



MS4
£4.00

GCE MARKING SCHEME

**MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced**

SUMMER 2009

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2009 examination in GCE MATHEMATICS . They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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Mathematics C1 May 2009

Solutions and Mark Scheme

1. (a) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1
 Gradient of $BC = \frac{3}{4}$ (or equivalent) A1
- (b) A correct method for finding C M1
 $C(3, 8)$ A1
- (c) Use of $m_{AB} \times m_L = -1$ to find gradient of L M1
 A correct method for finding the equation of L using candidate's coordinates for C and candidate's gradient for L . M1
 Equation of L : $y - 8 = -\frac{4}{3}(x - 3)$ (or equivalent) A1
 (f.t. candidate's coordinates for C and candidate's gradient for L)
 Equation of L : $4x + 3y - 36 = 0$ (convincing, c.a.o.) A1
- (d) (i) Substituting $y = 0$ in equation of L M1
 $D(9, 0)$ A1
- (ii) A correct method for finding the length of CD (AC) M1
 $CD = 10$ (f.t. candidate's coordinates for C and D) A1
- (iii) $AC = 5$ (f.t. candidate's coordinates for C) A1
 $\tan \hat{CAD} = \frac{CD}{AC} = 2$ (or $\frac{10}{5}$ or equivalent)
 (f.t. candidate's derived values for CD and AC) B1

2. (a) $\frac{8-\sqrt{7}}{\sqrt{7}-2} = \frac{(8-\sqrt{7})(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)}$ M1

Numerator: $8\sqrt{7} + 16 - 7 - 2\sqrt{7}$ A1

Denominator: $7 - 4$ A1

$\frac{8-\sqrt{7}}{\sqrt{7}-2} = \frac{6\sqrt{7}+9}{3} = 2\sqrt{7}+3$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $\sqrt{7}-2$

(b) $\sqrt{50} = 5\sqrt{2}$ B1

$\sqrt{3} \times \sqrt{6} = 3\sqrt{2}$ B1

$-\frac{14}{\sqrt{2}} = -7\sqrt{2}$ B1

$\sqrt{50} + (\sqrt{3} \times \sqrt{6}) - \frac{14}{\sqrt{2}} = \sqrt{2}$ (c.a.o.) B1

3. $\frac{dy}{dx} = 4x + 6$ (an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = -1$ in candidate's expression for $\frac{dy}{dx}$ m1

Gradient of tangent at $P = 2$ (c.a.o.) A1

y-coordinate at $P = 3$ B1

Equation of tangent at P : $y - 3 = 2[x - (-1)]$ (or equivalent) (f.t. one slip provided both M1 and m1 awarded) A1

4. (a) (i) $a = -2.5$ (or equivalent) B1

$b = 1.75$ (or equivalent) B1

(ii) Greatest value $= -b$ (or equivalent) B1

(b) $x^2 - x - 7 = 2x + 3$ M1

An attempt to collect terms, form and solve quadratic equation m1

$x^2 - 3x - 10 = 0 \Rightarrow (x-5)(x+2) = 0 \Rightarrow x = 5, x = -2$ (both values, c.a.o.) A1

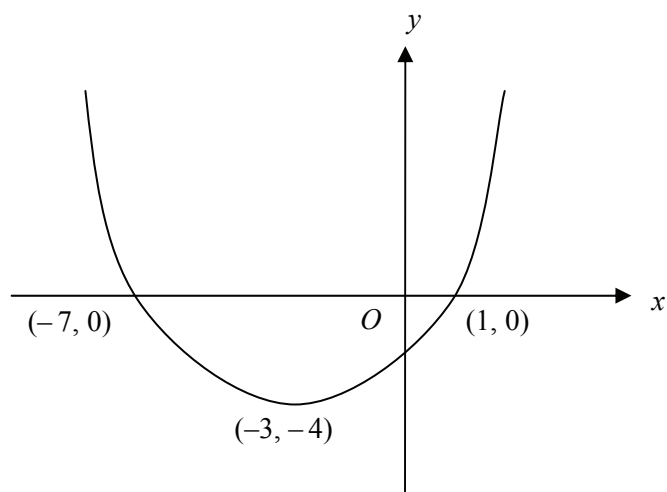
When $x = 5, y = 13$, when $x = -2, y = -1$ (both values f.t. one slip) A1

The line $y = 2x + 3$ intersects the curve $y = x^2 - x - 7$ at the points $(-2, -1)$ and $(5, 13)$ (f.t. candidate's points) E1

5. (a) $y + \delta y = 4(x + \delta x)^2 - 5(x + \delta x) - 3$ B1
 Subtracting y from above to find δy M1
 $\delta y = 8x\delta x + 4(\delta x)^2 - 5\delta x$ A1
 Dividing by δx and letting $\delta x \rightarrow 0$ M1
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 8x - 5$ (c.a.o.) A1
- (b) Required derivative $= 7 \times \frac{3}{4} \times x^{-1/4} - 2 \times (-4) \times x^{-5}$ B1, B1
6. (a) An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1
 $b^2 - 4ac = (2k)^2 - 4(k+1)(k-1)$ A1
 $b^2 - 4ac = 4$ (c.a.o.) A1
 candidate's value for $b^2 - 4ac > 0$ ($\Rightarrow$ two distinct real roots) A1
- (b) Finding critical values $x = -2, x = \frac{3}{5}$ B1
 $-2 \leq x \leq \frac{3}{5}$ **or** $\frac{3}{5} \geq x \geq -2$ **or** $[-2, \frac{3}{5}]$ **or** $-2 \leq x$ and $x \leq \frac{3}{5}$
or a correctly worded statement to the effect that x lies between -2 and $\frac{3}{5}$ (both inclusive)
 (f.t. critical values $\pm 2, \pm \frac{3}{5}$) B2
 Note: $-2 < x < \frac{3}{5}$,
 $-2 \leq x, x \leq \frac{3}{5}$
 $-2 \leq x \leq \frac{3}{5}$
 $-2 \leq x$ or $x \leq \frac{3}{5}$
 all earn B1
7. (a) $\left(x + \frac{2}{x}\right)^4 = x^4 + 4x^3\left(\frac{2}{x}\right) + 6x^2\left(\frac{2}{x}\right)^2 + 4x\left(\frac{2}{x}\right)^3 + \left(\frac{2}{x}\right)^4$
 (three terms correct) B1
 (all terms correct) B2
- $\left(x + \frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (three terms correct) B1
 (all terms correct) B2
- (- 1 for incorrect further 'simplification')
- (b) A correct equation in n , including ${}^nC_2 = 55$ M1
 $n = 11, -10$ (c.a.o.) A1
 $n = 11$ (f.t. $n = 10$ from $n = -11, 10$) A1

8. (a) Use of $f(-1) = -3$ M1
 $-a - 1 + 6 + 5 = -3 \Rightarrow a = 2$
A1
- (b) Attempting to find $f(r) = 0$ for some value of r M1
 $f(2) = 0 \Rightarrow x - 2$ is a factor A1
 $f(x) = (x - 2)(8x^2 + ax + b)$ with one of a, b correct M1
 $f(x) = (x - 2)(8x^2 + 2x - 3)$ A1
 $f(x) = (x - 2)(4x + 3)(2x - 1)$ (f.t. only $8x^2 - 2x - 3$ in above line) A1
- Special case**
Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 3 marks

9. (a)



Concave up curve and y -coordinate of minimum = -4

x -coordinate of minimum = -3

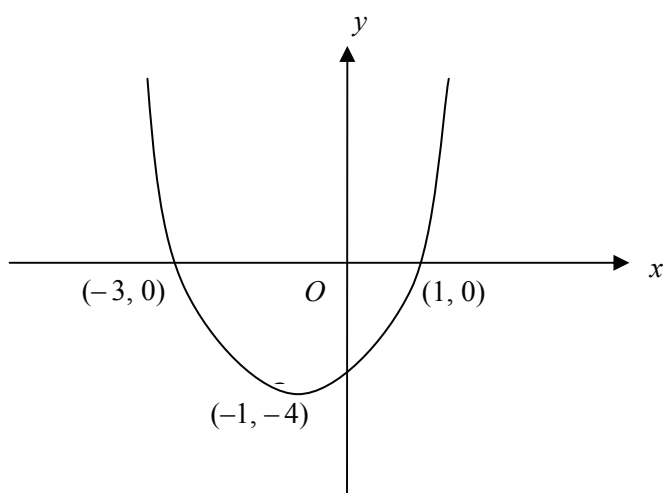
Both points of intersection with x -axis

B1

B1

B1

(b)



Concave up curve and y -coordinate of minimum = -4

x -coordinate of minimum = -1

Both points of intersection with x -axis

B1

B1

B1

10. (a) $\frac{dy}{dx} = 3x^2 - 6x + 3$ B1
 Putting derived $\frac{dy}{dx} = 0$ M1
 $3(x - 1)^2 = 0 \Rightarrow x = 1$ A1
 $x = 1 \Rightarrow y = 6 \Rightarrow$ stationary point is at (1, 6) (c.a.o) A1
- (b) **Either:**
 An attempt to consider value of $\frac{dy}{dx}$ at $x = 1^-$ and $x = 1^+$ M1
 $\frac{dy}{dx}$ has same sign at $x = 1^-$ and $x = 1^+ \Rightarrow$ (1, 6) is a point of inflection A1
Or:
 An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = 1$, $x = 1^-$ and $x = 1^+$ M1
 $\frac{d^2y}{dx^2} = 0$ at $x = 1$ and $\frac{d^2y}{dx^2}$ has different signs at $x = 1^-$ and $x = 1^+$
 $\Rightarrow$ (1, 6) is a point of inflection A1
Or:
 An attempt to find the value of y at $x = 1^-$ and $x = 1^+$ M1
 Value of y at $x = 1^- < 6$ and value of y at $x = 1^+ > 6 \Rightarrow$ (1, 6) is a point of inflection A1
Or:
 An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 1$ M1
 $\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = 1 \Rightarrow$ (1, 6) is a point of inflection A1

Mathematics C2 May 2009

Solutions and Mark Scheme

- 1.
- | | | | |
|--|--|-------------|-------------------------|
| | 0 | 0.5 | |
| | 0.1 | 0.43173647 | |
| | 0.2 | 0.408628001 | |
| | 0.3 | 0.392507416 | (3 values correct) B1 |
| | 0.4 | 0.379873463 | (5 values correct) B1 |
| | Correct formula with $h = 0.1$ | | M1 |
| | $I \approx \frac{0.1}{2} \times \{0.5 + 0.379873463 + 2(0.43173647 + 0.408628001 + 0.392507416)\}$ | | |
| | $I \approx 0.167280861$ | | |
| | $I \approx 0.167$ | | (f.t. one slip) A1 |
| | Special case for candidates who put $h = 0.08$ | | |
| | 0 | 0.5 | |
| | 0.08 | 0.438050328 | |
| | 0.16 | 0.416666666 | |
| | 0.24 | 0.401622886 | |
| | 0.32 | 0.389759395 | |
| | 0.4 | 0.379873463 | (all values correct) B1 |
| | Correct formula with $h = 0.08$ | | M1 |
| | $I \approx \frac{0.08}{2} \times \{0.5 + 0.379873463 + 2(0.438050328 + 0.416666666 + 0.401622886 + 0.389759395)\}$ | | |
| | $I \approx 0.16688288$ | | |
| | $I \approx 0.167$ | | (f.t. one slip) A1 |

Note: Answer only with no working earns 0 marks

2. (a) $5 \cos^2 \theta + 2 = 3(1 - \cos^2 \theta) - 2 \cos \theta$
 (correct use of $\sin^2 \theta = 1 - \cos^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation
 in $\cos \theta$, either by using the quadratic formula or by getting the
 expression into the form $(a \cos \theta + b)(c \cos \theta + d)$,
 with $a \times c =$ candidate's coefficient of $\cos^2 \theta$ and $b \times d =$ candidate's constant
 m1
 $8 \cos^2 \theta + 2 \cos \theta - 1 = 0 \Rightarrow (4 \cos \theta - 1)(2 \cos \theta + 1) = 0$
 $\Rightarrow \cos \theta = \frac{1}{4}, -\frac{1}{2}$ A1
 $\theta = 75.5(225)^\circ, 284.4(775)^\circ, 120^\circ, 240^\circ$ (75.5°, 284.5°) B1
 (120°) B1
 (240°) B1
 Note: Subtract 1 mark for each additional root in range for each branch,
 ignore roots outside range.
 $\cos \theta = +, -, \text{f.t. for 3 marks, } \cos \theta = -, -, \text{f.t. for 2 marks}$
 $\cos \theta = +, +, \text{f.t. for 1 mark}$

- (b) $2x + 12^\circ = -32, 212^\circ, 328^\circ$ (one value) M1
 $x = 100^\circ, 158^\circ$ (one value) A1
 (two values) A1

Note: Subtract 1 mark for each additional root in range,
 ignore roots outside range.

