

984/01

**MATHEMATICS S2**

**STATISTICS 2**

P.M. MONDAY, 11 June 2007

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The weights, in kg, of babies born during a particular week in a hospital are given below.

3.25   3.38   3.04   3.59   3.42   3.13   3.38   3.09   3.15

- (a) Assuming that this is a random sample from a normal distribution with mean  $\mu$  and standard deviation 0.15, calculate a 90% confidence interval for  $\mu$ . [5]
- (b) How many observations would be required to halve the width of this 90% confidence interval? [2]

2. The independent random variables  $X$  and  $Y$  are Poisson distributed with means 2 and 3 respectively.

- (a) (i) Show that  $E(X^2) = 6$  and evaluate  $E(Y^2)$ .  
 (ii) Deduce the value of  $E(X^2Y^2)$ . [5]

- (b) The random variable  $U$  is defined by

$$U = XY.$$

Determine the standard deviation of  $U$ . [4]

3. The weights of apples may be assumed to be normally distributed with mean 75 grams and standard deviation 5 grams.

- (a) (i) Find the probability that a randomly chosen apple weighs less than 80 grams.  
 (ii) Find the upper quartile of the weights of apples. [6]

- (b) The weights of plums may be assumed to be normally distributed with mean 56 grams and standard deviation 4 grams. Calculate the probability that the combined weight of 3 plums exceeds the combined weight of 2 apples. [6]

4. A circle has radius  $R$  cm, where  $R$  is a continuous random variable that is uniformly distributed on the interval  $[0, 5]$ .

- (a) State the probability density function of  $R$ . [1]
- (b) Find the expected area of the circle. [4]
- (c) Find the probability that the area of the circle is greater than  $25 \text{ cm}^2$ , giving your answer correct to three decimal places. [4]

5. Jim is a darts player. When he throws a dart, he claims to be able to hit the ‘bull’ with probability 0.75. His friends believe that the probability is less than this.

- (a) State suitable hypotheses to test Jim’s claim. [1]
- (b) They decide to set up a trial in which Jim throws 20 darts and they define  $X$  to be the number of darts hitting the ‘bull’.
- (i) Taking the critical region to be  $X \leq k$ , find the value of  $k$  for which the significance level is nearest to 10%.
- (ii) The actual value of the probability of Jim hitting the ‘bull’ is 0.5. With the value of  $k$  found in (i), find the probability of reaching an incorrect conclusion. [7]

6. A plumber knows that the number of emergency calls received per day follows a Poisson distribution with mean  $\mu = 2$ .

- (a) Calculate the probability that, in a 7-day period, he receives
- (i) exactly 10 calls,
- (ii) more than 12 calls. [5]
- (b) Wishing to increase the value of  $\mu$ , he increases his advertising budget.
- (i) State suitable hypotheses for investigating whether or not this achieves the desired result.
- (ii) In the first 7-day period after increasing the budget, he receives 20 emergency calls. Calculate and interpret the  $p$ -value of this result.
- (iii) In the next 100-day period, he receives 230 emergency calls. Calculate an approximate  $p$ -value of this result and interpret it. [12]

7. A scientist wishes to determine whether or not there is a difference in the acidity levels of two different liquids. He therefore makes five independent measurements of the acidity level of each liquid with the following results.

Liquid 1	6.31	6.38	6.33	6.34	6.35
Liquid 2	6.28	6.31	6.29	6.35	6.30

You may assume that these are random samples from normal distributions with common standard deviation 0.025.

- (a) (i) State suitable hypotheses.
- (ii) Calculate the  $p$ -value of the above measurements and interpret your value in context. [10]
- (b) Find a 95% confidence interval for the difference in the acidity levels of the two liquids. [3]