C4

1. (*a*) $f(x) \equiv \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$8 - x - x^2 \equiv A(x-2)^2 + Bx(x-2) + Cx$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 2$, $C = 1$, $B = -3$ (2 coefficients, c.a.o.) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = \frac{-2}{x^2} + \frac{3}{(x-2)^2} - \frac{2}{(x-2)^3}$ (at least one of first two terms) B1

(third term) B1

(f.t. candidates values for *A*, *B*, *C*)

$f'(1) = 3$ (c.a.o.) B1

2. $10x + 4x\frac{dy}{dx} + 4y - 3y^2\frac{dy}{dx} = 0$ $\left(4x\frac{dy}{dx} + 4y\right)$ B1

$\left(-3y^2\frac{dy}{dx}\right)$ B1

$\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - (-2) = -4(x - 1)$

$\left(\text{f.t. candidate's value for } \frac{dy}{dx}\right)$ A1

3. (*a*) $2(2\cos^2\theta - 1) = 9\cos\theta + 7$

(correct use of $\cos 2\theta = 2\cos^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = coefficient of $\cos^2\theta$ and $b \times d$ = constant m1

$4\cos^2\theta - 9\cos\theta - 9 = 0 \Rightarrow (4\cos\theta + 3)(\cos\theta - 3) = 0$

$\Rightarrow \cos\theta = -\frac{3}{4}$, ($\cos\theta = 3$) (c.a.o.) A1

$\theta = 138{\cdot}59°, 221{\cdot}41°$ B1 B1

Note: Subtract (from final two marks) 1 mark for each additional root in range from $4\cos\theta + 3 = 0$, ignore roots outside range.

$\cos\theta = -$, f.t. for 2 marks, $\cos\theta = +$, f.t. for 1 mark (*b*) (i) $R = 13$ B1

Correctly expanding $\sin(x - \alpha)$ and using either $13\cos\alpha = 5$ **or** $13\sin\alpha = 12$ **or** $\tan\alpha = \frac{12}{5}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 67{\cdot}38°$ (c.a.o) A1

(ii) Least value of $\dfrac{1}{5\sin x - 12\cos x + 20} = \dfrac{1}{13 \times (\pm1) + 20}$.

(f.t. candidate's value for R) M1

Least value = $\dfrac{1}{33}$ (f.t. candidate's value for R) A1

Corresponding value for $x = 157{\cdot}38°$ (o.e.)

(f.t. candidate's value for α) A1

4.

Volume = $\pi \displaystyle\int_{\pi/6}^{\pi/3} \sin^2 x \, dx$ B1

Use of $\sin^2 x = \dfrac{(\pm1 \pm \cos 2x)}{2}$ M1

Correct integration of candidate's $\dfrac{(\pm1 \pm \cos 2x)}{2}$ A1

Correct substitution of correct limits in candidate's integrated expression M1

Volume = $\dfrac{\pi^2}{12} = 0{\cdot}822(467...)$ (c.a.o.) A1

5. $\left(1 - \dfrac{x}{4}\right)^{1/2} = 1 - \dfrac{x}{8} - \dfrac{x^2}{128}$

$\left(1 - \dfrac{x}{8}\right)$ B1

$\left(- \dfrac{x^2}{128}\right)$ B1

$|x| < 4$ or $-4 < x < 4$ B1

$\dfrac{\sqrt{3}}{2} \approx 1 - \dfrac{1}{8} - \dfrac{1}{128}$ (f.t. candidate's coefficients) B1

$\sqrt{3} \approx \dfrac{111}{64}$. (convincing) B1

6. (*a*) Use of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and at least one of $\frac{dx}{dt} = -\frac{2}{t^2}$, $\frac{dy}{dt} = 4$ correct M1

$\frac{dy}{dx} = -2t^2$ (o.e.) A1

Equation of tangent at P: $y - 4p = -2p^2\left(x - \frac{2}{p}\right)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y = -2p^2x + 8p$ (convincing) A1

(*b*) Substituting $x = 2$, $y = 3$ in equation of tangent M1

$4p^2 - 8p + 3 = 0$ A1

$p = \frac{1}{2}, \frac{3}{2}$ (both values, c.a.o.) A1

Points are (4, 2), ($^4/_3$, 6) (f.t. candidate's values for p) A1

7. (*a*) $\int x^3 \ln x \, dx = f(x) \ln x - \int f(x)\, g(x) \, dx$ M1

$f(x) = \frac{x^4}{4}$, $g(x) = \frac{1}{x}$ A1, A1

$\int x^3 \ln x \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + c$ (c.a.o.) A1

(*b*) $\int x(2x-3)^4 \, dx = \int f(u) \times u^4 \times k \, du$

($f(u) = pu + q$, $p \neq 0$, $q \neq 0$ and $k = {}^1/_2$ or 2) M1

$\int x(2x-3)^4 \, dx = \int \frac{(u+3)}{2} \times u^4 \times \frac{du}{2}$ A1

$\int (au^5 + bu^4) \, du = \frac{au^6}{6} + \frac{bu^5}{5}$ ($a \neq 0$, $b \neq 0$) B1

Either: Correctly inserting limits of –1, 1 in candidate's $\frac{au^6}{6} + \frac{bu^5}{5}$

or: Correctly inserting limits of 1, 2 in candidate's

$\frac{a(2x-3)^6}{6} + \frac{b(2x-3)^5}{5}$ m1

$\int_1^2 x(2x-3)^4 \, dx = \frac{3}{10}$ (c.a.o.) A1

8. (*a*) $\frac{dV}{dt} = -kV^2$ B1

(*b*) $\int \frac{dV}{V^2} = -\int k\,dt$ (o.e.) M1

$-\frac{1}{V} = -kt + c$ A1

$c = -\frac{1}{12000}$ (c.a.o.) A1

$V = \frac{12000}{12000kt + 1} = \frac{12000}{at + 1}$ (convincing) A1

(*c*) Substituting $t = 2$ and $V = 9000$ in expression for V M1

$a = \frac{1}{6}$ A1

Substituting $t = 4$ in expression for V with candidate's value for a M1

$V = 7200$ (c.a.o) A1

9. (*a*) $(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = 18$ B1

$|2\mathbf{i} - 2\mathbf{j} + \mathbf{k}| = \sqrt{9}$, $|\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}| = \sqrt{81}$ (one correct) B1

Correctly substituting in the formula $\mathbf{a.b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos\theta$ M1

$\theta = 48{\cdot}2°$ (c.a.o.) A1

(*b*) (i) $\mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $2 - \lambda = -1 + \mu$

$-2 - 2\lambda = -4 + \mu$

$1 + 7\lambda = -2 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = -1$, $\mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $3\mathbf{i} - 6\mathbf{k}$ (f.t. one slip) A1

10. Assume that positive real numbers a, b exist such that $a + b < 2\sqrt{ab}$.

Squaring both sides we have: $(a + b)^2 < 4ab \Rightarrow a^2 + b^2 + 2ab < 4ab$ B1

$a^2 + b^2 - 2ab < 0 \Rightarrow (a - b)^2 < 0$ B1

This contradicts the fact that a, b are real and thus $a + b \geq 2\sqrt{ab}$ B1