3. (a) $\frac{\sin \hat{ACB}}{16} = \frac{\sin 23^\circ}{9}$
 (substituting the correct values in the correct places in the sin rule) M1
 $\hat{ACB} = 44^\circ, 136^\circ$ (both values) A1
- (b) (i) Use of angle sum of a triangle = 180° M1
 $\hat{BAC} = 21^\circ$
 (f.t. candidate's values for $\hat{ACB}$ provided acute value $< 67^\circ$) A1
- (ii) Area of triangle $ABC = \frac{1}{2} \times 16 \times 9 \times \sin 21^\circ$
 (substituting 16, 9 and candidate's value for $\hat{BAC}$
 in the correct places in the area formula) M1
 Area of triangle $ABC = 25.8 \text{ cm}^2$.
 (f.t. candidate's acute value for $\hat{BAC}$) A1

4. (a) $S_n = a + [a + d] + \dots + [a + (n - 1)d]$
 (at least 3 terms, one at each end) B1
 $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots + a$
 Either:
 $2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \dots + [a + a + (n - 1)d]$ Or:
 $2S_n = [a + a + (n - 1)d] + \dots$ (n times) M1
 $2S_n = n[2a + (n - 1)d]$
 $S_n = \frac{n}{2}[2a + (n - 1)d]$ (convincing) A1
- (b) $a + 7d = 46$ B1
 $\frac{9}{2} \times [2a + 8d] = 225$ B1
 An attempt to solve the candidate's equations simultaneously by eliminating one unknown M1
 $a = -3, d = 7$ (both values) (c.a.o.) A1
- (c) $a = 3, d = 4$ B1
 $S_n = \frac{n}{2}[2 \times 3 + (n - 1)4]$ (f.t. candidate's d) M1
 $S_n = n(2n + 1)$ (c.a.o.) A1
5. (a) $r = \frac{108}{36} = 3$ (c.a.o.) B1
 $t_7 = \frac{36}{3^2}$ (f.t. candidate's value for r) M1
 $t_7 = 4$ (c.a.o.) A1
- (b) (i) $ar = 9$ B1
 $\frac{a}{1 - r} = 48$ B1
 An attempt to solve these equations simultaneously by eliminating a M1
 $16r^2 - 16r + 3 = 0$ (convincing) A1
 (ii) $r = \frac{1}{4}, \frac{3}{4}$ B1
 $a = 36, 12$ (c.a.o.) B1

6. (a) $5 \times \frac{x^{-2}}{-2} - 3 \times \frac{x^{5/4}}{5/4} + c$ (Deduct 1 mark if no c present) B1,B1

(b) (i) $6 + 4x - x^2 = x + 2$ M1

An attempt to rewrite and solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b =$ candidate's constant m1

$(x - 4)(x + 1) = 0 \Rightarrow x = 4, y = 6$ at A (both values) A1

Note: Answer only with no working earns 0 marks

(ii) **Either:**

$$\text{Total area} = \int_0^4 (6 + 4x - x^2) dx - \int_0^4 (x + 2) dx$$

(use of integration) M1

$$= [4x + (3/2)x^2 - (1/3)x^3]_0^4$$

(correct integration) B3

$$= 16 + 24 - 4^3/3$$

(f.t. candidate's limits in at least one integral) m1

Correct subtraction of integrals with correct use of 0 and candidate's x_A as limits m1

$$= \frac{56}{3}$$

(c.a.o.) A1

Or:

Area of trapezium = 16

(f.t. candidate's coordinates for A) B1

$$\text{Area under curve} = \int_0^4 (6 + 4x - x^2) dx$$

(use of integration) M1

$$= [6x + 2x^2 - (1/3)x^3]_0^4$$

(correct integration) B2

$$= 24 + 32 - 64/3$$

(f.t. candidate's limits) m1

$$= \frac{104}{3}$$

Finding total area by subtracting values m1

$$\text{Total area} = \frac{104}{3} - 16 = \frac{56}{3}$$

(c.a.o.) A1

7. (a) Let $p = \log_a x$, $q = \log_a y$
Then $x = a^p$, $y = a^q$ (the relationship between log and power) B1
 $\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indices) B1
 $\log_a x/y = p - q$ (the relationship between log and power)
 $\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1
- (b) $(5 - 2x) \log 3 = \log 7$ (taking logs on both sides) M1
An attempt to isolate x (no more than 1 slip) m1
 $x = 1.614$ (c.a.o.) A1
Note: Candidates who write down $x = 1.614$ without explanation are awarded M0 m0 A0
- (c) $\log_a (x - 3) + \log_a (x + 3) = \log_a [(x - 3)(x + 3)]$ (addition law) B1
 $2 \log_a (x - 2) = \log_a (x - 2)^2$ (power law) B1
 $(x - 3)(x + 3) = (x - 2)^2$ (removing logs) M1
 $x = 3.25$ (c.a.o.) A1
8. (a) $A(3, -1)$ B1
A correct method for finding the radius M1
Radius = 5 A1
- (b) Gradient $AP = \frac{\text{inc in } y}{\text{inc in } x}$ M1
Gradient $AP = \frac{2 - (-1)}{7 - 3} = \frac{3}{4}$ (f.t. candidate's coordinates for A) A1
Use of $m_{\text{tan}} \times m_{\text{rad}} = -1$ M1
Equation of tangent is:
 $y - 2 = -\frac{4}{3}(x - 7)$ (f.t. candidate's gradient for AP) A1
- (c) Distance between centres of C_1 and $C_2 = 17$ B1
(f.t. candidate's coordinates for A)
Use of the fact that distance between centres =
sum of the radii + the shortest possible length of the line QR M1
Shortest possible length of the line $QR = 4$ (f.t. one slip) A1
9. (a) $\frac{1}{2} \times 13 \times 13 \times \theta = 60$ M1
 $\theta = 0.71$ A1
- (b) $QR = 13\phi$, $RS = 13(\pi - \phi)$, (at least one value) B1
 $13\phi = 13(\pi - \phi) \pm 7$ (or equivalent) M1
 $\phi = 1.84$ (c.a.o.) A1

Mathematics C3 Summer 2009

Solutions and Mark Scheme

1. $h = 0.2$ ($h = 0.2$, correct formula) M1

$$\text{Integral} \approx \frac{0.2}{3} [3 + 3 \cdot 7191397 + 4(3 \cdot 1189742 + 3 \cdot 4779304) + 2(3 \cdot 2778041)]$$

(3 values) B1

(2 values) B1

$$\approx 2.6442$$

A1

[Special case: B1 for 6 correct values, at least 5 decimal places:

3, 3.09206, 3.20936, 3.35287, 3.52292, 3.71914]

2. (a) $\theta = 90^\circ$
 $\cos \theta + \cos 3\theta = 0 + 0 = 0$ (choice of θ and evaluating one side) B1
 $2 \cos 2\theta \cos 4\theta = 2 \times (-1) \times 1 = -2$ (other side) B1
 $(\cos \theta + \cos 3\theta \neq 2 \cos 2\theta \cos 4\theta)$

- (b) $-9 + \cot^2 \theta = \operatorname{cosec} \theta - \operatorname{cosec}^2 \theta$
 $-9 + \operatorname{cosec}^2 \theta - 1 = \operatorname{cosec} \theta - \operatorname{cosec}^2 \theta$ ($\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$) M1
 $2 \operatorname{cosec}^2 \theta - \operatorname{cosec} \theta - 10 = 0$
 $(2 \operatorname{cosec} \theta - 5)(\operatorname{cosec} \theta + 2) = 0$ (reasonable attempt to solve) M1
 $\sin \theta = \frac{2}{5}, -\frac{1}{2}$ A1
 $\theta = 23.6, 156.4, 210^\circ, 330^\circ$ (23.6, 156.4) B1
(210°) B1
(330°) B1

[Notes: 1) Lose 1 mark for each additional value in range,
1 for each branch
2) Allow to nearest degree
3) F.T. for $\sin \theta = +, -$ 3 marks
F.T. for $\sin \theta = -, -$ 2 marks
F.T. for $\sin \theta = +, +$ 1 mark]

$$3. \quad (a) \quad 3x^2 + 2y \frac{dy}{dx} + \tan 2y + 2x \sec^2 2y \frac{dy}{dx} = 0 \quad \left(2y \frac{dy}{dx} \right) \quad \text{B1}$$

$$\left(\tan 2y + k \sec^2 2y \frac{dy}{dx}; \quad k = 1, 2 \right) \quad \text{B1}$$

$$(k = 2) \quad \text{B1}$$

$$\frac{dy}{dx} = -\frac{3x^2 + \tan 2y}{2y + 2x \sec^2 2y} \quad (\text{All correct. F.T. for last B1 if first two Bs gained}) \quad \text{B1}$$

$$(b) \quad (i) \quad \frac{dy}{dt} = \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2} \quad \left(\frac{(3+2t)f(t) - (1+4t)g(t)}{(3+2t)^2} \right) \quad \text{M1}$$

$$[f(t) = 4, g(t) = 2] \quad \text{A1}$$

(Give additional mark for simplification here, see last A1 of part (ii))

$$(ii) \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad (\text{attempt to use } \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}) \quad \text{M1}$$

$$= \frac{(3+2t)(4) - (1+4t)(2)}{(3+2t)^2} \cdot \frac{1}{(3+2t)} \quad \text{A1}$$

$$= \frac{10}{(3+2t)^3} \quad (\text{This may have been given in (i).}) \quad \text{A1}$$

(Penalise faulty simplification on last line)

4. (a) $(f'(x) = 0)$
 $2(2x-3)e^{2x} + 2e^{2x} - 4$ (Attempt to find $f'(x)$, - 4 present) M1
 $((2x-3)f(x) + e^{2x}g(x))$ M1
 $(f(x) = ke^{2x}, k = 1, 2; g(x) = 2)$ A1
 $(k = 2)$ A1

$$(4x-6+2)e^{2x} - 4$$

$$(4x-4)e^{2x} - 4 = 0$$

$$(x-1)e^{2x} - 1 = 0$$

(equate $f'(x)$ to zero) M1
(result – convincing) A1

- (b)

$\underline{x}$	$\underline{f'(x)}$	
1	- 1	(Attempt to find values or signs) M1
2	53.6	(correct values) A1

Change of sign indicates presence of root between 1 and 2.

$$x_0 = 1.1, x_1 = 1.1108..., x_2 = 1.1084..., x_3 = 1.1089... \quad (x_1) \quad \text{B1}$$

$$x_3 \approx 1.1089 \quad (x_3 \text{ correct to 4 decimal places}) \quad \text{B1}$$

Check 1.10895, 1.10885

$\underline{x}$	$\underline{f'(x)}$	
1.10885	- 0.00008	(Attempt to find values) M1
1.10895	+ 0.001	(Correct signs or values) A1

Change of sign indicates presence of root
so root is 1.1089 correct to 4 decimal places. A1

Note: (b) must involve ‘change of sign’ (o.e.) at least once.

5. (a) $\frac{4x}{3+2x^2}$ $\left(\frac{f(x)}{3+2x^2}, f(x) \neq 1 \right)$ M1
 $(f(x) = 4x)$ A1

(b) $\frac{x^2}{1+x^2} + 2x \tan^{-1} x$ $(x^2 f(x) + g(x) \tan^{-1} x)$ M1
 $\left(f(x) = \frac{1}{1+x^2}, g(x) = 2x \right)$ A1

(c) $10(5+7x^2)^9 \cdot 14x$ $(10(5+7x^2)^9 \cdot f(x))$ M1
 $(f(x) = 14x)$ A1
 $= 140x(5+7x^2)^9$ (Simplified answer) A1
(F.T. for $f(x) = 7x$, i.e. $70x$)

6. (a) $9x-7 \leq 3, \quad x \leq \frac{10}{9}$ B1
and
 $9x-7 \geq -3$ M1
 $x \geq \frac{4}{9}$ (answer must involve the word 'and' o.e.) A1
 $\frac{4}{9} \leq x \leq \frac{10}{9}$ (Note: lose A1 for the omission of '=')

Alternative Scheme: $(9x-7)^2 \leq 9$

$$81x^2 - 126x + 49 \leq 9$$

$$(9x-10)(9x-4)$$

B1

Any method

M1

Correct answer

A1

(b) $5|x|+1=9$
 $5|x|=8$ $(a|x|=b; a=5, b=8)$ B1
 $x = \pm \frac{8}{5}$ (both answers, F.T. a, b) B1

7. (a) Penalise the absence of constant one mark once only in part (a)

$$(i) \quad -\frac{1}{5}\cos 5x (+C) \quad \left(k \cos 5x; \quad k = \pm \frac{1}{5}, -5\right) \text{ M1}$$

$$\left(k = -\frac{1}{5}\right) \text{ A1}$$

$$(ii) \quad -\frac{3}{4(2x+7)^2} (+C) \quad \left(\frac{k}{(2x+7)^2}; \quad k = -\frac{3}{2}, \pm \frac{3}{4}\right) \text{ M1}$$

$$\left(k = -\frac{3}{4}\right) \text{ A1}$$

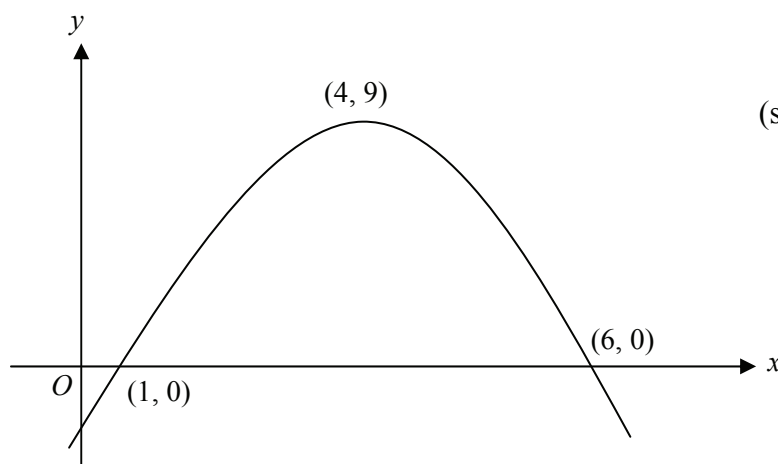
$$(b) \quad \left[\frac{2}{5}\ln|5x+3|\right]_0^3 \quad \left(k \ln|5x+3|; \quad || \text{ may be omitted}; \quad k = 2, \frac{2}{5}\right) \text{ M1}$$

$$\left(k = \frac{2}{5}\right) \text{ A1}$$

$$= \frac{2}{5}[\ln 18 - \ln 3] \quad (k(\ln 18 - \ln 3); \text{ F.T. for any } k \text{ except } k = 1) \text{ M1}$$

$$\approx 0.717... \text{ A1}$$

8.



