C3

1. (*a*)

1	1·386294361		
1·25	1·517870719		
1·5	1·658228077		
1·75	1·802122256	(5 values correct)	B2
2	1·945910149	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$I \approx \frac{0{\cdot}25}{3} \times \{1{\cdot}386294361 + 1{\cdot}945910149 + 4(1{\cdot}517870719 + 1{\cdot}802122256) + 2(1{\cdot}658228077)\}$

$I \approx 19{\cdot}92863256 \div 12$

$I \approx 1{\cdot}66071938$

$I \approx 1{\cdot}6607$ (f.t. one slip) A1

Note: Answer only with no working earns 0 marks

(*b*) $\displaystyle\int_1^2 \ln\left(\frac{1}{3+x^2}\right) dx \approx -1{\cdot}6607$ (f.t. candidate's answer to (*a*)) B1

2. $2\operatorname{cosec}^2\theta + 3(\operatorname{cosec}^2\theta - 1) + 4\operatorname{cosec}\theta = 9$

(correct use of $\cot^2\theta = \operatorname{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\operatorname{cosec}\theta + b)(c\operatorname{cosec}\theta + d)$, with $a \times c$ = coefficient of $\operatorname{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$5\operatorname{cosec}^2\theta + 4\operatorname{cosec}\theta - 12 = 0 \Rightarrow (5\operatorname{cosec}\theta - 6)(\operatorname{cosec}\theta + 2) = 0$

$\Rightarrow \operatorname{cosec}\theta = \frac{6}{5}, \operatorname{cosec}\theta = -2$

$\Rightarrow \sin\theta = \frac{5}{6}, \sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 56{\cdot}44°, 123{\cdot}56°$ B1

$\theta = 210°, 330°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks

$\sin\theta = +, +$, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(2x^3) = 6x^2$, $\frac{d}{dx}(2x) = 2$, $\frac{d}{dx}(25) = 0$ B1

$\frac{d}{dx}(x^2 \cos y) = x^2(-\sin y)\frac{dy}{dx} + 2x(\cos y)$ B1

$\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{6x^2 + 2x\cos y + 2}{x^2 \sin y - 4y^3}$ (c.a.o.) B1

(*b*) (i) candidate's *x*-derivative = $3t^2$

candidate's *y*-derivative = $4t + 20t^3$

(one term correct B1, all three terms correct B2)

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{4 + 20t^2}{3t}$ (c.a.o.) A1

(ii) $\frac{dy}{dx} = 5 \Rightarrow 20t^2 - 15t + 4 = 0$

(f.t. candidate's expression for $\frac{dy}{dx}$ from (i)) B1

Considering $b^2 - 4ac$ for candidate's quadratic M1

$b^2 - 4ac = 225 - 320 < 0$ and hence no such real value of *t* exists

(f.t. candidate's quadratic) A1

4. (*a*) $f'(x) = (11) \times g(x) - 6x$

where $g(x) =$ **either** $\frac{2}{1 + (2x)^2}$ **or** $\frac{1}{1 + (2x)^2}$ **or** $\frac{2}{1 + 2x^2}$ M1

$f'(x) = 11 \times \frac{2}{1 + 4x^2} - 6x$ A1

$f'(x) = 0 \Rightarrow 12x^3 + 3x - 11 = 0$ (convincing) A1

(*b*) $x_0 = 0{\cdot}9$

$x_1 = 0{\cdot}884366498$ (x_1 correct, at least 5 places after the point) B1

$x_2 = 0{\cdot}886029122$

$x_3 = 0{\cdot}885852598$

$x_4 = 0{\cdot}885871344 = 0{\cdot}88587$ (x_4 correct to 5 decimal places) B1

Let $h(x) = 12x^3 + 3x - 11$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}885865$,

$x = 0{\cdot}885875$ M1

$h(0{\cdot}885865) = -1{\cdot}42 \times 10^{-4} < 0$, $h(0{\cdot}885875) = 1{\cdot}70 \times 10^{-4} > 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}88587$ correct to five decimal places A1

