



GCE AS/A level

983/01

MATHEMATICS S1

Statistics

A.M. TUESDAY, 15 June 2010

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The independent events A and B are such that

$$P(A) = 0.6, \quad P(B) = 0.3.$$

Find the value of

- (a) $P(A \cup B)$, [3]
 (b) $P(A \cup B')$. [3]

2. The random variable X has mean 4 and variance 2. The random variable Y is given by

$$Y = 3X - 1.$$

- (a) Find the mean and variance of Y . [4]
 (b) Hence find the value of $E(Y^2)$. [2]

3. The number of customers arriving at a village shop during an interval of length t minutes may be assumed to have a Poisson distribution with mean $0.1t$.

- (a) Find the probability that the number of customers arriving between 10 a.m. and 11 a.m. is
 (i) exactly 3,
 (ii) less than 5. [5]
 (b) Given that the probability of no customers arriving during an interval of t minutes is equal to 0.25, find the value of t correct to two decimal places. [4]

4. Alan and Bill play a game with darts in which they throw a dart at the 'bull' on the dartboard alternately, starting with Alan, and the winner is the first to hit the 'bull'. Each time they throw a dart at the 'bull', Alan hits it with probability 0.2 and Bill hits it with probability 0.3.

Find the probability that

- (a) Bill wins the game with his first throw, [2]
 (b) Bill wins the game with his second throw, [2]
 (c) Bill wins the game. [4]

5. Jack is taking part in a quiz programme. For each question in the quiz, four alternative answers are given, only one of which is correct. Jack has probability 0.6 of knowing the correct answer to a question, and when he does not know the correct answer he chooses one of the four answers at random.

- (a) Calculate the probability that Jack gives the correct answer to a question. [3]
 (b) Given that Jack gave the correct answer to a question, find the probability that he knew the correct answer. [3]

6. The probability distribution of the discrete random variable X is given by

$$\begin{aligned} P(X = x) &= kx & \text{for } x = 1, 3, 5, 7, \\ P(X = x) &= 0 & \text{otherwise.} \end{aligned}$$

(a) Show that $k = \frac{1}{16}$. [2]

(b) Determine

(i) $E(X)$,

(ii) $E\left(\frac{1}{X}\right)$. [5]

(c) Given that X_1, X_2 are two independent values of X , determine

(i) $P(X_1 + X_2 = 6)$,

(ii) $P(X_1 = X_2)$. [7]

7. Sheila buys two biased dice in a shop. Each time either of the dice is thrown, the probability of obtaining a six is 0.2.

(a) She throws one of the dice 50 times. Determine the probability that she obtains

(i) exactly 12 sixes,

(ii) at least 10 sixes. [5]

(b) She now throws the two dice simultaneously 200 times. Use a Poisson approximation to find the probability that between 5 and 10 (both inclusive) double sixes are obtained. [5]

8. The continuous random variable X has probability density function f given by

$$\begin{aligned} f(x) &= kx(1 - x^2) & \text{for } 0 \leq x \leq 1, \\ f(x) &= 0 & \text{otherwise,} \end{aligned}$$

where k is a constant.

(a) Show that $k = 4$. [3]

(b) Calculate $E(X)$. [4]

(c) (i) Find an expression for $F(x)$, valid for $0 \leq x \leq 1$, where F denotes the cumulative distribution function of X .

(ii) Evaluate $P(0.25 \leq X \leq 0.75)$.

(iii) Find the median of X . [9]