(shape and coordinates of
one point) B1
(another point) B1
(another point) B1

**Note: 3 correct points, no
graph: B2
2 or less correct points,
no graph: B0**

Special Case: All correct with left x translation: B1

9. (a) Domain of $fg = (0, \infty)$ B1
Range = $(0, \infty)$ B1

(b) $3e^{2\ln 4x} = 12$ ($fg(x)$, correct order and equating) M1
 $3e^{\ln 16x^2} = 12$ (correct attempt to use laws of logs) m1
 $48x^2 = 12$ A1
 $x = \pm \frac{1}{2}$ (F.T. $12x^2 = 12$) A1
 $x = \frac{1}{2}$ (since domain $(0, \infty)$) A1

[**Alternatively:**

$3e^{2\ln 4x} = 12$ (correct order and equating) M1
 $e^{2\ln 4x} = 4$ A1
 $2\ln 4x = \ln 4$ (taking logs) m1
 $\ln 4x = \ln 2$ (o.e.) A1
 $x = \frac{1}{2}$ A1]

$$10. \quad (a) \quad f'(x) = \frac{-2 \times -1}{(3x^2 + 2)^2} \times 6x \quad \left(\frac{f(x)}{(3x^2 + 2)^2}; \quad f(x) \neq \pm 1, \pm 2 \right) \quad \text{M1}$$

$$= \frac{12x}{(3x^2 + 2)^2} \quad (f(x) = 12x) \quad \text{A1}$$

$$\text{Since } 12x > 0, \quad \frac{1}{(3x^2 + 2)^2} > 0, \quad f'(x) > 0 \quad (12x > 0) \quad \text{B1}$$

$$\left(\frac{1}{(3x^2 + 2)^2} > 0 \right) \quad \text{B1}$$

$$(b) \quad \text{Range is } (0, 1) \quad \text{B1}$$

$$(c) \quad \text{Let } y = 1 - \frac{2}{3x^2 + 2} \quad \left(y - 1 = \frac{-2}{(\quad)} \right) \quad \text{M1}$$

$$y - 1 = \frac{-2}{3x^2 + 2}$$

$$\frac{2}{1 - y} = 3x^2 + 2$$

$$\frac{2}{1 - y} - 2 = 3x^2 \quad \text{A1}$$

$$x = \pm \sqrt{\frac{2}{3} \left(\frac{1}{1 - y} - 1 \right)} \quad \text{o.e. (must have } \pm \sqrt{\quad} \text{)} \quad \text{A1}$$

$$x = \sqrt{\frac{2}{3} \left(\frac{y}{1 - y} \right)} \quad (\text{simplified form not required}) \quad \text{o.e. (domain } x > 0 \text{)} \quad \text{B1}$$

$$f^{-1}(x) = \sqrt{\frac{2}{3} \left(\frac{x}{1 - x} \right)} \quad (\text{simplified form not required})$$

$$(\text{F.T. candidate's expression}) \quad \text{B1}$$

$$\text{Domain of } f^{-1} \text{ is } (0, 1) \quad \text{o.e. (F.T. from (a))}$$

$$\text{Range of } f^{-1} \text{ is } (0, \infty) \quad \text{B1}$$

Mathematics C4 Summer 2009

Solutions and Mark Scheme

1. (a) $\frac{3x}{(1+x)^2(2+x)} \equiv \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2+x}$ (correct form) M1

$3x \equiv A(1+x)(2+x) + B(2+x) + C(1+x)^2$ (correct attempt to clear fractions and substitute for x) M1

$x = -1 \quad -3 = B(1)$

$B = -3$

$x = -2 \quad -6 = C(-1)^2$ (2 constants) A1

$C = -6$

$x^2 \quad 0 = A + C$

$A = 6$

(3rd constant) A1
(F.T. one slip)

(b) $\int_0^1 \left(\frac{6}{1+x} - \frac{3}{(1+x)^2} - \frac{6}{2+x} \right) dx$

$= \left[6\ln(1+x) + \frac{3}{1+x} - 6\ln(2+x) \right]_0^1$ $\left(\frac{3}{1+x} \right)$ B1

(F.T. candidate's equivalent work)

(logs) B1, B1

$= 6\ln 2 + \frac{3}{2} - 6\ln 3 - 6\ln 1 - 3 + 6\ln 2$

≈ 0.226

(must be at least 3 decimal places) C.A.O. B1

2. $3 \times 2 \sin \theta \cos \theta = 2 \sin \theta$ (Use of $\sin 2\theta = 2 \sin \theta \cos \theta$) M1

$\sin \theta = 0$ A1

or $3 \cos \theta = 1$

$\cos \theta = \frac{1}{3}$ A1

$\theta = 0^\circ, 180^\circ, 360^\circ$) (F.T. one slip) B1

$70.5^\circ, 289.5^\circ$) B1

No workings shown – no marks

3. (a) $R = 2$ B1

$\tan \alpha = \sqrt{3}, \alpha = 60^\circ$ (any method) M1

A1

(b) $2 \cos(\theta - 60^\circ) = 1$

$\cos(\theta - 60^\circ) = \frac{1}{2}$ (F.T. R and α) M1

$\theta - 60^\circ = -60^\circ, 60^\circ, 300^\circ$ (one value) A1

$\theta = 0^\circ, 120^\circ, 360^\circ$ (A2 for 3 answers, A1 for 2 answers) A2

A0 for 1 answer, lose 1 for more than 3 answers)

4. Volume $= \pi \int_0^{\frac{\pi}{8}} \cos^2 2x \, dx$ (must contain limits) B1

$= (\pi) \int_0^{\frac{\pi}{8}} \frac{1 + \cos 4x}{2} \, dx$ ($\cos^2 2x = a + b \cos 4x; a, b \neq 0$) M1

$\left(a = \frac{1}{2}, b = \frac{1}{2} \right)$ A1

$= (\pi) \left[\frac{x}{2} + \frac{\sin 4x}{8} \right]_0^{\frac{\pi}{8}}$ A1

$= (\pi) \left(\frac{\pi}{16} + \frac{1}{8} - 0 - 0 \right)$ (correct use of limits) m1

$= \frac{\pi}{2} \left(\frac{\pi}{8} + \frac{1}{4} \right)$ or 1.0095 (C.A.O.) A1

[If substitution used, marks are gained after

$\frac{1}{2} \cos^2 u = a + b \cos 2u$ M1]

5. (a) $\frac{dy}{dx} = \frac{3t^2}{2t}$ $\left(\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}}\right)$ M1

$= \frac{3t}{2}$ (simplified form) A1

Equation of tangent is

$$y - p^3 = \frac{3}{2}p(x - p^2)$$
(use of any method) M1

$$2y - 2p^3 = 3px - 3p^3$$

$$3px - 2y = p^3$$
(convincing) A1

(b) Substitute $x = q^2$, $y = q^3$ (substitution of $x = q^2$, $y = q^3$ and $p = 2$) M1

$$3pq^2 - 2q^3 = p^3$$

When $p = 2$,

$$6q^2 - 2q^3 = 8$$

$$q^3 - 3q^2 + 4 = 0$$
(convincing) A1

$$(q+1)(q^2 - 4q + 4) = 0$$
(attempt to solve) M1

$$q = -1 \text{ or } q = 2$$
A1

$$\text{Disregard } q = 2 \text{ (as this relates to point } P)$$
A1

[Alternatively:

$$\frac{y - q^3}{x - q^2} = 3$$
(must have gradient 3) M1

$$q^3 - 3q^2 + 4 = 0$$
(convincing) A1]

6. (a) $\int (x+3)e^{2x} dx = (x+3)\frac{e^{2x}}{2} - \int 1 \cdot e^{2x} dx$

$((x+3)f(x) - \int Af(x)dx; f(x) \neq k, A = 1, 3)$ M1

$(f(x) = ke^{2x})$ A1

$\left(k = \frac{1}{2}, A = 1\right)$ A1

$$= (x+3)\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$$
C.A.O. (must contain C) A1

$$\begin{aligned}
 (b) \quad \int_3^2 -\frac{1}{2u^{\frac{1}{2}}} du & \quad \left(\frac{k}{u^{\frac{1}{2}}} \right) \text{ M1} \\
 & \quad \left(k = -\frac{1}{2} \right) \text{ A1} \\
 & = \left[-u^{\frac{1}{2}} \right]_3^2 \\
 & = \sqrt{2} + \sqrt{3} \approx 0.318 \quad \begin{array}{l} \text{(integration, any } k, \text{ no limits) A1} \\ \text{(correct use of limits) m1} \\ \text{C.A.O. (either answer) A1} \end{array}
 \end{aligned}$$

Answer only gains 0 marks

$$7. \quad (a) \quad \frac{dP}{dt} = -kP^3 \quad (\text{allow } \pm k) \text{ B1}$$

$$\begin{aligned}
 (b) \quad \int \frac{dP}{P^3} &= -\int k \, dt \quad (\text{separation of variables \& attempt to integrate } \frac{1}{P^n}, \text{ any } n) \text{ M1} \\
 -\frac{1}{2P^2} &= -kt + C \quad (C \text{ may be omitted, } n \neq 1) \text{ A1}
 \end{aligned}$$

$$t = 0, P = 20 \quad (\text{attempt to find } C) \text{ M1}$$

$$\therefore -\frac{1}{800} = C \quad (\text{F.T. similar work}) \text{ A1}$$

$$\therefore -\frac{1}{2P^2} = -kt - \frac{1}{800}$$

$$\therefore \frac{1}{P^2} = 2kt + \frac{1}{400}$$

$$\frac{1}{P^2} = At + \frac{1}{400} \quad (A = 2k) \quad (\text{convincing}) \text{ A1}$$

$$(c) \quad t = 1, P = 10$$

$$\frac{1}{100} = A + \frac{1}{400} \quad (\text{attempt to find } A) \text{ M1}$$

$$\therefore A = \frac{3}{400} \quad \text{A1}$$

$$\frac{1}{25} = \frac{3}{400} + \frac{1}{400} \quad (\text{substitute } p = 5) \text{ m1}$$

$$\begin{aligned}
 \frac{15}{400} &= \frac{3}{400} t \\
 t &= 5 \quad (\text{F.T. one slip}) \text{ A1}
 \end{aligned}$$

8. (a) (i) $\mathbf{AB} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ B1

Equation of AB is

$$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \quad (\mathbf{r} = \mathbf{a} + \lambda\mathbf{AB}, \text{ o.e.}) \quad \begin{array}{l} \text{M1} \\ \text{A1} \end{array}$$

[Alternative:

$$(1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$$

($\mathbf{a}, \mathbf{b}$ substituted) M1

$$\mathbf{r} = \dots\dots\dots$$

A1

(all correct) A1]

(ii) Assume AB and L intersect. Equate coefficients of $\mathbf{i}, \mathbf{j}$ (o.e.).

$$(3 + \lambda) = 5 + 3\mu \quad (\text{F.T. candidate's values}) \quad \text{M1}$$

$$4 - 2\lambda = 6 - 2\mu \quad \text{A1}$$

Solve for λ, μ , (attempt to solve for λ, μ) m1

$$\lambda = -\frac{5}{2}, \mu = -\frac{3}{2} \quad (\text{one value; F.T. one slip}) \quad \text{A1}$$

Check $\mathbf{k}$ coefficient (o.e.)

$$\text{L.H.S.} = 7 + 3\lambda = -\frac{1}{2} \quad (\text{attempt to check}) \quad \text{m1}$$

$$\text{R.H.S.} = 1 + \mu = -\frac{1}{2}$$

(Terms check so lines intersect)

$$\text{Point of intersection is } \mathbf{i} + 9\mathbf{j} - \frac{1}{2}\mathbf{k}. \quad \text{C.A.O.} \quad \text{A1}$$

