GCE MARKING SCHEME

MATHEMATICS
AS/Advanced

SUMMER 2010

Mathematics S2

1.   (a) $z = \dfrac{120-106}{8} = 1.75$ (Accept $\pm$)   M1A1

                Prob $= 0.0401$ A1

    (b)   Distribution of total weight T is N(1060, 640)   M1A1A1

                $z = \dfrac{1000-1060}{\sqrt{640}} = -2.37$ M1A1

                $P(T < 1000) = 0.0089$ A1

                [No FT on incorrect variance]

2.   (a)   Under H_0, mean $= 15$ si B1

                $P(X \leq 9) = 0.0699$ (Accept 0.0778 from Normal app)   B1

                $P(X \geq 22) = 0.0531$ (Accept 0.0465 from Normal app)   B1

                Sig level $= 0.0699 + 0.0531 = 0.123$ M1A1

                [FT one slip but treat Normal apps as incorrect here]

    (b) X is now Po(150) which is approx N(150,150)   B1

                [FT their mean]

                $z = \dfrac{169.5-150}{\sqrt{150}}$ M1A1

                [Award M1A0 for incorrect continuity correction]

                  $= 1.59$ A1

                Prob from tables $= 0.0559$ A1

                p-value $= 0.1118$ B1

                Insufficient evidence to reject H_0 (Accept 'Accept H_0').   B1

                [No c/c gives $z = 1.63$, prob $= 0.0516$ and p-value $= 0.1032$

                Incorrect c/c gives $z = 1.67$. prob $= .0475$ and pv $= 0.095$]

3. (a) $\bar{x} = \dfrac{11.5 + 11.7 + 11.6}{3}$ (= 11.6) B1

SE of $\bar{X} = \dfrac{0.2}{\sqrt{3}}$ (= 0.115...) B1

95% conf limits are
$11.6 \pm 1.96 \times 0.2/\sqrt{3}$ M1A1
[M1 correct form, A1 1.96]
giving [11.4,11.8] cao A1

(b) $H_0 : \mu = 12; H_1 : \mu > 12$ B1

$\bar{y} = \dfrac{12.1 + 12.2 + 12.4 + 12.1}{4}$ (=12.2) B1

Test stat = $\dfrac{12.2 - 12}{\sqrt{0.2^2 / 4}}$ A1

[Award M1 only if there is division by 4 in the denominator]
[FT on slip in calculating $\bar{y}$]
= 2.0 1
p-value = 0.0228 A1
Strong evidence for thinking that μ exceeds 12. B1

(c) SE of $\bar{y} - \bar{x} = \sqrt{\dfrac{0.2^2}{3} + \dfrac{0.2^2}{4}}$ (= 0.152..) M1A1

90% confidence limits are
$12.2 - 11.6 \pm 1.645 \times 0.152\ldots$ m1A1
giving [0.35,0.85] cao A1

4. (a) $E(X) = 3 \Rightarrow np = 3$ B1
Using Var$(X) = E(X^2) - [E(X)]^2$ M1
$np(1 - p) = 11.1 - 9 = 2.1$ A1
Solving,

$1 - p = 0.7$ so $p = 0.3$ m1A1
$n = 10$ A1

(b) $E(Y) = 6$ B1
$E(XY) = 3 \times 6 = 18$ B1
$E(Y^2) = 3.6 + 6^2 = 39.6$ M1A1
$E(X^2 Y^2) = 11.1 \times 39.6 = 439.56$ M1A1
Var$(XY) = 439.56 - 18^2 = 115.56$ m1A1

5. (a) $A = 0.5 \times PQ \cos\theta . PQ \sin\theta = 8\sin\theta\cos\theta = 4\sin 2\theta$ B1

(b) $P(A \le 2) = P(4\sin 2\theta \le 2)$ M1

$\qquad = P(2\theta \le \sin^{-1}[1/2])$ A1

$\qquad = P(\theta \le \pi/12)$ [Accept 15°]   A1

$\qquad = \dfrac{\pi/12}{\pi/4}$ M1

$\qquad = 1/3$ A1

(c) $f(\theta) = \dfrac{4}{\pi}$ [Only award if quoted or used in (c)]   B1

$\text{E}(A) = \displaystyle\int_0^{\pi/4} \dfrac{4}{\pi} \times 4\sin 2\theta\, \mathrm{d}\theta$ M1

$\qquad = \dfrac{8}{\pi}\left[-\cos 2\theta\right]_0^{\pi/4}$ A1

$\qquad = \dfrac{8}{\pi}$ A1

6. (a) $H_0 : p = 0.75$ versus $H_1 : p < 0.75$ B1

(b) (i)   Under H_0, X (No germ) is B(50,0.75)   B1

and Y (No not germ) is B(50,0.25)   (si) B1

Sig level $= P(X < 30)\, |\, H_0)$ M1

$= P(Y > 20\, |\, H_0) = 0.0063$ A1

[Award B1B0M1A0 if Normal approx used]

(ii)   Required prob $= P(X \ge 30\, |p = 0.5)$ M1

$= P(Y \le 20) = 0.1013$ A1

[Award M1A0 if Normal approx used]

(c) X is now B(200,0.75) which is approx N(150,37.5)   B1B1

$z = \dfrac{140.5 - 150}{\sqrt{37.5}}$ M1

$= -1.55$ A1

[Award M1A0 for incorrect continuity correction]
p-value $= 0.0606$

     A1

[No c/c gives p = 0.0516, incorrect c/c gives p = 0.0436]
Insufficient evidence to doubt the statement on the packet   B1