MATHEMATICS C4

1. *(a)* Let $\dfrac{1}{x^2(2x-1)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{2x-1}$ — M1 (Correct form)

$1 \equiv Ax(2x-1) + B(2x-1) + Cx^2$ — M1 (correct clearing and attempt to substitute)

$\underline{x=0} \quad 1 = B(-1) \quad \therefore B = -1$

$\underline{x=\tfrac{1}{2}} \quad 1 = C\tfrac{1}{4} \quad \therefore C = 4$ — A1 (2 constants C.A.O.)

$\underline{x^2} \quad 0 = 2A + C \quad \therefore A = -2$ — A1 (third constant, F.T. one slip)

(no need for display)

(b) $\int \left(\dfrac{-2}{x} - \dfrac{1}{x^2} + \dfrac{4}{2x-1}\right) dx$

$= -2\ln|x| + \dfrac{1}{x} + 2\ln|2x-1|$ — B1,B1.B1

$(+C)$

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2. $2x + x\dfrac{dy}{dx} + y + 4y\dfrac{dy}{dx} = 0$ — B1 $(x\dfrac{dy}{dx} + y)$

B1 $(4y\dfrac{dy}{dx})$

$\dfrac{dy}{dx} = 5$ — B1 (C.A.O.)

Gradient of normal $= -\dfrac{1}{5}$ — M1 ($\dfrac{-1}{candidate's\ \dfrac{dy}{dx}}$, *numerical value*)

Equation of normal is $y - 1 = -\tfrac{1}{5}(x+3)$ — A1 (F.T. candidate's value)

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3. *(a)* $R\sin\alpha = 2$, $R\cos\alpha = 3$ — B1 ($R = \sqrt{13}$)

$R = \sqrt{13}$, $\alpha = \tan^{-1}\left(\tfrac{2}{3}\right) = 33.7° or 34°$ — M1 (correct method for α)

A1 ($\alpha = 34°$)

(b) $\cos(x - 33.7°) = \dfrac{1}{\sqrt{13}}$

$x - 33.7° = 73.9°, 286.1°$ — B1 (one value)

$x = 107.6°, 319.8°$ — B1, B1 — **6**

4. Volume $= \pi \int_1^4 \left(x + \frac{3}{\sqrt{x}} \right)^2 dx$ — B1

$= \pi \int_1^4 \left(x^2 + 6\sqrt{x} + \frac{9}{x} \right) dx$ — M1 (attempt to square, at least 2 correct terms)
A1 (all correct)

$= \pi \left[\frac{x^3}{3} + 4x^{\frac{3}{2}} + 9\ln x \right]_1^4$ — A3 (integration of 3 terms, F.T. similar work $Ax^2 + B\sqrt{x} + \frac{c}{x}$)

$= \pi[49 + 9\ln x] \simeq 193.1$
or 61.48π — A1 (C.A.O.)

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5. *(a)* $\frac{dy}{dx} = \frac{-2\sin 2t}{4\cos t}$ — M1 ($\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}$)
B1 ($4\cos t$)
M1 ($k\sin 2t, k = -1, \pm 2, -\frac{1}{2}$)
A1 ($k = -2$)

$= \frac{-4\sin t\cos t}{4\cos t} = -\sin t$ — M1 (correct use of formula)
A1 (C.A.O.)

(b) Equation of tangent is $y - \cos 2p = -\sin p(x - 4\sin p)$ — M1 ($y - y_1 = m(x - x_1)$)

$x\sin p + y = \cos 2p + 4\sin^2 p$

$= 1 - 2\sin^2 p + 4\sin^2 p$ — M1 (attempt to use correct formula)

$= 1 + 2\sin^2 p$ — A1

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6. *(a)* $\int (3x+1)e^{2x}dx = (3x+1)\frac{e^{2x}}{2} - \int \frac{3}{2}e^{2x}dx$ M1 ($f(x)(3x+1) - \int 3f(x)dx$)

A1 ($f(x) = ke^{2x}, k = 1, \frac{1}{2}, 2$)