(dependent on M1, m1 earlier)

(b) $(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 0$ (correct method of finding scalar product) M1

$$6 - 2 - 4 = 0 \quad \text{A1}$$

(therefore vectors are perpendicular)

9. $(1+4x)^{\frac{1}{2}} = 1 + \frac{1}{2}(4x) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2}(4x)^2 + \dots$ (first line with possibly $4x^2$) M1
 $= 1 + 2x - 2x^2 + \dots$ (1 + 2x) A1
 $(-2x^2)$ A1

Valid for $|x| < \frac{1}{4}$ B1

$(1+4k+16k^2) = 1 + 2(k+4k^2) - 2(k+4k^2) + \dots$ (correct substitution for x
and attempt to evaluate) M1
 $= 1 + 2k + 8k^2 - 2k^2 + \dots$
 $= 1 + 2k + 6k^2 + \dots$ (F.T. quadratic in x) A1

[**Alternative:**

First principles with three terms

M1

Answer

A1]

10. $9k^2 = 3b^2$ B1
 $b^2 = 3k^2$ B1
 $(b^2 \text{ has a factor } 3)$
 $b \text{ has a factor } 3$ B1
 $a \text{ and } b \text{ have a common factor} - \text{contradiction}$ (must mention contradiction) B1
 $(\sqrt{3} \text{ is irrational})$

A/AS Level Maths – FP1 – June 2009 – Markscheme

$$\begin{aligned}
 1. \quad S_n &= \sum_{r=1}^n r^3 + 2 \sum_{r=1}^n r^2 + \sum_{r=1}^n r & \text{M1} \\
 &= \frac{n^2(n+1)^2}{4} + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} & \text{A1A1A1} \\
 &= \frac{n(n+1)(3n^2 + 3n + 8n + 4 + 6)}{12} & \text{m1} \\
 &= \frac{n(n+1)(n+2)(3n+5)}{12} & \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad \alpha + \beta &= -3 \\
 \alpha\beta &= 4 & \text{B1}
 \end{aligned}$$

Consider

$$\begin{aligned}
 \alpha + \beta + \alpha\beta &= 1 & \text{cao} & \text{M1A1} \\
 \alpha\beta + \alpha^2\beta + \alpha\beta^2 &= \alpha\beta + \alpha\beta(\alpha + \beta) & \text{M1} \\
 &= -8 & \text{cao} & \text{A1} \\
 \alpha^2\beta^2 &= 16 & \text{B1}
 \end{aligned}$$

The required equation is

$$\begin{aligned}
 x^3 - x^2 - 8x - 16 &= 0 & \text{M1A1} \\
 \text{[FT from above]}
 \end{aligned}$$

$$3. \quad (a) \quad \det \mathbf{A} = 1(6 - 5) + 2(3 - 4) + 3(10 - 9) = 2 & \text{M1A1}$$

$$\text{Cofactor matrix} = \begin{bmatrix} 1 & -1 & 1 \\ 11 & -7 & 1 \\ -7 & 5 & -1 \end{bmatrix} & \text{M1}$$

$$\text{Adjugate matrix} = \begin{bmatrix} 1 & 11 & -7 \\ -1 & -7 & 5 \\ 1 & 1 & -1 \end{bmatrix} & \text{A2}$$

[A1 for 1,2 or 3 errors]

$$\text{Inverse matrix} = \frac{1}{2} \begin{bmatrix} 1 & 11 & -7 \\ -1 & -7 & 5 \\ 1 & 1 & -1 \end{bmatrix} & \text{A1}$$

[Award 0 marks for answer only : FT from their adjugate]

$$(b) \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 11 & -7 \\ -1 & -7 & 5 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 13 \\ 13 \\ 22 \end{bmatrix} & \text{M1}$$

$$= \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} & \text{A1}$$

[FT from their inverse]

4. (a) $z = \frac{(9+7i)(3+i)}{(3-i)(3+i)}$ M1
 $= \frac{27-7+21i+9i}{9+1}$ A1A1
 $= 2+3i$ A1
 (b) $\text{mod}(z) = \sqrt{13}$, $\arg(z) = 0.983$ (56.3°) B1B1
 [FT on their z]

5. The statement is true for $n = 1$ since the formula gives $1/2$ which is correct.

B1

Let the statement be true for $n = k$, ie

$$\sum_{r=1}^k \frac{1}{r(r+1)} = \frac{k}{k+1}$$
 M1

Consider

$$\sum_{r=1}^{k+1} \frac{1}{r(r+1)} = \sum_{r=1}^k \frac{1}{r(r+1)} + \frac{1}{(k+1)(k+2)}$$
 M1

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$
 A1

$$= \frac{k(k+2)+1}{(k+1)(k+2)}$$
 A1

$$= \frac{(k+1)^2}{(k+1)(k+2)}$$
 A1

$$= \frac{k+1}{k+2}$$
 A1

True for $n = k \Rightarrow$ true for $n = k + 1$, hence proved by induction.

A1

6. (a) $\det(A) = \lambda(-4 - \lambda^2) + 3\lambda - 8 + 2(2\lambda + 3) = -(\lambda^3 - 3\lambda + 2)$ M1A1
 This is zero when $\lambda = 1$ (so the matrix is singular) B1
 $\lambda^3 - 3\lambda + 2 = (\lambda - 1)(\lambda^2 + \lambda - 2) = (\lambda - 1)^2(\lambda + 2)$ M1
 [For candidates using long division award M1 for 2 terms]
 Therefore $\lambda = 1$ is the only positive value giving a zero determinant.

A1

- (b)(i) $x + y + 2z = 2$ M1
 $-3y - 3z = -6$ A1
 $-2y - 2z = -4$ A1

[Award the M1 for 1 correct row]

The equations are consistent because the 3rd equation is a multiple of the 2nd

A1

- (ii) Put $z = \alpha$ M1
 $y = 2 - \alpha$, $x = -\alpha$ cao

A1

7. Putting $z = x + iy$, [Award for attempting to use] M1
 $\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x+2)^2 + y^2}$ A1
 $x^2 - 2x + 1 + y^2 = 4(x^2 + 4x + 4 + y^2)$ A1
 $x^2 + y^2 + 6x + 5 = 0$ [Accept any multiple] A1
 $(x+3)^2 + y^2 = 4$ M1
Centre $(-3, 0)$, radius = 2 A1A1
[FT from their circle]

8. (a) Reflection matrix = $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

Translation matrix = $\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$ B1

$\mathbf{T} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B1

$= \begin{bmatrix} 0 & -1 & h \\ -1 & 0 & k \\ 0 & 0 & 1 \end{bmatrix}$

(b)(i) We are given that

$\begin{bmatrix} 0 & -1 & h \\ -1 & 0 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ M1

giving

$h = 4$ A1

$k = 2$ A1

(ii) EITHER

A general point on the line is $(\lambda, 3\lambda + 2)$.

Consider

$\begin{bmatrix} 0 & -1 & 4 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda \\ 3\lambda + 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 - 3\lambda \\ 2 - \lambda \\ 1 \end{bmatrix}$ M1

$x = 2 - 3\lambda; y = 2 - \lambda$ A1

Eliminating λ , $3y - x = 4$ cao M1A1

OR

Let $(x, y) \rightarrow (x', y')$

M1

$$\begin{bmatrix} 0 & -1 & 4 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

M1

$$-y + 4 = x' \text{ or } y = 4 - x'$$

A1

$$-x + 2 = y' \text{ or } x = 2 - y'$$

$$y = 3x + 2 \rightarrow 4 - x' = 6 - 3y' + 2$$

M1

$$\text{equation of image is } 3y - x = 4 \quad \text{cao}$$

A1

9. (a)(i) $\ln f(x) = \ln(x^x) + \ln(e^{-2x}) = x \ln x - 2x$

B1

(ii) $\frac{f'(x)}{f(x)} = \ln x + \frac{x}{x} - 2 = \ln x - 1$

B1B1

$$f'(x) = f(x)(\ln x - 1) \quad (\text{so } a = 1, b = -1)$$

B1

(b) $f''(x) = f'(x)(\ln x - 1) + \frac{f(x)}{x}$

B1

(c) At a stationary point,

$$\ln x = 1$$

B1

$$x = e \quad (2.72)$$

B1

$$y = e^{-e} \quad (0.066)$$

B1

From (b), $f''(e) > 0$

B1

so it is a minimum.

B1

[Award 0 marks for answer only]

A/AS Maths – FP2 – June 2009 – Markscheme

1. (a) h is not continuous B1
 Any valid reason, eg f is not defined for $x = 0$, $f(x)$ jumps from large negative to large positive going through 0. B1
- (b)(i) Attempting to compare $g(x)$ with $g(-x)$. M1
 g is even. A1
- (ii) Attempting to compare $h(x)$ with $h(-x)$. M1
 h is odd. A1
2. $u = \tan x \Rightarrow du = \sec^2 x dx$ B1
 and $[0, \pi/6] \rightarrow [0, 1/\sqrt{3}]$ B1
- $$I = \int_0^{1/\sqrt{3}} \frac{du}{\sqrt{3 - (1 + u^2)}}$$
- M1
- [Must be correct here]
- $$= \int_0^{1/\sqrt{3}} \frac{du}{\sqrt{2 - u^2}}$$
- A1
- $$= \left[\sin^{-1} \left(\frac{u}{\sqrt{2}} \right) \right]_0^{1/\sqrt{3}}$$
- A1
- $$= 0.421$$
- A1
- Any valid reason, eg the denominator would be the square root of a negative number towards the upper limit. B1
3. Modulus = 16 B1
 Argument = $2\pi/3$ B1
 $-8 + 8\sqrt{3}i = 16[\cos(2\pi/3) + i\sin(2\pi/3)]$ B1
 $(-8 + 8\sqrt{3}i)^{1/4} = 16^{1/4}[\cos(2\pi/3 \times 1/4) + i\sin(2\pi/3 \times 1/4)]$ M1
 $= 2(\cos(\pi/6) + i\sin(\pi/6))$ A1
 Second root = $2(\cos(2\pi/3) + i\sin(2\pi/3))$ A1
 Third root = $2(\cos(7\pi/6) + i\sin(7\pi/6))$ A1
 Fourth root = $2(\cos(5\pi/3) + i\sin(5\pi/3))$ A1
 [FT from their modulus and argument]
4. The equation can be rewritten M1A1
 $\sin 2\theta + 2\sin 2\theta \cos \theta = 0$ A1
 $\sin 2\theta(1 + 2\cos \theta) = 0$
 $\sin 2\theta = 0$ gives $\theta = \frac{n\pi}{2}$ M1A1
 $\cos \theta = -\frac{1}{2}$ gives $\theta = 2n\pi \pm 2\pi/3$ M1A1

5. (a) Let $\frac{1}{(x+1)(x+2)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$
 $= \frac{A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)}{(x+1)(x+2)(x+3)}$ M1
- $x = -1$ gives $A = 1/2$; $x = -2$ gives $B = -1$; $x = -3$ gives $C = 1/2$ A1A1A1
- (b) $I = \left[\frac{1}{2} \ln(x+1) - \ln(x+2) + \frac{1}{2} \ln(x+3) \right]_0^5$ B1B1
- [B1 for 1 error]
- $= \frac{1}{2} (\ln 6 - \ln 49 + \ln 8 + \ln 4 - \ln 3)$ M1
- $= \frac{1}{2} \ln \left(\frac{6 \times 8 \times 4}{49 \times 3} \right)$ A1
- $= \ln \left(\frac{8}{7} \right)$ A1
6. (a) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{b \cos \theta}{a \sin \theta}$ M1A1
- Equation of tangent is
- $y - b \sin \theta = -\frac{b \cos \theta}{a \sin \theta} (x - a \cos \theta)$ M1A1
- $b x \cos \theta + a y \sin \theta = ab(\cos^2 \theta + \sin^2 \theta)$ A1
- whence the printed result.
- (b) Putting $y = 0$, P is $\left(\frac{a}{\cos \theta}, 0 \right)$ M1A1
- Putting $x = 0$, Q is $\left(0, \frac{b}{\sin \theta} \right)$ A1
- Therefore R is $\left(\frac{a}{2 \cos \theta}, \frac{b}{2 \sin \theta} \right)$ A1
- Eliminating θ ,
- $\cos \theta = \frac{a}{2x}; \sin \theta = \frac{b}{2y}$ M1
- $\frac{a^2}{4x^2} + \frac{b^2}{4y^2} = \cos^2 \theta + \sin^2 \theta = 1$ M1A1

