Mathematics C4 Summer 2009

Solutions and Mark Scheme

1. *(a)* $\frac{3x}{(1+x)^2(2+x)} \equiv \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2+x}$ (correct form) M1

$3x \equiv A(1+x)(2+x) + B(2+x) + C(1+x)^2$ (correct attempt to clear fractions and substitute for x) M1

$x = -1$ $\quad -3 = B(1)$

$B = -3$

$x = -2$ $\quad -6 = C(-1)^2$ (2 constants) A1

$C = -6$

x^2 $\quad 0 = A + C$

$A = 6$ (3^{rd} constant) A1

(F.T. one slip)

(b) $\int_0^1 \left(\frac{6}{1+x} - \frac{3}{(1+x)^2} - \frac{6}{2+x} \right) \mathrm{d}x$

$= \left[6\ln(1+x) + \frac{3}{1+x} - 6\ln(2+x) \right]_0^1$ $\left(\frac{3}{1+x} \right)$ B1

(F.T. candidate's equivalent work) (logs) B1, B1

$= 6\ln 2 + \frac{3}{2} - 6\ln 3 - 6\ln 1 - 3 + 6\ln 2$

$\approx 0\cdot 226$ (must be at least 3 decimal places) C.A.O. B1

2. $3\times 2\sin\theta\cos\theta = 2\sin\theta$ (Use of $\sin 2\theta = 2\sin\theta\cos\theta$) M1

$\sin\theta = 0$ A1

or $3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$ A1

$\theta = 0°, 180°, 360°$) (F.T. one slip) B1

$70\cdot5°, 289\cdot5°$) B1

No workings shown – no marks

3. *(a)* $R = 2$ B1

$\tan\alpha = \sqrt{3},\ \alpha = 60°$ (any method) M1

A1

(b) $2\cos(\theta - 60°) = 1$

$\cos(\theta - 60°) = \frac{1}{2}$ (F.T. R and α) M1

$\theta - 60° = -60°, 60°, 300°$ (one value) A1

$\theta = 0°,\ 120°, 360°$ (A2 for 3 answers, A1 for 2 answers A2

A0 for 1 answer, lose 1 for more than 3 answers)

4. Volume $= \pi\int_0^{\frac{\pi}{8}} \cos^2 2x \, \mathrm{d}x$ (must contain limits) B1

$= (\pi)\int_0^{\frac{\pi}{8}} \frac{1+\cos 4x}{2} \, \mathrm{d}x$ ($\cos^2 2x = a + b\cos 4x;\quad a, b \neq 0$) M1

$\left(a = \frac{1}{2}, b = \frac{1}{2}\right)$ A1

$= (\pi)\left[\frac{x}{2} + \frac{\sin 4x}{8}\right]_0^{\frac{\pi}{8}}$ A1

$= (\pi)\left(\frac{\pi}{16} + \frac{1}{8} - 0 - 0\right)$ (correct use of limits) m1

$= \frac{\pi}{2}\left(\frac{\pi}{8} + \frac{1}{4}\right)$ or $1\cdot0095$ (C.A.O.) A1

[If substitution used, marks are gained after

$\frac{1}{2}\cos^2 u = a + b\cos 2u$ M1]

5. *(a)* $\frac{dy}{dx} = \frac{3t^2}{2t}$ $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$ M1

$= \frac{3t}{2}$ (simplified form) A1

Equation of tangent is

$y - p^3 = \frac{3}{2}p(x - p^2)$ (use of any method) M1

$2y - 2p^3 = 3px - 3p^3$

$3px - 2y = p^3$ (convincing) A1

(b) Substitute $x = q^2$, $y = q^3$ (substitution of $x = q^2$, $y = q^3$ and $p = 2$) M1

$3pq^2 - 2q^3 = p^3$

When $p = 2$,

$6q^2 - 2q^3 = 8$

$q^3 - 3q^2 + 4 = 0$ (convincing) A1

$(q+1)(q^2 - 4q + 4) = 0$ (attempt to solve) M1

$q = -1$ or $q = 2$ A1

Disregard $q = 2$ (as this relates to point *P*) A1

[Alternatively:

