \times a$ A1
 $a = \underline{0.65 \text{ ms}^{-2}}$ cao A1
- (c) W.D. $= F.d$ used M1
 $= 1800 \times 800 = \underline{1440000 \text{ J}}$ cao A1

3. (a) PE at start of motion = $3 \times 9.8 \times (0.8 + 0.4)$ M1 A1
 $= 35.28 \text{ J}$
- EE at end of motion = $\frac{1}{2} \lambda \frac{(0.4)^2}{0.8}$ M1 A1
 $= 0.1 \lambda$
- Energy consideration = M1
 $0.1 \lambda = 35.28$ A1
 $\lambda = \underline{352.8 \text{ N}}$ A1 convincing
- (b) Hooke's Law $T = \frac{\lambda x}{l}$ used M1
 $= \frac{352.8 \times 0.4}{0.8}$
 $= \underline{176.4 \text{ N}}$ A1
- N2L dim cor. M1
 $T - 3g = 3a$ A1
 $176.4 - 3 \times 9.8 = 3a$
 $a = \underline{49 \text{ ms}^{-2}}$ A1

4.



- (a) Consider vertical motion
Using $v = u + at$ with $u = 22.4$, $a = (-)9.8$, $t = 2$. M1
 $v = 22.4 - 9.8 \times 2$ A1
 $v = 2.8 \text{ ms}^{-1}$ A1
Speed = $\sqrt{4.5^2 + 2.8^2}$ M1
 $= \underline{5.3 \text{ ms}^{-1}}$ A1
- (b) Using $s = ut + 0.5at^2$ with $s = (-)42$, $u = 22.4$, $a = (-)9.8$ M1
 $-42 = 22.4t - 4.9t^2$ A1
 $0.7t^2 - 3.2t - 6 = 0$
 $t = \frac{3.2 \pm \sqrt{3.2^2 + 4 \times 0.7 \times 6}}{2 \times 0.7}$ m1
 $T = \underline{6 \text{ s}}$ (other solution negative) A1
- (c) Horizontal distance between A and B = 4.5×6 M1
 $= \underline{27 \text{ m}}$ A1

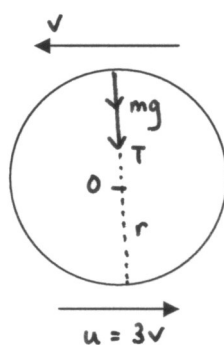
5. (a) **a, b perpendicular** $\Rightarrow$ **$\mathbf{a} \cdot \mathbf{b} = 0$** M1
 $\mathbf{a} \cdot \mathbf{b} = -2 + 13y - 50$ M1 A1
 $-2 + 13y - 50 = 0$
 $y = \underline{4}$ A1

(b) **a, b parallel** $\Rightarrow$ **$\mathbf{a} = -2\mathbf{b}$** M1
 $-2y = 13$
 $y = \underline{-6.5}$ A1

6. (a) Angular velocity $\omega = \frac{v}{r}$ used M1
 $\omega = \frac{3}{0.4}$
 $\omega = \underline{7.5 \text{ rad s}^{-1}}$ A1
 cao

(b) Tension in the string $T = \frac{mv^2}{r}$ oe M1
 $T = \frac{0.8 \times 3^2}{0.4} = \underline{18 \text{ N}}$ A1
 cao

7.



(a) Conservation of energy M1
 $0.5mu^2 = 0.5mv^2 + mg \times 2r$ A1 A1
 $9v^2 = v^2 + 9.8 \times 2 \times 0.9 \times 2$ B1
 $8v^2 = 35.28$
 $v = 2.1$
 and $u = 6.3$ convincing A1

(b) N2L towards centre O M1
 $T + mg = \frac{mv^2}{r}$ A1
 $T = \frac{3 \times 2.1^2}{0.9} - 3 \times 9.8$ A1
 $T = \underline{-14.7 \text{ N}}$ A1
 cao

(c) Object would not move in complete circles as T is negative,
 i.e. rod exerted a thrust which a string cannot exert. B1 E1