7. (a) $z^{-n} = \cos(-n\theta) + i\sin(-n\theta)$ M1
 $= \cos n\theta - i\sin n\theta$ A1
 $z^n + z^{-n} = \cos n\theta + i\sin n\theta + (\cos n\theta - i\sin n\theta)$ M1
 $= 2\cos n\theta$ AG
- (b) Using the result in (a),
 $2\cos 2\theta - 4\cos \theta + 3 = 0$ M1
 $2(2\cos^2 \theta - 1) - 4\cos \theta + 3 = 0$ M1
 $4\cos^2 \theta - 4\cos \theta + 1 = 0$ A1
 $(2\cos \theta - 1)^2 = 0$ M1
 $\cos \theta = \frac{1}{2}$ A1
 $\sin \theta = \pm \frac{\sqrt{3}}{2}$ A1
The roots are $\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$ A1
[The $\pm$ must be there for the last 2 marks; accept $\cos(\pi/3) \pm i\sin(\pi/3)$]

8. (a) By long division,

$$\begin{array}{r}
 x+4 \\
 x-1 \overline{) x^2+3x} \\
 \underline{x^2-x} \\
 4x \quad (\text{Must be } 2x \text{ or } 4x \text{ for M1}) \quad \text{M1A1} \\
 \underline{4x-4} \\
 4
 \end{array}$$

Therefore,

$$f(x) = x + 4 + \frac{4}{x-1} \quad \text{A1}$$

[FT 2 from above]

(b) $f'(x) = 1 - \frac{4}{(x-1)^2}$ B1

[FT their expression from (a)]

Stationary points occur when

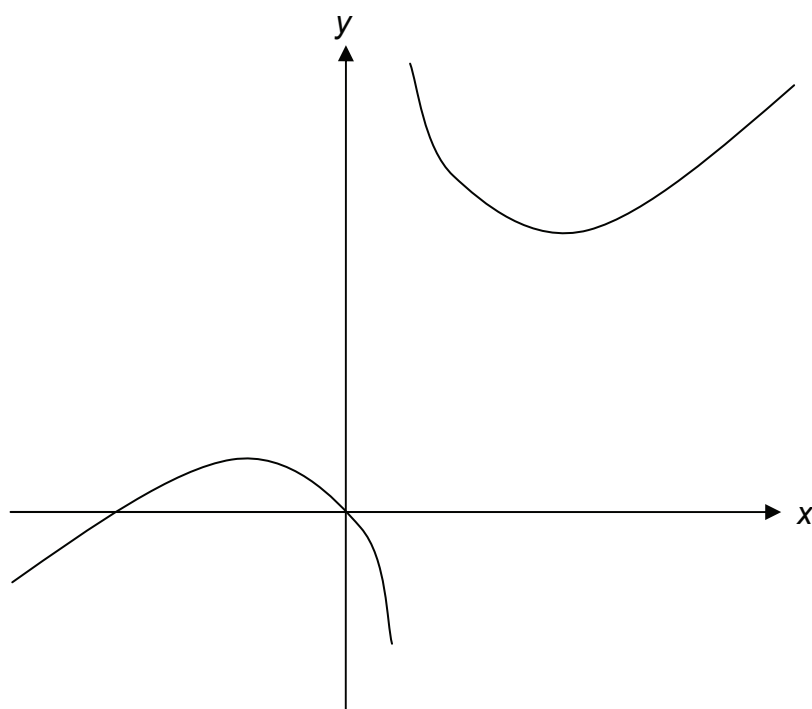
$$1 = \frac{4}{(x-1)^2} \quad \text{M1}$$

$$x = -1, 3 \quad \text{A1}$$

$$y = 1, 9 \quad \text{A1}$$

- (c) The asymptotes are $x = 1$ and $y = x + 4$.

B1B1



G2

- (d) Since $f(-3) = f(0) = 0$, part of the answer is $[-3, 0]$.
Consider

B1

$$\frac{x(x+3)}{(x-1)} = 10$$

M1

$$x^2 - 7x + 10 = 0$$

A1

$$x = 2, 5$$

A1

$$f^{-1}(A) = [-3, 0] \cup [2, 5]$$

A1

A/AS Level Mathematics – FP3 – June 2009 – Mark Scheme

1. The equation can be rewritten
 $1 + 2 \sinh^2 \theta = 6 \sinh \theta - 3$ M1A1
 $\sinh^2 \theta - 3 \sinh \theta + 2 = 0$ A1
 $(\sinh \theta - 1)(\sinh \theta - 2) = 0$ M1
 $\sinh \theta = 1, 2$ A1
 $\theta = \ln(1 + \sqrt{2}), \ln(2 + \sqrt{5})$ B1B1
2. $f(0) = 0$ B1
 $f'(x) = -\frac{e^x}{(2 - e^x)}; f'(0) = -1$ B1B1
 $f''(x) = -\frac{2e^x}{(2 - e^x)^2}; f''(0) = -2$ B1B1
 $f'''(x) = -\frac{2e^x(2 - e^x)^2 + 2e^x 2(2 - e^x)e^x}{(2 - e^x)^4}; f'''(0) = -6$ B1B1
 The Maclaurin series is
 $0 - x - \frac{2x^2}{2} - \frac{6x^3}{6} + \dots = -x - x^2 - x^3 + \dots$ M1A1
 [FT on their derivatives]
3. $dx = 2 \cosh u du; [0, 2] \rightarrow [0, \sinh^{-1} 1]$ B1B1
 $I = \int_0^{\sinh^{-1} 1} \frac{2 \cosh u du}{(4 \sinh^2 u + 4)^{3/2}}$ M1
 $= \int_0^{\sinh^{-1} 1} \frac{2 \cosh u du}{8 \cosh^3 u}$ A1
 $= \frac{1}{4} \int_0^{\sinh^{-1} 1} \sec^2 u d\theta$ A1
 $= \frac{1}{4} [\tanh u]_0^{\sinh^{-1} 1}$ A1
 $= \frac{1}{4} \tanh \sinh^{-1}(1)$ A1
 $= 0.18$ A1
 [Award 0 marks for answer only]

4. EITHER

$$2y \frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} \quad \text{B1}$$

$$\text{CSA} = 2\pi \int_0^a y \sqrt{1 + \frac{4a^2}{y^2}} dx \quad \text{M1A1}$$

$$= 2\pi \int_0^a \sqrt{4ax} \sqrt{1 + \frac{a}{x}} dx \quad \text{A1}$$

$$= 4\sqrt{a}\pi \int_0^a \sqrt{x+a} dx \quad \text{A1}$$

$$= 4\sqrt{a}\pi \cdot \frac{2}{3} [(x+a)^{3/2}]_0^a \quad \text{A1}$$

$$= \frac{8}{3} \sqrt{a}\pi [2^{3/2} a^{3/2} - a^{3/2}] \quad \text{A1}$$

= Answer given

OR

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a \quad \text{B1B1}$$

$$\text{CSA} = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad \text{M1}$$

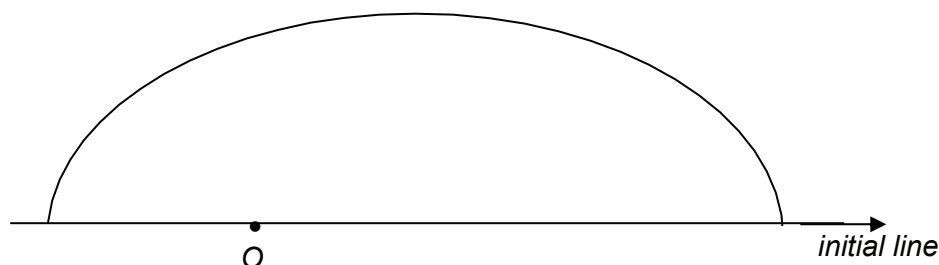
$$= 2\pi \int_0^1 2at \sqrt{4a^2 t^2 + 4a^2} dt \quad \text{A1}$$

$$= 8\pi a^2 \int_0^1 t \sqrt{t^2 + 1} dt \quad \text{A1}$$

$$= 8\pi a^2 \left[\frac{(t^2 + 1)^{3/2}}{3} \right]_0^1 \quad \text{M1A1}$$

= Answer given

5. (a)



G1

(b)
$$\text{Area} = \frac{1}{2} \int_0^{\pi/2} (2 + \cos \theta)^2 d\theta$$
 M1

$$= \frac{1}{2} \int_0^{\pi/2} (4 + 4 \cos \theta + \cos^2 \theta) d\theta$$
 A1

$$= \frac{1}{2} \int_0^{\pi/2} (4 + 4 \cos \theta + \frac{1}{2} + \frac{1}{2} \cos 2\theta) d\theta$$
 M1A1

$$= \frac{1}{2} \left[\frac{9\theta}{2} + 4 \sin \theta + \frac{1}{4} \sin 2\theta \right]_0^{\pi/2}$$
 A1

[Limits not required until here]

$$= \frac{9}{8} \pi + 2 \quad (5.53)$$
 A1

(c) Consider

$$y = r \sin \theta = (2 + \cos \theta) \sin \theta$$
 M1

$$\frac{dy}{d\theta} = -\sin^2 \theta + \cos \theta (2 + \cos \theta)$$
 A1

At a stationary point, $\frac{dy}{d\theta} = 0$ giving M1

$$2 \cos^2 \theta + 2 \cos \theta - 1 = 0$$
 A1

$$\cos \theta = \frac{\sqrt{3} - 1}{2}$$
 A1

$$\theta = 1.20, r = 2.37$$
 A1A1

6. (a) $I_n = \int_0^{\pi/4} \tan^{n-2} x \tan^2 x dx$ M1
- $$= \int_0^{\pi/4} \tan^{n-2} x (\sec^2 x - 1) dx$$
- m1A1
- $$= \frac{1}{n-1} [\tan^{n-1} x]_0^{\pi/4} - I_{n-2}$$
- A1A1
- $$= \frac{1}{n-1} - I_{n-2}$$
- (b) $I_0 = \int_0^{\pi/4} dx = [x]_0^{\pi/4} = \frac{\pi}{4}$ M1A1
- $$I_4 = \frac{1}{3} - I_2$$
- M1
- $$= \frac{1}{3} - (1 - I_0)$$
- A1
- $$= \frac{\pi}{4} - \frac{2}{3} \quad (0.119)$$
- A1
7. (a)(i) $f'(x) = \sinh x - x \cosh x, f'(0) = 0$ M1A1
 $f''(x) = -x \sinh x, f''(0) = 0$ A1
- (ii) Any valid method, eg looking at the behaviour of $f'(x)$ or $f''(x)$ around $x = 0$ or noting that f is an even function. M1
Concluding that it is not a stationary point of inflection A1
- b)(i) $2 \cosh \alpha - \alpha \sinh \alpha = 0$ M1

$$\frac{\alpha \sinh \alpha}{\cosh \alpha} = 2$$
 A1
whence $\alpha \tanh \alpha = 2$
- (ii) Let $f(\alpha) = \alpha \tanh \alpha - 2$
 $f(2) = -0.0719 \dots$ M1
 $f(2.1) = 0.0379 \dots$
The change of sign shows that the value of α lies between 2 and 2.1. A1
[Accept consideration of $f(\alpha) = 2 \cosh \alpha - \alpha \sinh \alpha$]
- (iii) Let $y = \frac{2}{\tanh x}$ M1

$$\frac{dy}{dx} = -\frac{2}{\tanh^2 x} \times \operatorname{sech}^2 x$$
 A1
 $= 0.137 < 1$ in modulus when $x = 2.05$ A1
therefore convergent.

(iv) Taking the initial value as 2.05, successive values are

2.05

2.067407830

B1

2.065063848

2.065374578

2.065333300

2.065338782

Thus $\alpha = 2.0653$ correct to 4dps.

