WJEC
CBAC

GCE AS/A level

981/01

MATHEMATICS M2
Mechanics 2

A.M. MONDAY, 2 June 2008

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Take g as 9·8 ms $^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. An elastic string, of natural length 0·3 m, supports a weight of 12 N hanging freely in equilibrium. The total length of the string is 0·55 m.

(a) Calculate the modulus of elasticity of the string. [3]

(b) Find the elastic energy stored in the string. [3]

2. The engine of a vehicle, of mass 900 kg, is working at a constant rate of 32 kW. The vehicle maintains a steady speed of 16 ms^{-1} up a hill which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{8}{49}$. Calculate the resistive force acting on the vehicle. [4]

3. A particle, of mass 5 kg, moves in a straight line under the action of a single force whose magnitude F N at time t s is given by

$$F = 15t^2 - 60t, \qquad t \geqslant 0.$$

(a) Find the acceleration of the particle when $t = 2$. [2]

(b) The velocity of the particle at time t s is denoted by v ms^{-1}. Given that $v = 35$ when $t = 0$, find an expression for v in terms of t. [4]

(c) Calculate the least value of the speed of the particle. [3]

(d) Determine the distance travelled by the particle between $t = 2$ and $t = 8$. [4]

4. In an event in the Winter Olympic Games, a competitor pushes a sled for a short time, then jumps onto the sled at a point A when the sled has a speed of 2 ms^{-1} and rides the sled downhill on a curved track. The altitude at A is 2232 m, the altitude at the finish is 2128 m and the length of the track from A to the finish is 1335 m. The competitor has a mass of 50 kg and her sled is of mass 40 kg. Her speed at the finish is 35 ms^{-1}.

(a) Calculate the work done against the resistance to motion from A to the finish. [6]

(b) Assuming the resistance is constant, calculate its magnitude. [2]

5. A particle is projected from point A on the horizontal ground such that its initial horizontal velocity is 12 ms^{-1} and its initial vertical velocity is 14 ms^{-1}. After it reaches its highest point and it is on its way down, it just clears a wall, which is 8·4 m high.

(a) Find the horizontal distance of the wall from the point A. [6]

(b) Find the speed and direction of motion of the particle as it clears the wall. [7]

6. A constant force $\mathbf{F} = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$ acts on a bead as it moves along a straight smooth wire from point A to point B. Point A has position vector $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and point B has position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find

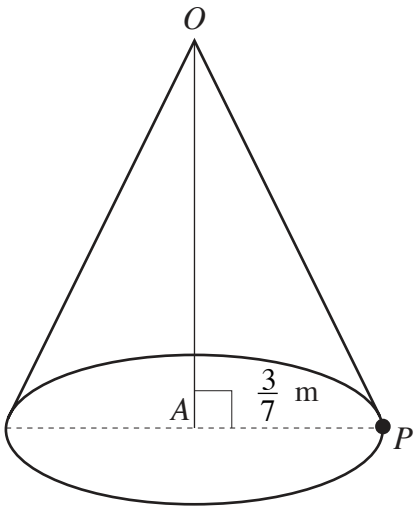
(a) the vector **AB**, [2]

(b) the work done by the force **F**. [3]

7. *(a)* A vehicle moves with velocity $\mathbf{v} = \sin(3t)\mathbf{i} + 2\cos(5t)\mathbf{j} + 3t^3\mathbf{k}$ at time t. Find an expression for the acceleration of the vehicle at time t. [3]

(b) Two vehicles A and B move on the same horizontal plane. At time t, A is at position $(-8t - 2)\mathbf{i} + (3t + 3)\mathbf{j}$ and B is at position $(-16t + 11)\mathbf{i} + (9t - 8)\mathbf{j}$. Determine the value of t when the distance between A and B is least, and calculate this distance. [7]

8. A particle P, of mass 4 kg, is tied to one end of a light inextensible string and the other end of the string is fastened to a fixed point O. The particle P moves with a uniform speed of $2\,\text{ms}^{-1}$ in a horizontal circle with centre A and radius $\frac{3}{7}$ m, as shown in the diagram.



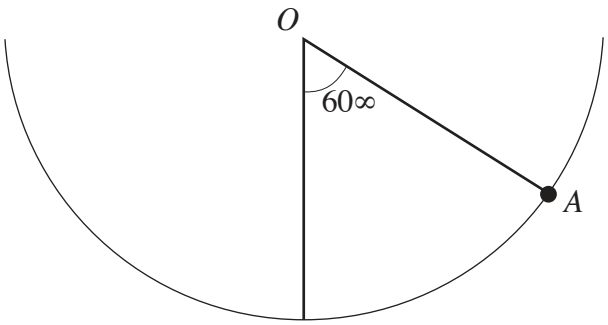
(a) Find the size of $A\widehat{O}P$. [6]

(b) Calculate the tension in the string. [1]

(c) Determine the length of the string. [1]

TURN OVER

9. A ball, of mass 2 kg, is attached to one end of a light inextensible string of length 0·5 m. The other end of the string is attached to a fixed point O. Initially, the ball is held at rest at a point A such that OA is inclined at an angle of 60° to the downward vertical through O, as shown in the diagram.



The ball is projected downwards from A with velocity $4\,\text{ms}^{-1}$ perpendicular to OA so that it starts describing a vertical circle centre O. When the string is inclined at an angle θ to the downward vertical, the speed of the ball is $v\,\text{ms}^{-1}$.

(a) Show that $v^2 = 9{\cdot}8\cos\theta + 11{\cdot}1$. [4]

(b) Find, in terms of θ, the tension in the string. [4]