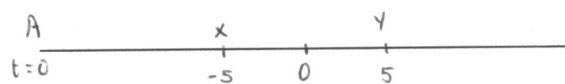
8. (a) $\mathbf{r}_A = (0\mathbf{i} + 3\mathbf{j} - 140\mathbf{k}) + t(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ M1 A1
 $\mathbf{r}_B = (-9\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}) + t(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$ A1
- (b) $\mathbf{r}_A - \mathbf{r}_B = (9 + 5t)\mathbf{i} + (7 - 8t)\mathbf{j} + (2t - 134)\mathbf{k}$ si M1 A1
 $AB^2 = (9 + 5t)^2 + (7 - 8t)^2 + (2t - 134)^2$ ft B1
 $AB^2 = 93t^2 - 558t + 18086$
- (c) At minimum distance $\frac{dAB^2}{dt^2} = 0$ M1
- $\frac{dAB^2}{dt^2} = 2(9 + 5t)(5) + 2(7 - 8t)(-8) + 2(2t - 134)(2)$ m1 A1
 $45 + 25t - 56 + 64t + 4t - 268 = 0$
 $93t - 279 = 0$
 $t = \underline{3\text{ s}}$ cao A1

MATHEMATICS M3

1.	(a)	(i)	N2L $F = ma$ $400 - 16v^2 = 800a$	used	M1 A1
			Divide by 800 and using $a = \frac{dv}{dt}$		
			$\frac{dv}{dt} = \frac{400 - 16v^2}{800}$	convincing	A1
			$\frac{dv}{dt} = \frac{25 - v^2}{50}$		
		(ii)	$50 \int \frac{dv}{25 - v^2} = \int dt$	sp.var.	M1
			$\frac{50}{2 \times 5} \ln \left \frac{5+v}{5-v} \right = t + C$		A2
			When $t = 0, x = 0.$ $C = 0$	ft	m1 A1
			$t = 5 \ln \left \frac{5+v}{5-v} \right $		
			When $v = 2,$		
			$t = 5 \ln \left \frac{7}{3} \right = \underline{4.24 \text{ s}}$	cao	A1
	(b)		$v \frac{dv}{dx} = \frac{25 - v^2}{50}$		M1
			$\frac{50}{2} \int \frac{2v}{25 - v^2} dv = \int dx$		m1
			$-25 \ln 25 - v^2 = x + C$		A2
			When $x = 0, v = 0.$ $C = -25 \ln 25 $	ft +/-	m1 A1
			When $v = 2,$ $x = 25 \ln 25 - 25 \ln 21$ $x = \underline{4.36 \text{ m}}$	cao	A1

2. (a) Period = $\frac{2\pi}{\omega} = 2 \times 4$ M1
 $\omega = 0.25\pi$ A1
Max speed = $a\omega = 3\pi$ M1
 $a = 3\pi / 0.25\pi$
 $a = \underline{12 \text{ m}}$ convincing A1

(b)

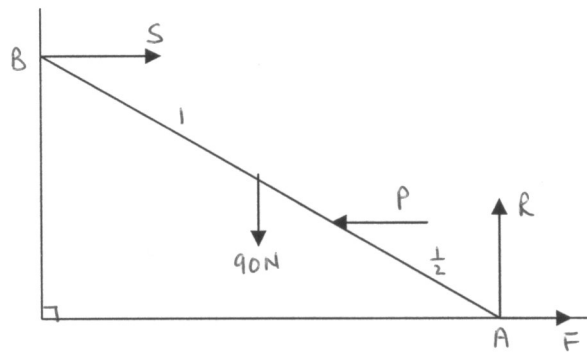


$x = -12 \cos(0.25t)$ +/- M1
When $t = \frac{2}{3}$ $x = -12 \cos(0.25\pi \times \frac{2}{3})$ m1
 $x = -12 \times \frac{\sqrt{3}}{2} = -6\sqrt{3}$ +/- A1
Distance of P from A when $t = \frac{2}{3} = 12 - 6\sqrt{3} = \underline{1.61 \text{ m}}$ cao A1

