



GCE AS/A level

984/01

MATHEMATICS S2
Statistics 2

P.M. FRIDAY, 6 June 2008

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. An ornithologist measured the weight, x kg, of each of five male birds and the weight, y kg, of each of six female birds of the same species. The results are summarised below.

$$\sum x = 32.6, \sum y = 36.78$$

You may assume that the weights are random samples from normal distributions with common standard deviation 0.25 kg. Find a 95% confidence interval for the difference in mean weights between male and female birds of the species. [6]

2. The random variable X has the binomial distribution $B(10, 0.4)$ and, independently, the random variable Y has the binomial distribution $B(30, 0.3)$.

(a) Show that $E(X^2) = 18.4$ and evaluate $E(Y^2)$. [5]

(b) The random variable U is defined by

$$U = XY.$$

Determine the mean and variance of U . [4]

3. A farm produces chicken eggs. The weights of these eggs may be assumed to be normally distributed with mean 50 grams and standard deviation 2 grams.

(a) (i) Find the probability that the weight of a randomly chosen egg is less than 49 grams.

(ii) Six eggs are chosen at random. Find the probability that exactly three of these eggs weigh less than 49 grams. [6]

(b) The farm also produces quail eggs whose weights may be assumed to be normally distributed with mean 18 grams and standard deviation 1.2 grams. Calculate the probability that the weight of a randomly chosen chicken egg is more than three times the weight of a randomly chosen quail egg. [5]

4. When a certain instrument is used to measure the concentration of a solution, the reading obtained (in appropriate units) is a normally distributed random variable with mean equal to the actual concentration and standard deviation 0.05. The concentration of a particular solution was measured eight times and the following independent readings were obtained.

$$6.02 \quad 6.10 \quad 5.98 \quad 6.04 \quad 6.07 \quad 5.94 \quad 6.05 \quad 6.12$$

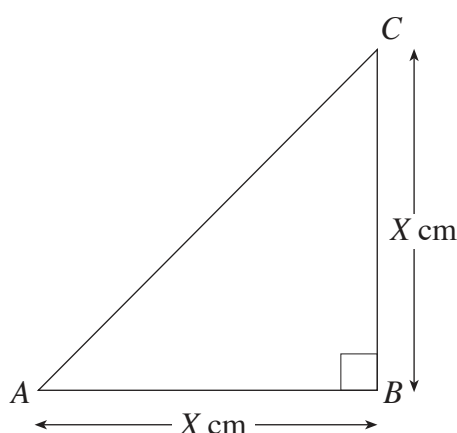
(a) Calculate a 99% confidence interval for the concentration of this solution. [5]

(b) Find the minimum number of measurements required to obtain a 99% confidence interval of width less than 0.04. [3]

5. The manager of a Garden Centre claims that each seed of a particular variety of flower has probability 0.6 of germinating, independently of all other seeds. Malcolm, however, thinks that the probability is less than 0.6 so he decides to plant a number of seeds and to count how many germinate.

- (a) State suitable hypotheses. [1]
- (b) Initially he plants 20 seeds and finds that 9 germinate. Calculate the p -value of this result. [4]
- (c) He now decides to plant 200 of these seeds. He finds that 101 germinate. Find an approximate p -value of this result and state your conclusion in context. [7]

6.



The above diagram shows an isosceles right-angled triangle ABC in which $AB = BC = X$ cm where X is a continuous random variable uniformly distributed on the interval $[4, 6]$.

- (a) Find the probability that
- (i) the length of AC exceeds 8 cm,
 - (ii) the area of the triangle ABC is less than 10 cm^2 . [8]
- (b) Find the expected value of the area of the triangle ABC . [4]
7. The random variable X has a Poisson distribution with unknown mean μ . It is required to test whether $\mu = 2.5$ against a two-sided alternative.
- (a) State suitable hypotheses. [1]
- (b) Let S denote the sum of six randomly chosen values of X . It is decided to reject the null hypothesis if either $S \leq 8$ or $S \geq 23$.
- (i) Calculate the significance level of this test.
 - (ii) Given that the value of μ is actually 2, find the probability of reaching an incorrect conclusion. [8]
- (c) It is now decided to obtain a random sample of 100 values of X . It is found that the sum of these 100 values is 270. Find an approximate p -value of this sum and interpret your result. [8]