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WELSH JOINT EDUCATION COMMITTEE
CYD-BWYLLGOR ADDYSG CYMRU

General Certificate of Education
Advanced Subsidiary/Advanced

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MARKING SCHEMES

SUMMER 2007

MATHEMATICS

WJEC
CBAC

MATHEMATICS S2

1. *(a)* $\bar{x} = \dfrac{29.43}{9}$ (= 3.27) B1

SE of $\bar{X} = \dfrac{0.15}{\sqrt{9}}$ (= 0.05) B1

90% conf limits are

$3.27 \pm 1.645 \times 0.05$ M1A1

[M1 correct form, A1 1.645, FT on their mean and SE]

giving [3.19,3.35] A1

(b) We solve

$\dfrac{0.15}{\sqrt{n}} = 0.5 \times 0.05$ giving $n = 36$ (cao) M1A1

2. (a) (i) Using Var(X) = $E(X^2) - [E(X)]^2$ M1

$2 = E(X^2) - 4$ A1

$E(X^2) = 6$

Similarly,

$E(Y^2) = 12$ B1

(ii) $E(X^2Y^2) = E(X^2)E(Y^2) = 72$ M1A1

(b) Var(U) = $E(X^2Y^2) - [E(XY)]^2$ M1

$= 72 - (2 \times 3)^2$ A1

$= 36$ A1

SD = 6 A1

3. *(a)* (i) $z = \dfrac{80 - 75}{5} = 1$ M1A1

Prob = 0.8413 (cao) A1

(ii) $z = 0.674$ B1

UQ = 75 + 0.674 × 5 M1

= 78.4 A1

(b) Put $U = X_1 + X_2 - (Y_1 + Y_2 + Y_3)$

E(U) = –18 (accept ±) B1

Var(U) = $2 \times 5^2 + 3 \times 4^2 = 98$ M1A1

$z = \dfrac{18}{\sqrt{98}} = (\pm)1.82$ M1A1

[FT on mean and variance] Prob = 0.9656 (cao) A1

4. *(a)* $f(r) = 1/5 \quad (0 \le r \le 5, = 0 \quad \text{otherwise})$ B1

[Accept labelled sketch]

(b) $E(A) = \int_0^5 \pi r^2 . \frac{1}{5} \mathrm{d}r$ M1A1

$= \frac{\pi}{15}\left[r^3\right]_0^5$ A1

$= \frac{25\pi}{3}$ (cao) A1

Alternative solution

$E(R^2) = \mathrm{Var}(R) + [E(R)]^2$ M1

$= \frac{1}{12} \times 5^2 + \left(\frac{5}{2}\right)^2$ A1A1

$\mathrm{E}(A) = \frac{25\pi}{3}$ (cao) A1

(c) $P(\pi R^2 > 25) = P(R > \sqrt{\frac{25}{\pi}})$ M1A1

$= \frac{5 - \sqrt{25/\pi}}{5}$ m1

$= 0.436$ A1

5. *(a)* $H_0 : p = 0.75$ versus $H_1 : p < 0.75$ B1

(b) (i) Under H_0, X is B(20,0.75) (si) B1

and Y (No of misses) is B(20,0.25)(si) M1A1

Using tables, we find that $k = 12$ B2

(ii) We require

$\mathrm{P}(X > 12 \,|\, p = 0.5)$ M1

$= 0.132$ A1

[Do not accept the use of a normal approximation in this question]

6. *(a)* X is Poi(14) (si) B1

(i) $P(X = 10) = \mathrm{e}^{-14} \times \frac{14^{10}}{10!} = 0.0663$ M1A1

(ii) $P(X > 12) = 0.6415$ M1A1

(b) (i) $\mathrm{H}_0 : \mu = 2$ versus $\mathrm{H}_1 : \mu > 2$ (Accept 14) B1

(ii) p-value = $\mathrm{P}(X \ge 20)$ M1

$= 0.0765$ A1

We cannot conclude that the mean has increased. B1

[Do not accept the use of a normal approximation in (a) or (b)(ii)]

(iii) Under H_0, X is Po(200) ≈ N(200,200) M1A1

$$z = \frac{229.5 - 200}{\sqrt{200}}$$ M1A1A1

$= 2.09$ A1

p-value = 0.0183 A1

Strong evidence that the mean has increased. B1

[No c/c gives $z = 2.12$, $p = 0.0170$; wrong c/c gives $z = 2.16$, $p = 0.0154$]

7. *(a)* (i) $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ B1

(ii) $\bar{x}_1 = \frac{31.71}{5}$ (6.342) B1

$\bar{x}_2 = \frac{31.53}{5}$ (6.306) B1

SE of difference of means = $\sqrt{\frac{0.025^2 \times 2}{5}}$ M1

$= 0.0158$ A1

$$z = \frac{6.342 - 6.306}{0.0158}$$ M1

$= 2.28$ A1

Prob from tables = 0.0113 B1

p-value = 0.0226 B1

Strong evidence of a difference in acidity levels. B1

(b) 95% confidence limits are

$6.342 - 6.306 \pm 1.96 \times 0.0158$ M1A1

giving [0.005, 0.067] A1