(c) $v = \frac{dx}{dt}$ used M1
 $v = -12 \cdot -\sin(0.25t) \cdot (0.25\pi)$
 $v = 3\pi \sin(0.25t)$ ft A1
When $t = \frac{2}{3}$, $v = 3\pi \sin(0.25\pi \times \frac{2}{3}) = \underline{1.5\pi \text{ ms}^{-1}}$ cao A1

(d) At X, $x = -5$ M1
 $-5 = -12 \cos(0.25\pi)$
 $t_X = \underline{1.4528 \text{ s}}$ ft ω A1
At Y, $x = 5$
 $5 = -12 \cos(0.25\pi)$
 $t_Y = \underline{5.5472 \text{ s}}$ ft ω A1
Therefore required time = $t_Y - t_X = 1.0944 = \underline{1.09 \text{ s}}$ cao A1

3. (a)



B1 B1

(b) Resolve vertically upwards
 $R = 90$

M1
 A1

Resolve horizontally to the right
 $S + F = P$

ft diagram

M1
 A1

Moments about A

$$P \times 0.5 \cos \theta + 90 \sin \theta = S \times 2 \cos \theta$$

$$P + 180 \tan \theta = 4S$$

$$P - 0.6 \times 90 = S$$

$$F = \mu R$$

M1
 A1 A1

B1

Therefore

$$\begin{aligned}
 P + 180 \times 0.8 &= 4(P - 54) \\
 P + 144 &= 4P - 216 \\
 3P &= 360 \\
 P &= \underline{120 \text{ N}}
 \end{aligned}$$

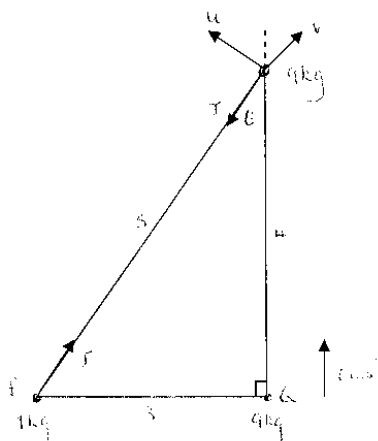
m1

A1

(c) I have assumed that the ladder is a rigid rod.

B1

4.



$\cos \theta = 0.8,$	$\sin \theta = 0.6$	si	B1
$u = 6 \sin \theta$			M1
$u = 6 \times 0.6 = 3.6 \text{ ms}^{-1}$		si	A1

Impulse = change in momentum

For P		M1
$J = 7v$		A1

For Q		M1
$9 \times 6 \cos \theta - J = 9v$		A1

Adding $54 \times 0.8 = 16v$		m1
$v = 2.7 \text{ ms}^{-1}$		A1
Speed of $P = \underline{2.7 \text{ ms}^{-1}}$		

Speed of $Q = \sqrt{2.7^2 + 3.6^2}$		M1
$= \underline{4.5 \text{ ms}^{-1}}$	ft 2.7(c)	A1

$J = 7 \times 2.7$		
$= \underline{18.9 \text{ Ns}}$	ft 2.7(c)	A1

Required angle $\alpha = \tan^{-1}\left(\frac{u}{v}\right)$		M1
$= \tan^{-1}\left(\frac{3.6}{2.7}\right)$		
$= \underline{53.13^\circ}$	ft	A1

5.	(a)	N2L			M1
		$(6120 - 80t) - (120 + 40v) = 800a$			A1
		$6000 - 40v - 80t = 800a$			A1
		Divide by 40			
		$150 - \frac{dx}{dt} - 2t = 20 \frac{d^2x}{dt^2}$			B1
		$20 \frac{d^2x}{dt^2} + \frac{dx}{dt} = 150 - 2t$			a1
	(b)	Auxiliary equation	$20m^2 + m = 0$		M1
			$m = 0, -0.05$	both	A1
		Complementary Function is	$x = A + Be^{-0.05t}$	ft m	B1
		Particular integral, try $x = at^2 + bt$			M1
		$\frac{dx}{dt} = 2at + b, \frac{d^2x}{dt^2} = 2a$			
		$20(2a) + (2at + b) = 150 - 2t$			A1
		$2a = -2$		comp.coef.	m1
		$a = -1$			
		$-40 + b = 150$			
		$b = 190$		both cao	A1
		General solution is $x = A + B^{-0.05t} - t^2 + 190t$			B1
		When $t = 0, x = 0 \Rightarrow A + B = 0$			m1
		$\frac{dx}{dt} = -0.05Be^{-0.05t} - 2t + 190$		ft	B1
		When $t = 0, \frac{dx}{dt} = 0 \Rightarrow 0 = -0.05B + 190$			m1
		$B = 3800$			
		$A = -3800$		both cao	A1
		Therefore $x = 3800(e^{-0.05t} - 1) - t^2 + 190t$			