$\frac{y - q^3}{x - q^2} = 3$ (must have gradient 3) M1

$q^3 - 3q^2 + 4 = 0$ (convincing) A1 **]**

6. *(a)* $\int (x+3)e^{2x}\,dx = (x+3)\frac{e^{2x}}{2} - \int 1.e^{2x}\,dx$

$\left((x+3)f(x) - \int Af(x)\,dx;\ f(x) \neq k,\ A = 1, 3\right)$ M1

$(f(x) = k\,e^{2x})$ A1

$\left(k = \frac{1}{2},\ A = 1\right)$ A1

$= (x+3)\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$ C.A.O. (must contain *C*) A1 *(b)* $\int_3^2 -\frac{1}{2u^{\frac{1}{2}}}\,\mathrm{d}u$ $\left(\frac{k}{u^{\frac{1}{2}}}\right)$ M1

$\left(k=-\frac{1}{2}\right)$ A1

$=\left[-u^{\frac{1}{2}}\right]_3^2$ (integration, any k, no limits) A1

(correct use of limits) m1

$=\sqrt{2}+\sqrt{3}\approx 0\cdot 318$ C.A.O. (either answer) A1

Answer only gains 0 marks

7. *(a)* $\frac{\mathrm{d}P}{\mathrm{d}t}=-kP^3$ (allow $\pm k$) B1

(b) $\int\frac{\mathrm{d}P}{p^3}=-\int k\,\mathrm{d}t$ (separation of variables & attempt to integrate $\frac{1}{p^n}$, any n) M1

$-\frac{1}{2p^2}=-kt+C$ (C may be omitted, $n\neq 1$) A1

$t=0,\ P=20$ (attempt to find C) M1

$\therefore\ -\frac{1}{800}=C$ (F.T. similar work) A1

$\therefore\ -\frac{1}{2p^2}=-kt-\frac{1}{800}$

$\therefore\ \frac{1}{p^2}=2kt+\frac{1}{400}$

$\frac{1}{p^2}=At+\frac{1}{400}$ $(A=2k)$ (convincing) A1

(c) $t=1,\ P=10$

$\frac{1}{100}=A+\frac{1}{400}$ (attempt to find A) M1

$\therefore\ A=\frac{3}{400}$ A1

$\frac{1}{25}=\frac{3}{400}+\frac{1}{400}$ (substitute $p=5$) m1

$\frac{15}{400}=\frac{3}{400}t$

$t=5$ (F.T. one slip) A1

8. *(a)* (i) $\mathbf{AB} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ B1

Equation of AB is

$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ ($\mathbf{r} = \mathbf{a} + \lambda\mathbf{AB}$, o.e.) M1
A1

[Alternative:
$(1-\lambda)\mathbf{a} + \lambda\mathbf{b}$ (**a**, **b** substituted) M1
$\mathbf{r} = \ldots\ldots\ldots\ldots$ A1
(all correct) A1 **]**

(ii) Assume AB and L intersect. Equate coefficients of **i**, **j** (o.e.).

$(3+\lambda) = 5 + 3\mu$ (F.T. candidate's values) M1
$4 - 2\lambda = 6 - 2\mu$ A1

Solve for λ, μ, (attempt to solve for λ, μ) m1
$\lambda = -\frac{5}{2}$, $\mu = -\frac{3}{2}$ (one value; F.T. one slip) A1

Check **k** coefficient (o.e.)
L.H.S. $= 7 + 3\lambda = -\frac{1}{2}$ (attempt to check) m1
R.H.S. $= 1 + \mu = -\frac{1}{2}$
(Terms check so lines intersect)
Point of intersection is $\mathbf{i} + 9\mathbf{j} - \frac{1}{2}\mathbf{k}$. C.A.O. A1
(dependent on M1, m1 earlier)

(b) $(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}).(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 0$ (correct method of finding scalar product) M1
$6 - 2 - 4 = 0$ A1
(therefore vectors are perpendicular)

9. $(1+4x)^{\frac{1}{2}} = 1+\frac{1}{2}(4x)+\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(4x)^2+...$ (first line with possibly $4x^2$) M1

$= 1+2x-2x^2+....$ $(1+2x)$ A1

$(-2x^2)$ A1

Valid for $|x|<\frac{1}{4}$ B1

$(1+4k+16k^2) = 1+2(k+4k^2)-2(k+4k^2)+...$ (correct substitution for x and attempt to evaluate) M1

$= 1+2k+8k^2-2k^2+....$

$= 1+2k+6k^2+....$ (F.T. quadratic in x) A1

[**Alternative:**

First principles with three terms M1

Answer A1]

10. $9k^2 = 3b^2$ B1

$b^2 = 3k^2$ B1

(b^2 has a factor 3)

b has a factor 3 B1

a and b have a common factor – contradiction (must mention contradiction) B1

($\sqrt{3}$ is irrational)