B1

(c)
$$\text{Area} = \int_0^{2.0653} (2 \cosh x - x \sinh x) dx$$
 M1

$$= [2 \sinh x]_0^{2.0653} - [x \cosh x]_0^{2.0653} + \int_0^{2.0653} \cosh x dx$$
 A1A1A1

$$= [3 \sinh x - x \cosh x]_0^{2.0653}$$
 A1

$$= 3.365$$
 A1



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GCE MARKING SCHEME

**MATHEMATICS - M1-M3 & S1-S3
AS/Advanced**

SUMMER 2009

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2009 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

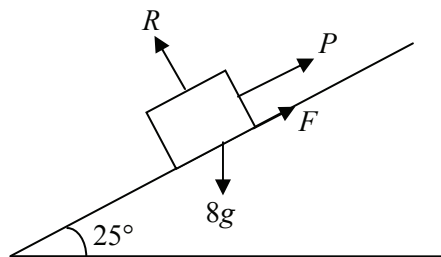
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M1	1
M2	5
M3	8
 S1	 12
S2	16
S3	19

Mathematics M1 (June 2009)
Final Markscheme

- 1.(a) Using $v = u + at$ with $u = 14.7$, $a = (-)9.8$, $t = 2$. M1
 $v = 14.7 - 9.8 \times 2$ A1
 $v = -4.9$
Speed = 4.9 ms^{-1} A1
- 1.(b) Using $v^2 = u^2 + 2as$ with $u = 14.7$, $a = (-)9.8$, $s = (-)70.2$. M1
 $v^2 = 14.7^2 + 2 \times (-9.8) \times (-70.2)$ A1
 $v = \underline{39.9 \text{ ms}^{-1}}$ cao A1
- 1.(c) Using $s = ut + \frac{1}{2}at^2$ with $u = 14.7$, $a = (-)9.8$, $s = 3.969$. M1
 $3.969 = 14.7t - \frac{1}{2} \times 9.8 \times t^2$ A1
 $t^2 - 3t + 0.81 = 0$ attempt to solve m1
 $(t - 0.3)(t - 2.7) = 0$
 $t = 0.3, 2.7$
Therefore required length of time $= 2.7 - 0.3$
 $= \underline{2.4 \text{ s}}$ cao A1
- 2.(a) N2L $5g - T = 5a$ dim. correct M1 A1
 $T - 2g = 2a$ dim. correct M1 A1
Adding $3g = 7a$ m1
 $a = \underline{3g/7} = (4.2) \text{ ms}^{-2}$ cao A1
 $T = 2 \times 9.8 + 2 \times 4.2$
 $= \underline{28 \text{ N}}$ cao A1
- 2.(b) Magnitude of acceleration of objects A and B are equal. B1
- 3.(a) N2L applied to lift and person $900g - T = 900a$ dim corr. M1
 $900 \times 9.8 - 8550 = 900a$ A1
 $a = \underline{0.3 \text{ ms}^{-2}}$ cao A1
- 3.(b) N2L applied to person $65g - R = 65a$ M1
 $R = 65(9.8 - 0.3)$ A1
 $R = \underline{617.5 \text{ N}}$ ft c's a A1

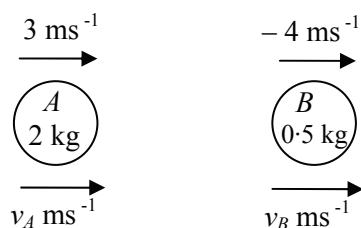
- 4.(a) At $t = 10$, acceleration = $\frac{20-5}{30}$ M1
 $= 0.5 \text{ ms}^{-2}$ cao A1
 At $t = 40$, acceleration = 0 B1
- 4.(b) Using $v = u + at$ with $u = 5$ $t = 20$, $a = 0.5$ M1
 $v = 5 + 0.5 \times 20$
 $v = 15 \text{ ms}^{-1}$ ft acce if > 0 A1
- 4.(c) Distance = $\frac{1}{2}(5+20) \times 30 + 20 \times 40 + \frac{1}{2} \times 20 \times 50$ method for distance M1
 any correct area B1
 correct expression A1
 Distance = 8875 m cao A1

5.



- (a) $R = 8g \cos 25^\circ (=71.05)$ si M1 A1
 $F = 0.3 \times 8g \cos 25^\circ (=21.32)$ si m1
 N2L up slope dim correct, all forces, $a = 0$ M1
 $P + F = 8g \sin 25^\circ$ A1
 $P = 8 \times 9.8 \sin 25^\circ - 2.4 \times 9.8 \cos 25^\circ$
 $P = \underline{11.82 \text{ N}}$ cao A1
- (b) N2L down slope dim correct, all forces M1
 $P - F - 8g \sin 25^\circ = 8a$ A1 A1
 $P = 8g \sin 25^\circ + 2.4g \cos 25^\circ + 8 \times 0.6$
 $P = \underline{59.25 \text{ N}}$ cao A1

6.



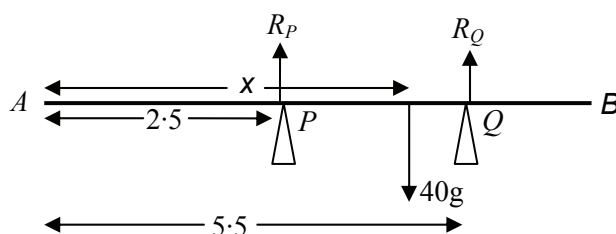
- (a) conservation of Momentum M1
 $2 \times 3 - 0.5 \times 4 = 2v_A + 0.5v_B$ A1
 $4v_A + v_B = 8$

Restitution M1
 $v_B - v_A = -e(-4 - 3)$ A1
 $= \frac{2}{7} \times 7$
 $v_B - v_A = 2$

Subtracting $5v_A = 6$ dep. on both M's m1
 $v_A = \underline{1.2\text{ms}^{-1}}$ cao A1
 $v_B = \underline{3.2\text{ms}^{-1}}$ cao A1

- (b) Impulse on B = Change in momentum of B. used M1
 $I = 0.5(3.2 - (-4))$
 $I = \underline{3.6\text{Ns}}$ ft v_A, v_B A1 B1

7.



- (a) Resolve vertically $R_P + R_Q = 40g$ oe M1
 $R_P = R_Q = R$ $R + R = 40g$
 $R = 20g = (196)\text{N}$ A1

- (b) Moments about A moments both sides, dim correct M1
 $2.5 R_P + 5.5 R_Q = x \times 40(g)$ A1 B1
 $8 \times 20g = 40g \times x$
 $x = \underline{4\text{m}}$ cao A1

OR If $R_P = R_Q$, C must be the midpoint of PQ. M1
 Therefore $x = 2.5 + 0.5(5.5 - 2.5)$ B1 A1
 $= \underline{4\text{m}}$ A1

8.	Resolve in one direction to obtain component of resultant	M1
	$X = 7\cos 30^\circ - 2\cos 60^\circ - 5\cos 50^\circ$	A1
	$X = 1.8482$	

	Resolve in perpendicular direction	M1
	$Y = 5\cos 40^\circ + 7\cos 60^\circ - 2\cos 30^\circ$	A1
	$Y = 5.5982$	

	$\text{Resultant}^2 = 1.8482^2 + 5.5982^2$	m1
	$\text{Resultant} = \underline{5.9\text{ N}}$	cao A1

9.(a)		Area	from AE	from AB	
	Square	36	3	3	
	Triangle	12	3	$6 + \frac{4}{3} = \frac{22}{3}$	
	The sign	48	x	y	
					B1 B1 B1

Distance of centre of mass from $AE = x = \underline{3}$	B1
--	----

Moments about AB	M1
--------------------	----

$48y = 12 \times \frac{22}{3} + 36 \times 3$	ft areas, y's	A1
--	---------------	----

$y = \frac{49}{12} = \underline{4.083\text{ cm}}$	cao	A1
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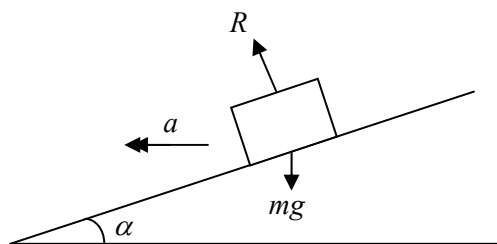
(b)	$\tan \theta = \frac{3}{\frac{49}{12}}$	ft x, y	M1 A1
	$\theta = \underline{36.3^\circ}$	ft x, y	A1

Mathematics M2 (June 2009)
Final Markscheme

1.(a)	$a = \frac{d}{dt}(\cos 2t - 3 \sin 2t)$ $= -2 \sin 2t - 3 \cos 2t$ When $t = \pi$ $a = -2 \sin 2\pi - 3 \cos \pi$ $a = 3 \text{ ms}^{-2}$	attempted	M1
			A1 A1
1.(b)	$x = \int \cos 2t - 3 \sin t \, dt$ $x = 0.5 \sin 2t + 3 \cos t + C$ When $t = 0, x = 4$ $4 = 3 + C$ $C = 1$ $x = 0.5 \sin 2t + 3 \cos t + 1$ $x = \underline{3.62 \text{ m}}$	attempted	M1
		used	A1 A1
		ft	m1
		ft	A1
		cao	A1
2.(a)	Using Hooke's Law $80 = \frac{\lambda(0.25 - 0.2)}{0.25}$ $\lambda = \underline{400 \text{ N}}$	cao	M1 A1
			A1
2.(b)	Energy at start $= \frac{1}{2} \times 400 \times \frac{0.05^2}{0.25} (= 2)$ Energy at end $= \frac{1}{2} \times 0.36 v^2 (= 0.18 v^2)$ Conservation of energy $0.18 v^2 = 2$ $v = 3\frac{1}{3} \text{ ms}^{-1}$	si	M1 A1
		used	M1
		cao	A1
3.(a)	Resolve perpendicular to plane $R = mg \cos \alpha$ $R = 3.5 \times 9.8 \times 0.8 = 27.44$ $F = \mu R$ $F = 0.25 \times 27.44 = 6.86$ Work done against friction $= 6.86 \times 2$ $= \underline{13.72 \text{ J}}$	attempted	M1
			A1
		used	m1
		ft c's F	M1
			A1
3.(b)	K. E. at start $= 0.5 m u^2$ $= 1.75 u^2$ P. E. at end $= mgh = 3.5 \times 9.8 \times 2 \sin \alpha$ $= 3.5 \times 9.8 \times 2 \times 0.6$ $= 41.16$ Work-energy Principle $1.75 u^2 = 41.16 + 13.72$ $u^2 = 31.36$ $u = \underline{5.6 \text{ ms}^{-1}}$	used	M1
		cao	M1 A1
			A1

4.(a)	N2L applied to particle $F - 1500 = 5000a$ $F = 2500 \text{ N}$ $F = \frac{P}{v} = \frac{P}{12}$ $P = \underline{30000 \text{ W}}$	3 terms cao	M1 A1 M1 A1
4.(b)	Since maximum velocity, $a = 0$ $F = 1500$ $F = \frac{45 \times 1000}{v}$ $\frac{45000}{v} = 1500$ $v = \underline{30 \text{ ms}^{-1}}$	si cao	M1 A1 M1 A1
5.(a)	Initial horizontal velocity Time to reach wall	$= 17.5 \cos \alpha = 17.5 \times 0.6$ $= 10.5$ $= \frac{25 \cdot 2}{10 \cdot 5}$ $= \underline{2.4 \text{ s}}$	B1 M1 A1
5.(b)	Initial vertical velocity Using $s = ut + \frac{1}{2}at^2$ with $u = 14$ (c), $a = (-)9.8$, $t = 2.4$ $= 14 \times 2.4 - 4.9 \times 2.4^2$ $= \underline{5.376 \text{ m}}$	$= 17.5 \sin \alpha = 17.5 \times 0.8$ $= 14$ ft if M1 in (a) awarded	B1 M1 A1 A1
5.(c)	Using $v = u + at$ with $u = 14$ (c), $a = (-)9.8$, $v = 0$ $0 = 14 - 9.8t$ $t = \frac{10}{7} \text{ s}$	ft if M1 in (a) awarded	M1 A1 A1
6.(a)	velocity Momentum	$= \frac{dr}{dt}$ $= -8t\mathbf{i} + (6t - 5)\mathbf{j}$ $= m\mathbf{v} = -16t\mathbf{i} + 2(6t - 5)\mathbf{j}$	used A1 A1
6.(b)	acceleration Magnitude	$= \frac{dv}{dt}$ $= -8\mathbf{i} + 6\mathbf{j}$ $= \sqrt{8^2 + 6^2}$ $= \underline{10 \text{ ms}^{-2}}$	used constant since independent of t M1 M1 A1
6.(c)	Velocity is perpendicular to acceleration when $\mathbf{v} \cdot \mathbf{a} = 0$ $\mathbf{v} \cdot \mathbf{a} = (-8t\mathbf{i} + (6t - 5)\mathbf{j}) \cdot (-8\mathbf{i} + 6\mathbf{j})$ $= 64t + 6(6t - 5)$ $100t - 30$ $t = \underline{0.3 \text{ s}}$	ft $\mathbf{v}$, $\mathbf{a}$ cao	M1 M1 A1 A1

7.