MATHEMATICS S1

1. (a) $P(A \cap B) = 0.6 \times 0.3$ B1
 $P(A \cup B) = 0.6 + 0.3 - 0.6 \times 0.3$ M1A1
 $= 0.72$ A1
- (b) $P(A' \cap B') = (1 - 0.6)(1 - 0.3)$ or $1 - 0.72$ M1A1
 $= 0.28$ A1
- (c) $P(A | A \cup B) = \frac{P(A)}{P(A \cup B)}$
 [FT their $P(A \cup B)$ from (a)]
 $= \frac{0.6}{0.72}$ B1B1
 $= \frac{5}{6}$ B1
 [Award final B1 only if previous 2 B1s are awarded]
2. (a) (i) $P(X = 5) = e^{-4.5} \times \frac{4.5^5}{5!} = 0.171$ M1A1
- (ii) $P(X \leq 2) = e^{-4.5} (1 + 4.5 + \frac{4.5^2}{2})$ M1A1
 $= 0.17$ (cao) A1
- (b) $P(3 \leq X \leq 7) = 0.9134 - 0.1736$ or $0.8264 - 0.0866$ B1B1
 $= 0.740$ (cao) B1
3. $E(Y) = 5a - b = 0$ M1A1
 $\text{Var}(Y) = 4a^2 = 1$ M1A1
 $a = \frac{1}{2}, b = \frac{5}{2}$ (cao) A1A1
4. (a) There are 36 possible pairs B1
 $P(A) = \frac{15}{36} \left(\frac{5}{12} \right)$ M1A1
- (b) $P(B) = \frac{10}{36} \left(\frac{5}{18} \right)$ M1A1
- (c) $P(A \cap B) = \frac{5}{36}$ M1A1
 Attempting to compare $P(A \cap B)$ with $P(A)P(B)$ or $P(A | B)$ with $P(A)$ or $P(B | A)$ with $P(B)$ M1
 $P(A)P(B) = \frac{25}{216} \neq P(A \cap B)$ or $P(A | B) = 0.5 \neq P(A)$ or $P(B | A) = 1/3 \neq P(B)$ A1
 A and B are therefore not independent. A1
 (FT from previous line)

5.	(a)	(i)	X is $B(5, 0.4)$	(parameters si)	B1
		(ii)	Mean = 2 SD = $\sqrt{5 \times 0.4 \times 0.6} = 1.10$		B1 M1A1
		(iii)	$P(X \geq 3) = 0.317$		M1A1
	(b)		Y is $B(24, 0.05)$ and therefore approx $Po(1.2)$. $P(Y \leq 2) = 0.88$		M1A1 M1A1
6.	(a)		Sum of probs = 1 so $k(1 + 4 + 9 + 16) = 1 \Rightarrow k = \frac{1}{30}$		M1 A1
	(b)		$E(X) = \frac{1}{30}(1 + 8 + 27 + 64)$ $= \frac{10}{3}$		M1A1 A1
			$E(X^2) = \frac{1}{30}(1 + 16 + 81 + 256) (= 11.8)$ $\text{Var}(X) = 354/30 - (10/3)^2$ $= 31/45 \quad (0.69) \text{ (cao)}$		M1A1 M1 A1
	(a)		$E\left(\frac{1}{X}\right) = \frac{6}{5} \int_1^2 \frac{1}{x} \cdot x(x-1) dx$ $= \frac{6}{5} \left[\frac{x^2}{2} - x \right]_1^2$ $= \frac{3}{5}$		M1A1 A1 A1
	(b)	(i)	$F(x) = \int_1^x \frac{6}{5} (y^2 - y) dy$ $= \frac{6}{5} \left[\frac{y^3}{3} - \frac{y^2}{2} \right]_1^x$ $= \frac{2}{5} x^3 - \frac{3}{5} x^2 + \frac{1}{5}$		M1A1 A1 A1
		(ii)	$P(X \leq 1.75) = F(1.75) = 0.506$	(81/160)	M1A1
		(iii)	The median is less than 1.75 because the answer to (ii) > 0.5 (FT on answer to (ii))	(oe)	B1 B1

