

981/01

MATHEMATICS M2

Mechanics 2

A.M. THURSDAY, 7 June 2007

(1 $\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9.8 ms^{-2} .

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

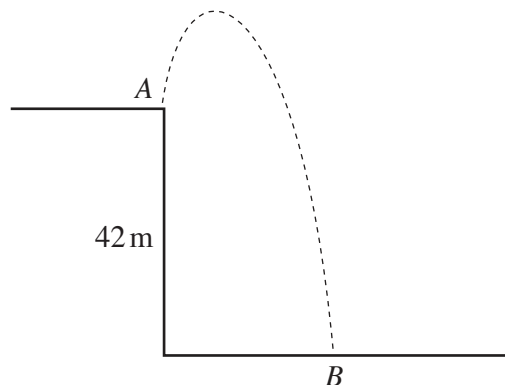
You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle P is projected from the origin O so that it moves along the x -axis. At time t s after projection, the velocity of the particle, $v \text{ ms}^{-1}$, is given by

$$v = 3t^2 - 24t + 45.$$

- (a) Show that P first comes to instantaneous rest when $t = 3$. [2]
 - (b) Find an expression for the acceleration of P at time t s. [2]
 - (c) Find an expression for the displacement of P from O at time t s. [3]
 - (d) Find the distance travelled by the particle in the first 3 seconds of its motion. [2]
 - (e) Find the distance travelled by the particle in the first 4 seconds of its motion. [2]
2. A car of mass 900 kg can produce a maximum power of 45 kW. The car experiences a constant resistance to motion of magnitude 1800 N.
- (a) Calculate the maximum speed of the car when travelling on a horizontal road. [3]
 - (b) The car travels up a slope inclined at an angle of 4° to the horizontal. Assuming maximum power is employed, calculate, correct to two decimal places, the acceleration of the car at the instant when its speed is 15 ms^{-1} . [5]
 - (c) The car travels a distance of 800 m. Calculate the work done against resistance. [2]
3. The end A of a light elastic string AB , of natural length 0.8 m, is fixed. A particle P , of mass 3 kg, is attached to the end B of the string. Initially, P is held at rest at the point A . It is then released and allowed to fall. The greatest extension of the string in the subsequent motion is 0.4 m.
- (a) Show that the modulus of elasticity of the string is 352.8 N. [7]
 - (b) Find the tension in the string when P is at its lowest point and deduce the magnitude of the acceleration of P in this position. [5]

4. A stone is projected from point A on the top of a vertical cliff and it hits the sea at point B . The height of A above sea level is 42 m.



The horizontal and vertical components of the stone's initial velocity are 4.5 ms^{-1} and 22.4 ms^{-1} respectively.

- (a) Find the speed of the stone 2 s after projection. [5]
- (b) Calculate the time of flight of the stone. [4]
- (c) Determine the distance of B from the foot of the cliff. [2]

5. Vectors \mathbf{a} and \mathbf{b} are given by

$$\mathbf{a} = 2\mathbf{i} + 13\mathbf{j} - 10\mathbf{k},$$

$$\mathbf{b} = -\mathbf{i} + y\mathbf{j} + 5\mathbf{k}.$$

- (a) If \mathbf{a} and \mathbf{b} are perpendicular, find the value of y . [4]
- (b) If \mathbf{a} and \mathbf{b} are parallel, find the value of y . [2]

6. A particle of mass 0.8 kg is attached to one end of a light inextensible string of length 0.4 m . The other end of the string is fixed to a point O of a smooth horizontal surface. The particle moves on the surface with constant speed 3 ms^{-1} in a horizontal circle with centre O .

- (a) Find the angular velocity about O of the particle. [2]
- (b) Calculate the tension in the string. [2]

TURN OVER

7. A particle, of mass 3 kg, is attached to one end of a light rod of length 0.9 m. The other end of the rod is freely pivoted at a fixed point O . The particle moves in a vertical circle with centre O , such that its speed at the lowest point of its path is three times its speed at the highest point of its path.
- (a) Show that the speed of the particle at the lowest point of its path is 6.3 ms^{-1} . [5]
 - (b) Calculate the thrust in the rod when the particle is at the highest point of its path. [4]
 - (c) If a string replaced the rod, state, with a reason, whether the particle would still move in complete circles. [2]
8. A toy plane A is moving with constant velocity $(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) \text{ ms}^{-1}$ and at time $t = 0$, its position vector is $(3\mathbf{j} - 140\mathbf{k}) \text{ m}$. Another toy plane B is moving with constant velocity $(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \text{ ms}^{-1}$ and at time $t = 0$, its position vector is $(-9\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}) \text{ m}$.
- (a) Write down the position vectors of A and B at time $t \text{ s}$. [3]
 - (b) Find an expression for the square of the distance between A and B at time $t \text{ s}$. [3]
 - (c) Determine the time when A and B are closest together. [4]