



GCE AS/A level

976/01

MATHEMATICS C4
Pure Mathematics

P.M. FRIDAY, 18 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{8 - x - x^2}{x(x - 2)^2}.$$

- (a) Express $f(x)$ in terms of partial fractions. [4]

- (b) Use your result to part (a) to find the value of $f'(1)$. [3]

2. Find the equation of the normal to the curve

$$5x^2 + 4xy - y^3 = 5$$

at the point $(1, -2)$. [5]

3. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$2 \cos 2\theta = 9 \cos \theta + 7. \quad [5]$$

- (b) (i) Express $5 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

- (ii) Use your results to part (i) to find the least value of

$$\frac{1}{5 \sin x - 12 \cos x + 20}.$$

Write down a value for x for which this least value occurs. [6]

4. The region R is bounded by the curve $y = \sin x$, the x -axis and the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$. Find the volume generated when R is rotated through four right-angles about the x -axis. Give your answer correct to three decimal places. [5]

5. Expand $\left(1 - \frac{x}{4}\right)^{\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2 .

State the range of values of x for which your expansion is valid.

Hence, by writing $x = 1$ in your expansion, show that

$$\sqrt{3} \approx \frac{111}{64}. \quad [5]$$

6. The parametric equations of the curve C are

$$x = \frac{2}{t}, \quad y = 4t.$$

- (a) Show that the tangent to C at the point P with parameter p has equation

$$y = -2p^2x + 8p. \quad [4]$$

- (b) The tangent to C at the point P passes through the point $(2, 3)$. Show that P can be one of two points. Find the coordinates of each of these two points. [4]

7. (a) Find $\int x^3 \ln x \, dx$. [4]

(b) Use the substitution $u = 2x - 3$ to evaluate $\int_1^2 x(2x - 3)^4 \, dx$. [5]

8. The value, £ V , of a car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^2 .

(a) Write down a differential equation satisfied by V . [1]

(b) Given that $V = 12000$ when $t = 0$, show that

$$V = \frac{12000}{at + 1},$$

where a is a constant. [4]

(c) The value of the car at the end of two years is £9000. Find the value of the car at the end of four years. [4]

9. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \\ \mathbf{b} &= \mathbf{i} - 4\mathbf{j} + 8\mathbf{k},\end{aligned}$$

respectively.

(a) Find the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$. [4]

(b) (i) Write down the vector $\mathbf{AB}$.

(ii) Find the vector equation of the line AB . [3]

(c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} - 4\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}).$$

Show that the lines AB and L intersect and find the position vector of the point of intersection. [6]

10. Prove by contradiction the following proposition.

If a, b are positive real numbers, then $a + b \geq 2\sqrt{ab}$.

The first line of the proof is given below.

Assume that positive real numbers a, b exist such that $a + b < 2\sqrt{ab}$. [3]