$$8. \quad (a) \quad P(\text{Red}) = \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{2}{5} + \frac{1}{3} \times \frac{3}{5} = \frac{2}{5}$$

M1A1
A1

$$(b) \quad P(\text{Box A} \mid \text{Red}) = \frac{1/15}{2/5} = \frac{1}{6}$$

B1B1
B1

$$(c) \quad P(\text{Box B} \mid \text{Red}) = \frac{1}{3}$$

B1

$$P(\text{Box C} \mid \text{Red}) = \frac{1}{2}$$

B1

$$P(\text{Red}) = \frac{1}{6} \times 0 + \frac{1}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{2}{4} = \frac{1}{3}$$

M1A1

Alternative solution:

$$P(1^{\text{st}} \text{ ball red and } 2^{\text{nd}} \text{ ball from chosen box red}) = \frac{1}{3} \times \frac{2}{5} \times \frac{1}{4} + \frac{1}{3} \times \frac{3}{5} \times \frac{1}{2} = \frac{2}{15}$$

M1A1

$$P(2^{\text{nd}} \text{ ball from chosen box red} \mid 1^{\text{st}} \text{ ball red}) = \frac{2/15}{2/5} = 1/3$$

B1B1

MATHEMATICS S2

1. (a) $\bar{x} = \frac{29.43}{9} (= 3.27)$ B1
- SE of $\bar{X} = \frac{0.15}{\sqrt{9}} (= 0.05)$ B1
- 90% conf limits are
 $3.27 \pm 1.645 \times 0.05$ M1A1
- [M1 correct form, A1 1.645, FT on their mean and SE]
giving [3.19, 3.35] A1
- (b) We solve
 $\frac{0.15}{\sqrt{n}} = 0.5 \times 0.05$ giving $n = 36$ (cao) M1A1
2. (a) (i) Using $\text{Var}(X) = E(X^2) - [E(X)]^2$ M1
- $2 = E(X^2) - 4$ A1
- $E(X^2) = 6$
- Similarly,
 $E(Y^2) = 12$ B1
- (ii) $E(X^2Y^2) = E(X^2)E(Y^2) = 72$ M1A1
- (b) $\text{Var}(U) = E(X^2Y^2) - [E(XY)]^2$ M1
- $= 72 - (2 \times 3)^2$ A1
- $= 36$ A1
- SD = 6 A1
3. (a) (i) $z = \frac{80 - 75}{5} = 1$ M1A1
- Prob = 0.8413 (cao) A1
- (ii) $z = 0.674$ B1
- UQ = $75 + 0.674 \times 5$ M1
- $= 78.4$ A1
- (b) Put $U = X_1 + X_2 - (Y_1 + Y_2 + Y_3)$
- $E(U) = -18$ (accept $\pm$) B1
- $\text{Var}(U) = 2 \times 5^2 + 3 \times 4^2 = 98$ M1A1
- $z = \frac{18}{\sqrt{98}} = (\pm)1.82$ M1A1
- [FT on mean and variance] Prob = 0.9656 (cao) A1