A1 ($k = \frac{1}{2}$)

$= (3x+1)\frac{e^{2x}}{2} - \frac{3e^{2x}}{4}(+C)$ A1 (F.T. one slip)

(b) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{9 - 9\sin^2\theta}.3\cos\theta d\theta$ B1 (for first line, unsimplified)

B1 (simplified without limits)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 9\cos^2\theta d\theta$ B1 (limits)

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1+\cos 2\theta}{2}\right)d\theta$ M1 ($\cos^2\theta = a + b\cos 2\theta$)

A1 ($a = b = \frac{1}{2}$)

$= \left[\frac{9\theta}{2} + \frac{9\sin 2\theta}{4}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ M1 ($k\sin 2\theta, k = \pm\frac{b}{2}, 2b, b$)

A1 ($k = \frac{b}{2}$)

$= \frac{3\pi}{2} - \frac{9\sqrt{3}}{8}$ *or* 2.764 A1 (C.A.O.)

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7.	*(a)*	$\frac{dW}{dt} = kW \qquad (k > 0)$	B1
	(b)	$\int \frac{dW}{W} = \int k dt$	M1 (attempt to separate variables)
		$\ln W = kt + C$	A1 (allow absence of *C*)
		$t = 0, \quad W = 0.1, \quad C = \ln 0.1$	B1 (value of *C*)
		$\ln \frac{W}{0.1} = kt$	M1 (use of logs or exponentials)
		$\frac{W}{0.1} = e^{kt}$	
		$k = 3.0007$	B1 (value of k)
		$W = 0.1e^{3t}$	A1
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8.	*(a)(i)*	**AB** = **i** + 2**j** – 2**k**	B1 (**AB**)
	(ii)	Equation of AB is **r** = 4**i** – **j** +**k** + λ (**i** + 2**j** – 2**k**)	M1 (reasonable attempt to write equations) A1 (must contain **r**, F.T. candidate's **AB**)
	(b)	(Point of intersection is on both lines) Equate coeffs of **i** and **j** (any two of **i,j,k**)	M1 (attempt to write equations using candidate's equation)
		$1 + \mu = 4 + \lambda$	A1 (2 correct equations, F.T. candidate's equations)
		$\lambda = \frac{1}{3} \quad \left(\mu = \frac{10}{3}\right)$	M1 (attempt to solve)
			A1 (C.A.O.)
		Position vector is $\frac{13}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$	A1 (F.T. value of λ *or* μ)
	(c)	angle between $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$ is required	B1 (coeffs of λ and μ)
		$\lvert \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} \rvert = 3, \quad \lvert \mathbf{i} - \mathbf{j} + \mathbf{k} \rvert = \sqrt{3}$	B1 (for one modulus)
		$(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}).(\mathbf{i} - \mathbf{j} + \mathbf{k}) = \lvert \quad \rvert \times \lvert \quad \rvert \cos\theta$	M1 (use of correct formula)
		$1 - 2 - 2 = 3\sqrt{3}\cos\theta$	B1 (l.h.s. unsimplified)
		$\theta = 125.3°$	A1 (C.A.O.)
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9. $(1+3x)(1-2x)^{-\frac{1}{2}} = (1+3x)\left(1+\left(-\tfrac{1}{2}\right)(-2x)+\left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\tfrac{1}{2!}1(-2x)^2+\cdots\right)$

B1, B1 (unsimplified)

$= (1+3x)\left(1+x+\tfrac{3}{2}x^2+\cdots\right)$

$= 1+4x+\dfrac{9x^2}{2}+\cdots$ B1 $(1+4x)$

B1 $(\dfrac{9x^2}{2})$

Exparsim is valid for $|x| < \tfrac{1}{2}$ B1

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10. $(x^2+49<14x)$

$x^2-14x+49<0$ B1

$(x-7)^2<0$ B1

$x-7$ is not real B1

contradiction B1 (accept impossibility)

$\left(\therefore x+\dfrac{49}{x}\geq 14 \quad \textit{for all real and positive } x\right)$

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