5. (*a*) $\frac{dy}{dx} = \frac{1}{3} \times (9 - 2x)^{-2/3} \times f(x)$ ($f(x) \neq 1$) M1

$\frac{dy}{dx} = -\frac{2}{3} \times (9 - 2x)^{-2/3}$ A1

(*b*) $\frac{dy}{dx} = \frac{f(x)}{\cos x}$ (including $f(x) = 1$) M1

$\frac{dy}{dx} = \frac{\pm \sin x}{\cos x}$ A1

$\frac{dy}{dx} = -\tan x$ (c.a.o.) A1

(*c*) $\frac{dy}{dx} = x^3 \times f(x) + \tan 4x \times g(x)$ M1

$\frac{dy}{dx} = x^3 \times f(x) + \tan 4x \times g(x)$ (either $f(x) = 4\sec^2 4x$ or $g(x) = 3x^2$) A1

$\frac{dy}{dx} = x^3 \times 4\sec^2 4x + \tan 4x \times 3x^2$ (all correct) A1

(*d*) $\frac{dy}{dx} = \frac{(3x+2)^4 \times k \times e^{6x} - e^{6x} \times 4 \times (3x+2)^3 \times m}{[(3x+2)^4]^2}$

with either $k = 6$, $m = 3$ or $k = 6$, $m = 1$ or $k = 1$, $m = 3$ M1

$\frac{dy}{dx} = \frac{(3x+2)^4 \times 6 \times e^{6x} - e^{6x} \times 4 \times (3x+2)^3 \times 3}{[(3x+2)^4]^2}$ A1

$\frac{dy}{dx} = \frac{18x \times e^{6x}}{(3x+2)^5}$ (c.a.o.) A1

6. (*a*) (i) $\int \frac{9}{4x+3}\,dx = k \times 9 \times \ln|4x+3| + c$ $(k = 1, 4, {}^1/_4)$ M1

$\int \frac{9}{4x+3}\,dx = {}^9/_4 \times \ln|4x+3| + c$ A1

(ii) $\int 3e^{5-2x}\,dx = k \times 3 \times e^{5-2x} + c$ $(k = 1, -2, -{}^1/_2)$ M1

$\int 3e^{5-2x}\,dx = -{}^3/_2 \times e^{5-2x} + c$ A1

(iii) $\int \frac{5}{(7x-1)^3}\,dx = \frac{k \times 5 \times (7x-1)^{-2}}{-2} + c$ $(k = 1, 7, {}^1/_7)$ M1

$\int \frac{5}{(7x-1)^3}\,dx = \frac{5 \times (7x-1)^{-2}}{-2 \times 7} + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \cos\left(3x - \frac{\pi}{6}\right)dx = \left[k \times \sin\left(3x - \frac{\pi}{6}\right)\right]$ $(k = 1, 3, \pm {}^1/_3)$ M1

$\int \cos\left(3x - \frac{\pi}{6}\right)dx = \left[{}^1/_3 \times \sin\left(3x - \frac{\pi}{6}\right)\right]$ A1

$\int_0^{\pi/3} \cos\left(3x - \frac{\pi}{6}\right)dx = k \times \left[\sin\left(\frac{5\pi}{6}\right) - \sin\left(-\frac{\pi}{6}\right)\right]$

(A correct method for substitution of limits
f.t. only candidate's value for k, $k = 1, 3, \pm {}^1/_3$) m1

$\int_0^{\pi/3} \cos\left(3x - \frac{\pi}{6}\right)dx = \frac{1}{3}$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

7. (*a*) Choice of a, b, with one positive and one negative and one side correctly evaluated M1