4. (a) $f(r) = 1/5$ ($0 \leq r \leq 5$, $= 0$ otherwise) B1
[Accept labelled sketch]
- (b) $E(A) = \int_0^5 \pi r^2 \cdot \frac{1}{5} dr$ M1A1
 $= \frac{\pi}{15} [r^3]_0^5$ A1
 $= \frac{25\pi}{3}$ (cao) A1
- Alternative solution
 $E(R^2) = \text{Var}(R) + [E(R)]^2$ M1
 $= \frac{1}{12} \times 5^2 + \left(\frac{5}{2}\right)^2$ A1A1
 $E(A) = \frac{25\pi}{3}$ (cao) A1
- (c) $P(\pi R^2 > 25) = P(R > \sqrt{\frac{25}{\pi}})$ M1A1
 $= \frac{5 - \sqrt{25/\pi}}{5}$ m1
 $= 0.436$ A1
5. (a) $H_0 : p = 0.75$ versus $H_1 : p < 0.75$ B1
- (b) (i) Under H_0 , X is B(20, 0.75) (si) B1
and Y (No of misses) is B(20, 0.25) (si) M1A1
Using tables, we find that $k = 12$ B2
- (ii) We require
 $P(X > 12 | p = 0.5)$ M1
 $= 0.132$ A1
- [Do not accept the use of a normal approximation in this question]
6. (a) X is Poi(14) (si) B1
- (i) $P(X = 10) = e^{-14} \times \frac{14^{10}}{10!} = 0.0663$ M1A1
- (ii) $P(X > 12) = 0.6415$ M1A1
- (b) (i) $H_0 : \mu = 2$ versus $H_1 : \mu > 2$ (Accept 14) B1
- (ii) $p\text{-value} = P(X \geq 20)$ M1
 $= 0.0765$ A1
We cannot conclude that the mean has increased. B1
[Do not accept the use of a normal approximation in (a) or (b)(ii)]

(iii)	Under H_0 , X is $Po(200) \approx N(200, 200)$	M1A1
	$z = \frac{229.5 - 200}{\sqrt{200}}$	M1A1A1
	$= 2.09$	A1
	$p\text{-value} = 0.0183$	A1
	Strong evidence that the mean has increased.	B1
	[No c/c gives $z = 2.12$, $p = 0.0170$; wrong c/c gives $z = 2.16$, $p = 0.0154$]	
7.	(a)	
	(i) $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$	B1
	(ii) $\bar{x}_1 = \frac{31.71}{5}$ (6.342)	B1
	$\bar{x}_2 = \frac{31.53}{5}$ (6.306)	B1
	SE of difference of means = $\sqrt{\frac{0.025^2 \times 2}{5}}$	M1
	$= 0.0158$	A1
	$z = \frac{6.342 - 6.306}{0.0158}$	M1
	$= 2.28$	A1
	Prob from tables = 0.0113	B1
	$p\text{-value} = 0.0226$	B1
	Strong evidence of a difference in acidity levels.	B1
(b)	95% confidence limits are $6.342 - 6.306 \pm 1.96 \times 0.0158$ giving $[0.005, 0.067]$	M1A1 A1

MATHEMATICS S3

1. (a) $P(X = 3) = P(1,2) + P(2,1) = \frac{1}{6} \times \frac{2}{5} \times 2 = \frac{2}{15}$ M1A1
- (b) $P(X = 4) = P(2,2) + P(1,3) + P(3,1)$ M1
- $$= \frac{2}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{3}{5} \times 2 = \frac{4}{15}$$
- A1A1
- $$P(X = 5) = P(2,3) + P(3,2) = \frac{2}{6} \times \frac{3}{5} \times 2 = \frac{2}{5}$$
- M1A1
- $$P(X = 6) = P(3,3) = \frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$$
- B1
2. (a) $\Sigma x = 246$; $\Sigma x^2 = 6055.74$ si B1B1
- UE of $\mu = 24.6$ B1
- $$\text{UE of } \sigma^2 = \frac{6055.74}{9} - \frac{246^2}{9 \times 10}$$
- M1
- $$= 0.46$$
- A1
- (b) Use of a t -value M1
- 95% confidence limits for μ are
- $$\bar{x} \pm t \times \text{SE}$$
- m1
- $$24.6 \pm 2.262 \sqrt{\frac{0.46}{10}}$$
- A1
- giving [24.1, 25.1] cao A1
3. (a) $\hat{p} = \frac{120}{200}$ B1
- $$\text{SE} = \sqrt{\frac{0.6 \times 0.4}{200}}$$
- B1
- Approx 95% confidence limits are
- $$\hat{p} \pm z \times \text{SE}$$
- M1
- giving [[0.532, 0.668] cao A1
- (b) Yes because the interval is entirely above 0.5 B1
- (c) We require
- $$2 \times 1.96 \sqrt{\frac{0.6 \times 0.4}{n}} = 0.05$$
- M1A1
- $$n = 0.24 \left(\frac{3.92}{0.05} \right)^2 = 1475/1476$$
- M1A1
- [Accept the use of $p = 0.5$ giving $n = 1537$]

