WJEC
CBAC

GCE AS/A level

981/01

MATHEMATICS M2
Mechanics 2

A.M. FRIDAY, 11 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle P moves in a straight line so that its acceleration $a\,\text{ms}^{-2}$ at time $t\,\text{s}$, is given by

$$a = 3 - 4t.$$

At time $t = 0$, the particle P passes through the point O and its velocity is $-1\,\text{ms}^{-1}$.

(a) Find an expression for the velocity of P at time $t\,\text{s}$. [4]

(b) Find the values of t when P is instantaneously at rest. [2]

(c) Find the distance between the points at which P is instantaneously at rest. [4]

2. At time $t\,\text{s}$, the position vector $\mathbf{r}$ m of a particle P is given by

$$\mathbf{r} = (3t^2 + 1)\mathbf{i} + (13t - 2t^2)\mathbf{j}.$$

(a) Find the speed of P when $t = 2$. [4]

(b) Calculate the value of t when the velocity of P is perpendicular to the vector $2\mathbf{i} - \mathbf{j}$. [3]

(c) Show that the acceleration of P is constant and find its magnitude. [3]

(d) Find the angle between the direction of the acceleration of P and the direction of the velocity of P when $t = 2$. [3]

3. A particle P, of mass 3 kg, is attached to one end A of a light elastic string of natural length 2 m. The other end B of the string is attached to a point on the ceiling. The modulus of elasticity of the string is 294 N.

(a) The particle P is suspended in equilibrium. Calculate the extension of the string AB with A vertically below B. [3]

(b) The particle P is held at a distance of 1·2 m vertically below B and is then released. Determine the speed of P as it passes through the equilibrium position. [8]

4. The engine of a vehicle, of mass 1500 kg, works at a constant rate of 30 kW. The vehicle is moving up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{6}{49}$. The resistance to motion is a constant force of magnitude 600 N.

(a) Determine the acceleration of the vehicle when its speed is $8\,\text{ms}^{-1}$. [6]

(b) Find the maximum speed that can be attained by the vehicle. [4]

5. The point A is at the top of a vertical cliff 39·2 m above sea level. A pebble is projected from point A with speed $V\,\text{ms}^{-1}$ at an angle of 30° above the horizontal. The greatest height reached by the pebble is 4·9 m above A.

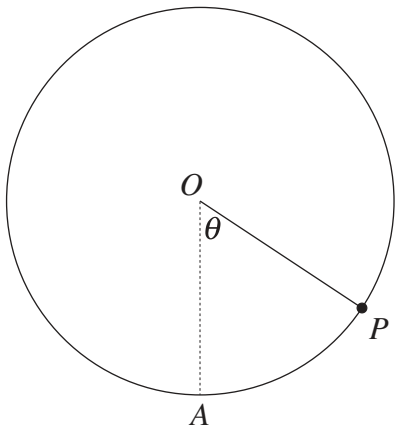
(a) Show that $V = 19{\cdot}6$. [4]

(b) Calculate the time taken for the pebble to reach the surface of the sea. [4]

(c) Find, correct to 3 significant figures, the speed of the pebble 3 s after projection. [5]

6. An athlete is cycling at a constant speed $v\,\text{ms}^{-1}$ in a horizontal circle, of radius 40 m, on a track that is banked at an angle of 30° to the horizontal. The combined mass of the bicycle and the athlete is 60 kg and the coefficient of friction between the bicycle tyres and the track is $\frac{1}{4}$. Find, correct to three significant figures, the greatest possible value of v. [7]

7. The diagram shows a particle P, of mass 3 kg, attached by a light inextensible string of length 2·5 m to a fixed point O. Initially, P is projected from its lowest point A with a horizontal speed of $13\,\text{ms}^{-1}$ so that it starts to move in a vertical circle with centre O.



(a) Find an expression, in terms of θ, for the speed of P when OP makes an angle θ with OA.

Find the speed of P when $\cos\theta = \frac{1}{2}$. [5]

(b) Find an expression, in terms of θ, for the tension in the string when OP makes an angle θ with OA. [4]

(c) Determine whether or not P describes complete circles. [2]