



GCE AS/A level

975/01

MATHEMATICS C3

Pure Mathematics

A.M. THURSDAY, 26 May 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_1^2 \ln(3+x^2) \, dx.$$

Show your working and give your answer correct to four decimal places. [4]

- (b) Use your answer to part (a) to deduce an approximate value for the integral

$$\int_1^2 \ln\left(\frac{1}{3+x^2}\right) \, dx. \quad [1]$$

2. Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$2\operatorname{cosec}^2\theta + 3\cot^2\theta + 4\operatorname{cosec}\theta = 9. \quad [6]$$

3. (a) Given that

$$2x^3 + x^2 \cos y + y^4 + 2x - 25 = 0,$$

find an expression for $\frac{dy}{dx}$ in terms of x and y . [4]

- (b) Given that

$$x = t^3, \quad y = 2t^2 + 5t^4,$$

(i) find and simplify an expression for $\frac{dy}{dx}$ in terms of t ,

(ii) show that there is no real value of t for which $\frac{dy}{dx} = 5$. [7]

4. (a) Show that $f(x) = 11\tan^{-1}2x - 3x^2$ has a stationary value when x satisfies

$$12x^3 + 3x - 11 = 0. \quad [3]$$

- (b) **You may assume** that the equation $12x^3 + 3x - 11 = 0$ has a root α between 0 and 1.

The recurrence relation

$$x_{n+1} = \left(\frac{11 - 3x_n}{12} \right)^{\frac{1}{3}}$$

with $x_0 = 0.9$ can be used to find α . Find and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to five decimal places and show this is the value of α correct to five decimal places. [5]

5. Differentiate each of the following with respect to x , simplifying your answer wherever possible.

(a) $(9-2x)^{\frac{1}{3}}$ (b) $\ln(\cos x)$ (c) $x^3 \tan 4x$ (d) $\frac{e^{6x}}{(3x+2)^4}$
 [2], [3], [3], [3]

6. (a) Find

(i) $\int \frac{9}{4x+3} dx$, (ii) $\int 3e^{5-2x} dx$, (iii) $\int \frac{5}{(7x-1)^3} dx$. [6]

(b) Evaluate $\int_0^{\frac{\pi}{3}} \cos\left(3x - \frac{\pi}{6}\right) dx$. [4]

7. (a) Show, by counter-example, that the statement

$$|a+b| \equiv |a| + |b|$$

is false. [2]

- (b) Solve the equation

$$|2x+1| = |3x-4|$$
 [3]

8. Given that $f(x) = \ln x$, sketch, on the same diagram, the graphs of $y = f(x)$ and $y = \frac{1}{2}f(x+3)$.

Label the coordinates of the point of intersection of each of the graphs with the x -axis. Indicate the behaviour of each of the graphs for large positive and negative values of y . [5]

9. The function f has domain $(-\infty, -\frac{1}{2})$ and is defined by

$$f(x) = e^{2x+1} - 3.$$

- (a) Find an expression for $f^{-1}(x)$. [4]

- (b) Write down the domain of f^{-1} . [2]

TURN OVER

10. The functions f and g have domains $(-\infty, 0)$ and $(6, \infty)$ respectively and are defined by

$$f(x) = x^2 - 19,$$

$$g(x) = 1 - \frac{1}{2}x.$$

(a) Write down the range of f and the range of g . [2]

(b) Write down the domain and range of fg . [2]

(c) (i) Write down an expression for $fg(x)$.

(ii) Hence, solve the equation

$$fg(x) = 2x - 26. \quad [4]$$