GCE AS/A level

MATHEMATICS C4
Pure Mathematics

A.M. THURSDAY, 12 June 2008

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$f(x) = \frac{1}{x^2(2x-1)} \quad ,$$

   (a)   express $f(x)$ in partial fractions, [4]

   (b)   find $\int f(x)dx$. [3]

2. Find the equation of the normal to the curve

$$x^2 + xy + 2y^2 = 8$$

   at the point (−3, 1). [5]

3. (a)   Express 3cos$x$ + 2sin$x$ in the form Rcos(x − α), where R and α are constants with $R > 0$ and $0° < \alpha < 90°$. [3]

   (b)   Find all values of x between 0° and 360° satisfying

$$3\cos x + 2\sin x = 1.$$ [3]

4. The region R is bounded by the curve $y = x + \dfrac{3}{\sqrt{x}}$, the x-axis and the lines $x = 1$, $x = 4$. Find the volume generated when R is rotated through four right-angles about the x-axis. [7]

5. The parametric equations of the curve C are $x = 4\sin t$, $y = \cos 2t$.

   (a)   Find $\dfrac{dy}{dx}$, simplifying your answer as much as possible. [6]

   (b)   Show that the equation of the tangent to C at the point P with parameter p is

$$x\sin p + y = 1 + 2\sin^2 p.$$ [3]

6. (a)   Find $\int (3x+1)\,e^{2x}dx.$ [4]

   (b)   Use the substitution $x = 3\sin\theta$ to show that

$$\int_{1\cdot5}^{3} \sqrt{9-x^2}\,dx = \int_{a}^{b} k\cos^2\theta\,d\theta \quad ,$$

   where the values of the constants a, b and k are to be found.

   Hence evaluate $\int_{1\cdot5}^{3} \sqrt{9-x^2}\,dx$. [8]

7. A neglected large lawn contains a certain type of weed. The area of the lawn covered by the weed at time t years is $W\,\text{m}^2$. The rate of increase of W is directly proportional to W.

   (a) Write down a differential equation that is satisfied by W. [1]

   (b) The area of the lawn covered by the weed initially is $0{\cdot}10\,\text{m}^2$ and one year later the area covered is $2{\cdot}01\,\text{m}^2$. Find an expression for W in terms of t. [6]

8. The position vectors of the points A and B are given by

$$\mathbf{a} = 4\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} + \mathbf{j} - \mathbf{k}.$$

   (a) (i) Write down the vector $\mathbf{AB}$.

   (ii) Find the vector equation of the line AB. [3]

   The vector equation of the line L is

$$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - \mathbf{j} + \mathbf{k}).$$

   (b) Given that the lines AB and L intersect, find the position vector of the point of intersection. [5]

   (c) Find the angle between the line AB and the line L. [5]

9. Expand $\dfrac{1 + 3x}{\sqrt{1 - 2x}}$ in ascending powers of x up to and including the term in x^2. State the range of x for which the expansion is valid. [5]

10. Prove by contradiction the following proposition.

   When x is real and positive,

$$x + \frac{49}{x} \geqslant 14 \ .$$

   The first line of the proof is given below.

   Assume that there is a positive and real value of x such that

$$x + \frac{49}{x} < 14 \ . $$
[4]