4.	(a)	UE of $\mu = 2.56$ UE of $\text{Var}(X) = 2.56$	B1 B1
	(b)	$\text{ESE} = \sqrt{\frac{2.56}{100}} = 0.16$	M1A1
	(c)	Approx 90% confidence limits for μ are $2.56 \pm 1.645 \times 0.16$ giving [2.30, 2.82]	M1A1 A1
	(d)	The 1.645 requires the normality of $\bar{X}$.	B1
5.	(a)	$H_0 : \mu_x = \mu_y$ versus $H_1 : \mu_x < \mu_y$	B1
	(b)	$\bar{x} = 42.82; \bar{y} = 43.4$	B1B1
		$s_x^2 = \frac{185855}{99} - \frac{4282^2}{100 \times 99} = 25.250..$	M1A1
		$s_y^2 = \frac{230347}{119} - \frac{5208^2}{120 \times 119} = 36.300..$ [Accept division by n]	A1
		Test stat = $\frac{43.4 - 42.82}{\sqrt{\frac{36.300..}{120} + \frac{25.250..}{100}}}$	M1A1A1
		= ($\pm$)0.78	cao A1
		$p\text{-value} = 0.2177$ Insufficient evidence to support the psychologist's theory	A1 B1
6.	(a)	X_1 is $B(n, 3\theta)$, X_2 is $B(n, 1 - 4\theta)$ (si)	B1B1
		$E(U_1) = \frac{n \times 3\theta}{3n} = \theta$	M1A1
		$E(U_2) = \frac{n - n(1 - 4\theta)}{4n} = \theta$	M1A1
	(b)	(i) $\text{Var}(U_1) = \frac{n \cdot 3\theta(1 - 3\theta)}{9n^2} = \frac{\theta(1 - 3\theta)}{3n}$	M1A1
		$\text{Var}(U_2) = \frac{n \cdot 4\theta(1 - 4\theta)}{16n^2} = \frac{\theta(1 - 4\theta)}{4n}$	M1A1
		$\frac{\text{Var}(U_1)}{\text{Var}(U_2)} = \frac{\theta(1 - 3\theta)}{3n} \cdot \frac{4n}{\theta(1 - 4\theta)} = \frac{4(1 - 3\theta)}{3(1 - 4\theta)}$	A1AG
	(ii)	U_2 is better because it has the smaller variance.	M1A1

7.	(a)	$\Sigma x = 105, \Sigma y = 262$	si	B1
		$S_{xx} = 700$		B1
		$S_{xy} = 830$		B1
		$b = \frac{700}{830}$		M1
		$= 1.1857$		A1
		$a = \frac{262 - 105 \times 1.1857}{7}$		M1
		$= 19.6429$		A1
	(b)			
	(i)	$H_0 : \beta = 1.2$ versus $H_1 : \beta \neq 1.2$		B1
	(ii)	$SE \text{ of } b = \frac{0.25}{\sqrt{2275 - 105^2 / 7}} = 0.009449$		M1A1
		Test statistic = $\frac{1.1857 - 1.2}{0.009449}$		M1A1
		$= -1.51$		A1
		Prob from tables = 0.0656		A1
		$p\text{-value} = 0.1312$		B1
		We conclude that $\beta = 1.2$.		B1

Welsh Joint Education Committee
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk/exams.html

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