

976/01

**MATHEMATICS C4**

**Pure Mathematics**

P.M. THURSDAY, 14 June 2007

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Express  $\frac{x+3}{x^2(x-1)}$  in terms of partial fractions. [4]

(b) Find  $\int \frac{x+3}{x^2(x-1)} dx$ . [2]

2. Find the equation of the tangent to the curve

$$x^5 + xy^2 + y^3 = 17$$

at the point  $(-1, 3)$ . [4]

3. Find all values of  $x$  in the range  $0^\circ \leq x \leq 360^\circ$  satisfying the equation  $4\cos x + 2\sin x = 3$ . [7]

4. Expand  $(1+4x)^{\frac{1}{2}} - \frac{1}{1+3x}$  as far as the term in  $x^2$ . For what range of values of  $x$  is your expansion valid? [7]

5. The region bounded by the curve  $y = \sqrt{e^{2x} + 1}$ , the  $x$ -axis and the lines  $x = 0$ ,  $x = 1$  is rotated through four right-angles about the  $x$ -axis. Find the volume of the solid generated, giving your answer correct to three decimal places. [4]

6. The parametric equations of the curve  $C$  are  $x = 2t$ ,  $y = t^2$ .

- (a) Show that the normal to  $C$  at the point  $P$  with parameter  $p$  has equation

$$x + py = p^3 + 2p. \quad [4]$$

- (b) The normal to  $C$  at the point  $P$  intersects the  $x$ -axis at  $A$  and the  $y$ -axis at  $B$ . Given that  $O$  is the origin and  $OA = 2OB$ , find the value of  $p$ . [4]

7. (a) Find  $\int x^2 \ln x \, dx$ . [4]

- (b) Use the substitution  $x = 2\sin\theta$  to show that

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} \, dx = \int_0^a k \sin^2 \theta \, d\theta,$$

where the values of  $a$  and  $k$  are to be determined.

Hence, or otherwise, evaluate  $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} \, dx$ . [8]

8. The price £ $P$  of an item at time  $t$  years is to be modelled as a continuous variable such that the rate of increase of  $P$  is directly proportional to  $P$ .
- (a) Write down a differential equation that is satisfied by  $P$ . [1]
- (b) Given that the price of the item at  $t = 0$  is £50, show that  $P = 50e^{kt}$ , where  $k$  is a positive constant. [5]
- (c) After seven years the price of the item is £65. Find the price of the item after sixteen years. [4]
9. (a) The position vectors of the points  $A$  and  $B$ , relative to a fixed origin  $O$ , are  $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$  and  $3\mathbf{i} + 6\mathbf{j} + \mathbf{k}$ , respectively.
- (i) Find  $\mathbf{AB}$ .
- (ii) Find the vector equation of the line  $AB$ .
- (iii) The vector equation of the line  $L$  is  $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ .  
Given that  $L$  and  $AB$  intersect, find the position vector of the point of intersection. [9]
- (b) Find the angle between the vectors  $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and  $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ . [6]
10. Complete the following proof by contradiction to show that, if  $n$  is a positive integer and  $3n + 2n^3$  is odd, then  $n$  is odd. [2]
- It is given that  $3n + 2n^3$  is odd.*  
*Assume that  $n$  is even so that  $n = 2k$ .*