Resolve vertically M1

$$\begin{aligned} R \cos \alpha &= mg \\ &= 1000 \times 9.8 \\ &= 9800 \end{aligned} \quad \text{A1}$$

Using N2L $R \sin \alpha = ma$ M1

$$a = \frac{v^2}{r} \quad \text{M1}$$

$$\begin{aligned} R \sin \alpha &= \frac{1000 \times 28^2}{250} \quad \text{si} \quad \text{A1} \\ &= 3136 \end{aligned}$$

Solving $\tan \alpha = \frac{3136}{9800}$ m1

$$\begin{aligned} &= 0.32 \\ \alpha &= \underline{17.74^\circ} \quad \text{cao} \quad \text{A1} \end{aligned}$$

8.(a) Conservation of energy used M1

$$\begin{aligned} 0.5 \times 5 \times 9^2 &= 0.5 \times 5 \times v^2 + 5g(2 - 2\cos\theta) \quad \text{A1 A1} \\ v^2 &= 81 - 39.2(1 - \cos\theta) \\ v^2 &= 41.8 + 39.2 \quad \text{A1} \end{aligned}$$

8.(b) N2L towards centre used M1

$$\begin{aligned} R - mg \cos \theta &= \frac{mv^2}{r} \quad \text{A1} \\ R &= 5 \times 9.8 \cos \theta + \frac{5}{2} (41.8 + 39.2 \cos \theta) \quad \text{m1} \\ &= 147 \cos \theta + 104.5 \quad \text{A1} \end{aligned}$$

8.(c) Particle leaves sphere when $R = 0$ oe M1

$$\begin{aligned} 147 \cos \theta + 104.5 &= 0, \quad \theta = 135.3^\circ \\ \text{Therefore particle will leave circle before reaching the top, i.e. particle} & \\ \text{will not complete circle.} & \quad \text{A1} \end{aligned}$$

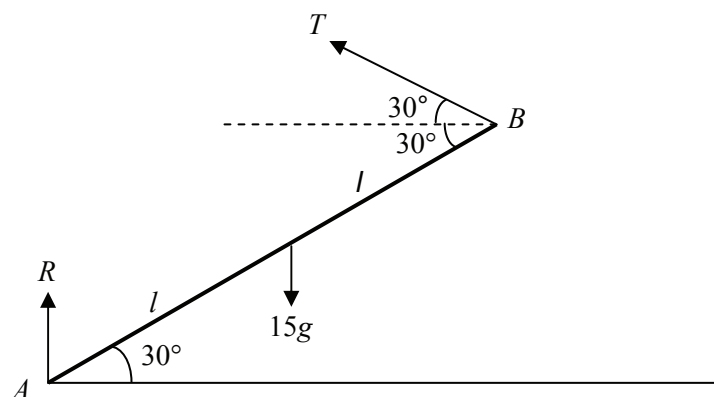
Mathematics M3 (June 2009)
Final Markscheme

1.(a)	Using N2L		M1
	$-0.2 - 0.03v = 9 \frac{dv}{dt}$		A1
	$900 \frac{dv}{dt} = -(20 + 3v)$		A1
1.(b)	$900 \int \frac{dv}{20+3v} = - \int dt$	sep. var.	M1
	$900 \cdot \frac{1}{3} \ln(20 + 3v) = -t (+ C)$		A1 A1
	When $t=0, v = 20$	used	m1
	$C = 300 \ln 80$		
	Therefore $t = 300 \ln(80) - 300 \ln(20 + 3v)$		A1
	$t = 300 \ln\left(\frac{80}{20 + 3v}\right)$		
1.(c)	When body is at rest, $v=0$	used	m1
	$t = 300 \ln(80) - 300 \ln(20)$		
	$= 300 \ln(4)$		
	$= \underline{416 \text{ s}}$	cao	A1
2.(a)	Amplitude = 24 cm = 0.24 m		B1
	Period = $2 \times 4 = 8 \text{ s}$		B1
	Therefore $\frac{2\pi}{\omega} = 8$		M1
	$\omega = \frac{\pi}{4}$		A1
	Speed of projection = $a\omega$	used o.e.	M1
	$= 0.24 \times \frac{\pi}{4}$		
	$= 0.06\pi = \underline{0.188 \text{ ms}^{-1}} (= 18.8 \text{ cms}^{-1})$	cao	A1
2.(b)	$x = 0.24 \sin\left(\frac{\pi}{4}t\right)$		M1
	$0.15 = 0.24 \sin\left(\frac{\pi}{4}t\right)$		m1
	$t = 0.86$	cao	A1
	Required time = $8 + 0.86$		
	$= \underline{8.86 \text{ s}}$	ft t and period	A1

2.(c)	$v = \frac{dx}{dt}$	used	M1
	$v = 0.06\pi \cos(\frac{\pi}{4}t)$	ft ω	A1
	When $t = 1.5$ $v = 0.06\pi \cos(\frac{\pi}{4} \times 1.5)$		m1
	$v = \underline{0.072 \text{ ms}^{-1}}$ ($= 7.2 \text{ cms}^{-1}$)	cao	A1
2.(d)	$v^2 = \omega^2 (a^2 - x^2)$		M1
	$v^2 = \frac{\pi^2}{4^2} (0 \cdot 24^2 - 0 \cdot 2^2)$		A1
	$v = \underline{0.104 \text{ ms}^{-1}}$ ($= 10.4 \text{ cms}^{-1}$)	cao	A1
3.	Apply N2L		M1
	$180 - 3v^2 = 75a$		A1
	$60 - v^2 = 25v \frac{dv}{dx}$		
	$25v \frac{dv}{dx} = 60 - v^2$		A1
	$25 \int \frac{v dv}{dx} = \int dx$	sep. var.	M1
	$-\frac{25}{2} \ln(60 - v^2) = x (+C)$		A1 A1
	When $x = 0, v = 0$	(accept limits) used	m1
	$-\frac{25}{2} \ln(60) = C$	cao	A1
	$x = \frac{25}{2} \ln\left(\frac{60}{60 - v^2}\right)$		
	When $x = 20$		
	$\ln\left(\frac{60}{60 - v^2}\right) = 20 \times \frac{2}{25} = 1.6$		
	$\frac{60}{60 - v^2} = e^{1.6}$	$x = 20$ and inversion	m1
	$60 = 60e^{1.6} - e^{1.6}v^2$		
	$v^2 = \frac{60(e^{1.6} - 1)}{e^{1.6}}$		
	$v = \underline{6.92 \text{ ms}^{-1}}$	cao	A1

4.(a)	Impulse = change in momentum $1.2 = 3v$ $v = \underline{0.4 \text{ ms}^{-1}}$	used cao	M1 A1
4.(b)	For Q $-I = 3v - 3 \times 0.4$ $I = 3v - 1.2$ For P $I = 5v$ Both equations correct Solving simultaneously $5v = 1.2 - 3v$ $8v = 1.2$ $v = \underline{0.15 \text{ ms}^{-1}}$ $I = \underline{0.75 \text{ Ns}}$	attempt P or Q cao cao	M1 A1 A1
4.(c)	Loss in energy $= 0.5 \times 3 \times 0.4^2 - 0.5 \times 8 \times 0.15^2$ $= \underline{0.15 \text{ J}}$	ft v's cao	M1 A1 A1 A1
5.(a)	N2L $(156 - 52x) - 4v = 2a$ $2a + 4v + 52x = 156$ $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 26x = 78$		M1 A1
5.(b)	Auxiliary Equation $m^2 + 2m + 26 = 0$ $m = -1 \pm 5i$ Complementary function is $x = e^{-t}(A\sin 5t + B\cos 5t)$ P. I. try $x = a$ $26a = 78$ $a = 3$ General solution is $x = e^{-t}(A\sin 5t + B\cos 5t) + 3$ When $t=0, x=0$ $0 = B + 3$ $B = -3$ $\frac{dx}{dt} = -e^{-t}(A\sin 5t + B\cos 5t) + e^{-t}(5A\cos 5t + 5B\sin 5t)$ When $t=0, \frac{dx}{dt} = 3$ $3 = 3 + 5A$ $A = 0$ $x = 3 - 3e^{-0.5} \cos 5t$ When $t = 0.5$ $x = 3 - 3e^{-0.5} \cos(5 \times 0.5)$ $x = \underline{4.46 \text{ m}}$	cao ft m if complex ft CF + PI subst. into GS ft similar exp. cao	M1 A1 B1 B1 B1 m1 A1 m1 A1

6.



Moments about A

$$15g \times l \cos 30^\circ = T \times 2l \cos 30^\circ$$

$$T = 75g$$

$$T = \underline{75.5 \text{ N}}$$

dim correct

M1

A1

A1

Resolve horizontally

$$T \cos 30^\circ = F$$

$$F = 73.5 \cos 30^\circ$$

$$F = 36.75\sqrt{3} \text{ N}$$

subst. for T

M1

A1

m1

Resolve vertically

$$R + T \sin 30^\circ = 15g$$

$$R = 15g - 73.5 \times 0.5$$

$$R = 110.25 \text{ N}$$

subst. for T

M1

A1

m1

$$F \leq \mu R$$

$$\mu \geq \frac{36.75\sqrt{3}}{110.25}$$

any correct expression

M1

A1

Therefore least value of μ is $0.577 \left(\frac{1}{\sqrt{3}} \right)$

cao

A1

- 1 (a) $P(\text{no teachers}) = \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7}$ or $\frac{\binom{7}{3}}{\binom{9}{3}} = \frac{5}{12}$ M1A1
- (b) $P(1 \text{ of each}) = \frac{2}{9} \times \frac{3}{8} \times \frac{4}{7} \times 6$ or $\frac{\binom{2}{1} \times \binom{3}{1} \times \binom{4}{1}}{\binom{9}{3}}$ M1A1
- $= \frac{2}{7}$ (cao) A1
- 2 (a)(i) $P(A \cup B) = P(A) + P(B) = 0.5$ M1A1
- (ii) $P(A \cap B) = P(A)P(B) = 0.2 \times 0.3$ M1
- $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ M1
- $= 0.2 + 0.3 - 0.2 \times 0.3 = 0.44$ A1
- (b) EITHER $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ M1
- $= 0.1$ A1
- $P(A|B) = \frac{P(A \cap B)}{P(B)}$ M1
- $= \frac{1}{3}$ A1
- [FT on $P(A \cap B)$ unless independence assumed]
- OR $P(A \cup B) = P(A) + P(B) - P(B)P(A|B)$ M1
- $0.4 = 0.2 + 0.3 - 0.3P(A|B)$ A1
- $P(A|B) = \frac{0.1}{0.3}$ M1
- $= \frac{1}{3}$ A1
- (c) Smallest value is 0.3 B1
- when A is a subset of B . B1
- 3 (a) Mean = 20, variance = 4. B1B1
- (b)(i) $E(Y) = 2 \times 20 - 3 = 37$ [FT from (a)] M1A1
- $\text{Var}(Y) = 4 \times 4 = 16$ M1A1
- (ii) $20a - b = 0$ and $4a^2 = 1$ B1B1
- The solution is $a = \frac{1}{2}$, $b = 10$ B1B1
- [No FT on equations, treat $aX + b$ as MR and accept $b = -10$]

- 4 (a) Mean = 2.4 si B1
- (i) Prob = $\frac{2.4^3}{6} e^{-2.4} = 0.2090$ M1A1
- (or $0.7787 - 0.5697$ or $0.4303 - 0.2213$)
- (ii) Prob = 0.4303 or $1 - 0.5697 = 0.4303$ M1A1
- (b) Prob of no fish = $e^{-0.6t} = 0.5$ M1A1
- EITHER
- $-0.6t \log_{10} e = \log_{10} 0.5$ so $t = 1.16$ M1A1
- OR
- $-0.6t = \ln(0.5)$ so $t = 1.16$ M1A1
- [No marks answer only]
- Special case:
- Using tables,
- $0.7 \approx 0.6t$ M1
- so $t \approx 1.17$ A1
- 5 (a) $P(+) = 0.05 \times 0.99 + 0.95 \times 0.02$ M1A1
- $= 0.0685$ (cao) (137/2000) A1
- (b) $P(\text{Dis} | +) = \frac{0.05 \times 0.99}{0.0685}$ B1B1
- $= 0.723$ (99/137) B1
- [FT from (a) – only award final B1 if previous 2 B marks awarded]
- 6 (a)(i) $E(X) = 0.1 \times 1 + 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 4 + 0.1 \times 5$ M1A1
- $= 3.1$ cao A1
- (ii) $E(X^2) = 0.1 \times 1 + 0.2 \times 4 + 0.3 \times 9 + 0.3 \times 16 + 0.1 \times 25$ (10.9) M1A1
- $\text{Var}(X) = 10.9 - 3.1^2 = 1.29$ [FT from $E(X^2)$ and $E(X)$] A1
- [Accept answers with no working]
- (b) The possibilities are 1,1 ; 2,2 ; 3,3 ; 4,4 ; 5,5 si B1
- Prob = $0.1^2 + 0.2^2 + 0.3^2 + 0.3^2 + 0.1^2 = 0.24$ M1A1
- [Award M1 if 4 or more correct probabilities seen]

7

(a)

The probability distribution for Ann is

No. of heads	0	1	2	3
Prob	1/8	3/8	3/8	1/8

B1(si)

and for Bob is

No. of heads	0	1	2
Prob	1/4	1/2	1/4

B1(si)

We now require

A = 3 OR A = 2 and B = 0 or 1 OR A = 1 and B = 0 si M1A1
(or B = 0 and A = 1, 2 or 3 OR B = 1 and A = 2 or 3 OR B = 2 and A = 3)

$$P(A > B) = \frac{1}{8} + \frac{3}{8} \times \frac{3}{4} + \frac{3}{8} \times \frac{1}{4} = \frac{1}{2} \quad \text{M1A1}$$

$$(\text{or } P(A > B) = \frac{1}{4} \times \frac{7}{8} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{8} = \frac{1}{2})$$

$$(b)(i) \quad P(\text{M wins 1}^{\text{st}} \text{ time}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \quad \text{M1A1}$$

$$(ii) \quad P(\text{M wins 2}^{\text{nd}} \text{ time}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} \quad \text{A1}$$

$$(iii) \quad P(\text{M wins}) = \frac{1}{4} + \frac{1}{16} + \dots \quad \text{M1A1}$$

$$= \frac{1/4}{1 - 1/4} = \frac{1}{3} \quad \text{M1A1}$$

8

(a)

$$E(X) = \frac{1}{2} \int_0^1 x(1+2x) dx \quad \text{M1A1}$$

$$= \frac{1}{2} \left[\frac{x^2}{2} + \frac{2x^3}{3} \right]_0^1 \quad \text{A1}$$

$$= \frac{7}{12} \quad \text{cao} \quad \text{A1}$$

$$(b) \quad F(x) = \frac{1}{2} \int_0^x (1+2u) du \quad \text{M1}$$

[Correct limits required but accept a solution with no limits which includes a constant of integration which is then shown to be zero]

$$= \frac{1}{2} [u + u^2]_0^x \quad \text{B1}$$

$$= \frac{1}{2} (x + x^2) \quad \text{A1}$$

(c) (i)	$\text{Prob} = F(0.5) - F(0.4)$	M1
	$= 0.095$	A1
	[FT from (b), accept with no working]	
(ii)	$\frac{1}{2}(m + m^2) = \frac{1}{2}$	M1
	$m = \frac{-1 \pm \sqrt{1+4}}{2}$	m1
	$= 0.618 \quad \left(\frac{\sqrt{5}-1}{2} \right)$	A1