Both sides of identity evaluated correctly A1

(*b*) Trying to solve $2x + 1 = 3x - 4$ M1

Trying to solve $2x + 1 = -(3x - 4)$ M1

$x = 5$, $x = 0{\cdot}6$ (both values) A1

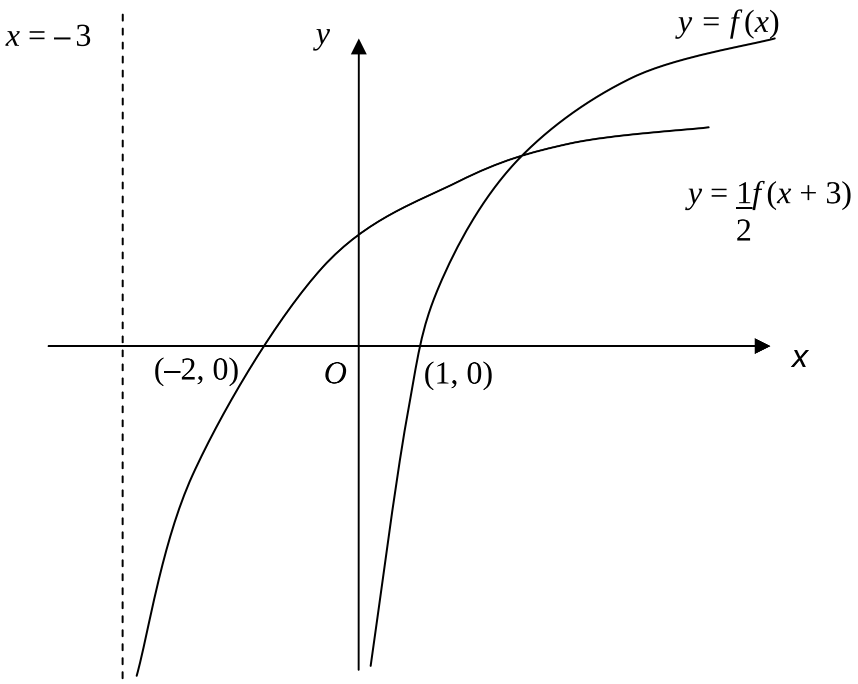
Alternative mark scheme

$(2x + 1)^2 = (3x - 4)^2$ (squaring both sides) M1

$5x^2 - 28x + 15 = 0$ (c.a.o.) A1

$x = 5$, $x = 0{\cdot}6$ (both values, f.t. one slip in quadratic) A1

8.



Correct shape, including the fact that the y-axis is an asymptote for $y = f(x)$ at $-\infty$ B1

$y = f(x)$ cuts x-axis at (1, 0) B1

Correct shape, including the fact that $x = -3$ is an asymptote for $y = \frac{1}{2} f(x + 3)$ at $-\infty$ B1

$y = \frac{1}{2} f(x + 3)$ cuts x-axis at (–2, 0) (f.t. candidate's x-intercept for $f(x)$) B1

The diagram shows that the graph of $y = f(x)$ is steeper than the graph of $y = \frac{1}{2} f(x + 3)$ in the first quadrant B1

9. (*a*) $y + 3 = e^{2x+1}$ B1

An attempt to express equation as a logarithmic equation and to isolate x M1

$x = \frac{1}{2}[\ln(y + 3) - 1]$ (c.a.o.) A1

$f^{-1}(x) = \frac{1}{2}[\ln(x + 3) - 1]$ (f.t. one slip in candidate's expression for x) A1

(*b*) $D(f^{-1}) = (a, b)$ with

$a = -3$ B1

$b = -2$ B1

10. (*a*) $R(f) = (-19, \infty)$ B1

$R(g) = (-\infty, -2)$ B1

(*b*) $D(fg) = (6, \infty)$ B1

$R(fg) = (-15, \infty)$ B1

(*c*) (i) $fg(x) = \left(1 - \frac{1}{2}x\right)^2 - 19$ B1

(ii) Putting expression for $fg(x)$ equal to $2x - 26$ and setting up a quadratic in x of the form $ax^2 + bx + c = 0$ M1

$\frac{1}{4}x^2 - 3x + 8 = 0 \Rightarrow x = 4, 8$ (c.a.o.) A1

Rejecting $x = 4$ and thus $x = 8$ (c.a.o.) A1