[If 2 roots are given, some indication of which to accept must be given]

A/AS level Maths - S2 June 2009 - Mark Scheme – Post Examiners' Conference

- 1 (a) Mean = 12 si B1
p-value = $P(X \geq 18 \mid \text{mean} = 12)$ M1
= 0.0630 A1
- (b) X is now $\text{Po}(100)$ which is approx $N(100, 100)$ B1

$$z = \frac{124.5 - 100}{10}$$
 M1
= 2.45 cao A1
p-value = 0.00714 (FT from z) A1
Very strong evidence for concluding that the mean has increased. B1
[FT from p-value]
- 2 (a) (i)
$$z = \frac{150 - 140}{8} = 1.25$$
 M1A1
Prob = 0.1056 (FT from z) A1
- (ii) Required prob = $0.1056^3 = 0.00118$ [FT from (i)] M1A1
- (b) $A - R$ is $N(145 - 140, 8^2 + 6^2)$ ie $N(5, 100)$ M1A1
 $P(A < R) = P(A - R < 0)$ M1

$$z = \frac{5}{\sqrt{100}} = (\pm)0.5$$
 A1
Prob = 0.3085 A1
[No FT on mean and variance]
- 3 (a) $\bar{x} = \frac{66.8}{10}$ (= 6.68) si B1
SE of $\bar{X} = \frac{0.1}{\sqrt{10}}$ (= 0.03162...) si B1
[Accept variance of mean]
99% conf limits are
 $6.68 \pm 2.576 \times 0.1/\sqrt{10}$ M1A1
[M1 correct form, A1 2.576, allow their mean and SE for the M mark]
giving [6.60, 6.76] A1
[FT on their mean, SE and z excluding the use of 0.1 as SE]
- (b)
$$z = \frac{6.74 - 6.68}{0.03162}$$
 M1
[FT on their SE]
= 1.90 A1
Conf level = $1 - 0.0287 \times 2 = 0.9426$ B1B1
[M0 for trial and improvement]

4	(a)	$H_0 : \mu_x = \mu_y$ versus $H_1 : \mu_x \neq \mu_y$	B1
	(b)	$\bar{x} = 15.8, \bar{y} = 16.2$	B1
		SE of difference of means = $\sqrt{\frac{0.5^2}{6} + \frac{0.5^2}{5}}$ (= 0.3027...)	B1
		[Accept variance]	
		$z = \frac{16.2 - 15.8}{0.3027}$	M1
		= 1.32 cao	A1
		Prob from tables = 0.0934 cao	A1
		p-value = $2 \times 0.0934 = 0.1868$ (FT from line above)	B1
		Mean times are equal (oe) (FT from p-value)	B1
5	(a)(i)	$E(X) = 8$	B1
	(ii)	$\text{Var}(X) = 4.8$	B1
		Using $\text{Var}(X) = E(X^2) - [E(X)]^2$	M1
		$E(X^2) = 4.8 + 64 = 68.8$	A1
	(b)	$\text{Var}(Y) = \mu$	B1
		using $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$	M1
		$\mu = 9.36 - \mu^2$	A1
		$\mu = 2.6$ cao	A1
	(c)	$E(U) = E(X)E(Y) = 20.8$	B1
		$E(U^2) = E(X^2)E(Y^2) = 643.968$	B1
		$\text{Var}(U) = E(X^2Y^2) - [E(XY)]^2$	M1
		$= 643.968 - 20.8^2 = 211.328$	A1
		[FT their values from (a) and (b) – allow a multiple of μ in first line]	
6	(a)(i)	$f(x) = \frac{1}{7}$	B1
	(ii)	$F(x) = \int_9^x \frac{1}{7} du$	M1
		$= \left[\frac{u}{7} \right]_9^x$	A1
		$= \frac{x-9}{7}$	A1

	(b)(i)	$E(Y) = \int_9^{16} \sqrt{x} \cdot \frac{1}{7} dx$	M1
		[no limits required for M mark]	
		$= \frac{1}{7} \times \left[x^{1.5} \cdot \frac{2}{3} \right]_9^{16}$	A1
		$= 3.52$	A1
	(ii)	The median m satisfies	
		$P(Y \leq m) = 0.5$	M1
		$P(\sqrt{X} \leq m) = 0.5$	A1
		$P(X \leq m^2) = 0.5$	A1
		$F(m^2) = \frac{m^2 - 9}{7} = 0.5$	M1
		$m = 3.54$	A1
		[FT their $F(x)$ from (a)]	
7	(a)	$H_0 : p = 0.7$ versus $H_1 : p > 0.7$	B1
	(b)	Under H_0 , X (No cured) is B(50,0.7)	B1
		and Y (No not cured) is B(50,0.3) (si)	B1
		p-value = $P(X \geq 40) \mid H_0$	M1
		$= P(Y \leq 10 \mid H_0) = 0.0789$	A1
		The new drug is no better	B1
	(c)(i)	X is now B(250,0.7) which is approx N(175,52.5)	M1A1
		$z = \frac{189.5 - 175}{\sqrt{52.5}}$	M1
		$= 2.00$	A1
		Sig level = 0.02275 (FT from their z)	A1
	(ii)	X is now B(250,0.8) which is approx N(200,40)	M1A1
		$z = \frac{189.5 - 200}{\sqrt{40}}$	M1
		$= -1.66$	A1
		Prob = 0.0485	A1

AS/A Maths - S3 – June 2009 – Markscheme (Post Examiners' Conference)

1. (a)(i) $P(X = 5) = \binom{5}{5} \div \binom{8}{5} \text{ or } \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{1}{4} = \frac{1}{56} \quad (0.018) \quad \text{M1A1}$
- $P(X = 4) = \binom{5}{4} \times \binom{3}{1} \div \binom{8}{5} = \frac{15}{56} \quad (0.27) \quad \text{A1}$
- $P(X = 3) = \binom{5}{3} \times \binom{3}{2} \div \binom{8}{5} = \frac{30}{56} \quad (0.54) \quad \text{A1}$
- $P(X = 2) = \binom{5}{2} \times \binom{3}{3} \div \binom{8}{5} = \frac{10}{56} \quad (0.18) \quad \text{A1}$
- (ii) $E(X) = 5 \times \frac{1}{56} + 4 \times \frac{15}{56} + 3 \times \frac{30}{56} + 2 \times \frac{10}{56} = \frac{25}{8}$ M1A1
 [FT on probabilities]
- (b) X is binomial B1
 with parameters $(5, 5/8)$ B1
 Mean value $= 5 \times \frac{5}{8} = \frac{25}{8}$ B1
 [Accept any correct method]
2. (a)(i) $\hat{p} = \frac{78}{120} = 0.65$ B1
- (ii) $\text{ESE} = \sqrt{\frac{0.65 \times 0.35}{120}} = 0.04354..$ M1A1
 [M mark needs square root term in denominator]
- (iii) 95% confidence limits are M1A1
 $0.65 \pm 1.96 \times 0.0435..$ A1
 giving $[0.56, 0.74]$
 [FT values from (i) and (ii) but M mark needs square root in denominator]
- (b) Sian is likely to be of above average intelligence. B1

- 3 (a) $H_0 : \mu = 1.5; H_1 : \mu \neq 1.5$ B1
- (b) $\bar{x} = \frac{121.2}{80} (= 1.515)$ si B1
- $s^2 = \frac{184.42}{79} - \frac{121.2^2}{79 \times 80} (= 0.01015\dots)$ si B1
- [Accept division by 80 giving 0.010025]
- Test stat = $\frac{1.515 - 1.5}{\sqrt{0.01015/80}}$ M1A1
- [Award M1 only if $\sqrt{n}$ term present]
- = 1.33 (1.34) A1
- Prob from tables = 0.0918 (0.0901) A1
- p-value = $2 \times 0.0918 = 0.1836/5$ (0.1802) (FT from line above) B1
- Accept H_0 (oe) (FT from line above) B1
- (c) The sample mean is (approximately) normal. B1
- The variance estimate is used in place of the actual variance. B1
- 4 (a) $\Sigma x = 61.1; \Sigma x^2 = 373.3412$ B1
- UE of $\mu = 6.11$ cao B1
- UE of $\sigma^2 = \frac{373.3412}{9} - \frac{61.1^2}{9 \times 10}$ M1
- = 0.002244... cao A1
- (b) DF = 9 si B1
- At the 95% confidence level, critical value = 2.262 B1
- The 95% confidence limits are
- $6.11 \pm 2.262 \sqrt{\frac{0.002244\dots}{10}}$ M1A1
- [Award the M mark only if t -value used – FT on t percentile]
- giving [6.08, 6.14] A1
- [FT from line above as long as $\sqrt{10}$ present]
- 5 (a) $\bar{x} = 65.5; \bar{y} = 67.0$ si B1B1
- $s_x^2 = \frac{258000}{59} - \frac{3930^2}{59 \times 60} (= 9.91525\dots)$ si M1A1
- $s_y^2 = \frac{269900}{59} - \frac{4020^2}{59 \times 60} (= 9.49152\dots)$ si A1
- [Accept division by 60 which gives 9.75 and 9.3333..]
- $SE = \sqrt{\frac{9.91525\dots}{60} + \frac{9.49152}{60}} (= 0.5687 \text{ or } 0.5640)$ M1A1
- The 90% confidence limits for the difference of means are
- $67.0 - 65.5 \pm 1.645 \times 0.5687$ M1A1
- [M mark requires $\sqrt{60}$, FT from earlier values]
- giving [0.6, 2.4] A1
- (b) Yes because the interval does not contain 0.

- 6 (a) $E(U) = \lambda\mu + (1-\lambda)\mu = \mu$ M1A1
- (b) $\text{Var}(U) = \lambda^2 \text{Var}(\bar{X}) + (1-\lambda)^2 \text{Var}(\bar{Y})$ M1
- $$= \lambda^2 \cdot \frac{\sigma_x^2}{m} + (1-\lambda)^2 \cdot \frac{\sigma_y^2}{n}$$
- m1A1
- (c)(i) $\frac{d\text{Var}(U)}{d\lambda} = 2\lambda \cdot \frac{\sigma_x^2}{m} - 2(1-\lambda) \cdot \frac{\sigma_y^2}{n}$ M1A1
- This equals zero when m1
- $$\frac{\lambda}{1-\lambda} = \frac{\sigma_y^2}{n} \cdot \frac{m}{\sigma_x^2}$$
- A1
- whence $\lambda = \frac{m\sigma_y^2}{m\sigma_y^2 + n\sigma_x^2}$ A1
- $$\frac{d^2\text{Var}(U)}{d\lambda^2} = \frac{2\sigma_x^2}{m} + \frac{2\sigma_y^2}{n} > 0 \text{ therefore minimum}$$
- B1
- [Accept other correct solutions, eg U is a quadratic in λ with positive λ^2 term]
- (ii) Min Var = $\frac{\sigma_x^2}{m} \cdot \frac{m^2\sigma_y^4}{(m\sigma_y^2 + n\sigma_x^2)^2} + \frac{\sigma_y^2}{n} \cdot \frac{n^2\sigma_x^4}{(m\sigma_y^2 + n\sigma_x^2)^2}$ M1A1
- $$= \frac{\sigma_x^2\sigma_y^2(m\sigma_y^2 + n\sigma_x^2)}{(m\sigma_y^2 + n\sigma_x^2)^2}$$
- A1
- whence the given result.
- 7 (a) $S_{xy} = 5590.5 - 105 \times 262.6 / 6 = 995$ B1
- $$S_{xx} = 2275 - 105^2 / 6 = 437.5$$
- B1
- $$b = \frac{995}{437.5} = 2.27$$
- M1A1
- $$a = \frac{262.6 - 105 \times 2.27..}{6}$$
- M1
- $$= 3.97$$
- A1
- [No marks for answer only]
- (b) SE of $b = \frac{0.5}{\sqrt{437.5}}$ (= 0.0239) M1A1
- $$z = \frac{2.27.. - 2.34}{0.0239}$$
- M1
- [Award M mark only if $\sqrt{S_{xx}}$ present]
- $$= -2.75$$
- A1
- p-value = 0.00298 A1
- Very strong evidence that $\beta < 2.34$ B1
- (c) It is clear from the data that β is approximately 2 because y increases by approx 10 when x increases by 5. B1



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