C4

1. (*a*) $f(x) \equiv \dfrac{A}{(x+2)^2} + \dfrac{B}{(x+2)} + \dfrac{C}{(x-3)}$ (correct form) M1

$x^2 + x + 13 \equiv A(x-3) + B(x+2)(x-3) + C(x+2)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3,\ C = 1,\ B = 0$ (all three coefficients correct) A2

(at least one coefficient correct) A1

(*b*) $\displaystyle\int f(x)\,dx = \frac{3}{(x+2)} + \ln(x-3)$ B1 B1

(f.t. candidates values for *A*, *B*, *C*)

$\displaystyle\int_6^7 f(x)\,dx = \left[\frac{3}{9} - \frac{3}{8}\right] - [\ln 4 - \ln 3] = 0{\cdot}246(015405)$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. $4x^3 - 2x^2\dfrac{dy}{dx} - 4xy + 2y\dfrac{dy}{dx} = 0$

$\left(-2x^2\dfrac{dy}{dx} - 4xy\right)$ B1

$\left(4x^3 + 2y\dfrac{dy}{dx}\right)$ B1

Either $\dfrac{dy}{dx} = \dfrac{4xy - 4x^3}{2y - 2x^2}$ **or** $\dfrac{dy}{dx} = 2$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 3$ in candidate's expression **and** the use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 3 = -\dfrac{1}{2}(x-1)$ $\left(\text{f.t. candidate's value for } \dfrac{dy}{dx}\right)$ A1

3. (*a*) $\frac{2\tan x}{1-\tan^2 x} = 4\tan x$ (correct use of formula for $\tan 2x$) M1

$\tan x = 0$ A1

$2\tan^2 x - 1 = 0$ A1

$x = 0°, 180°$ (both values) A1

$x = 35{\cdot}26°, 144{\cdot}74°$ (both values) A1

(*b*) $R = 25$ B1

Expanding $\cos(\theta - \alpha)$ and using either $25\cos\alpha = 7$ **or** $25\sin\alpha = 24$ **or** $\tan\alpha = \frac{24}{7}$ to find α (f.t. candidate's value for *R*) M1

$\alpha = 73{\cdot}74°$ (c.a.o.) A1

$\cos(\theta - \alpha) = \frac{16}{25} = 0{\cdot}64$ (f.t. candidate's value for *R*) B1

$\theta - \alpha = 50{\cdot}21°, -50{\cdot}21°$

(at least one value, f.t. candidate's value for *R*) B1

$\theta = 23{\cdot}53°, 123{\cdot}95°$ (c.a.o.) B1

4. (*a*) candidate's *x*-derivative = $-3\sin t$

candidate's *y*-derivative = $4\cos t$ (at least one term correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\frac{4\cos t}{3\sin t}$ (o.e.) (c.a.o.) A1

At *P*, $y - 4\sin p = -\frac{4\cos p}{3\sin p}(x - 3\cos p)$ (o.e.)

(f.t. candidate's expression for $\frac{dy}{dx}$) M1

$(3\sin p)y - 12\sin^2 p = (-4\cos p)x + 12\cos^2 p$

$(3\sin p)y = (-4\cos p)x + 12\cos^2 p + 12\sin^2 p$

$(3\sin p)y + (4\cos p)x - 12 = 0$ (convincing) A1

(*b*) (i) $A = (2\sqrt{3}, 0)$ B1

$B = (0, 8)$ B1

(ii) Correct use of Pythagoras Theorem to find *AB* M1

$AB = 2\sqrt{19}$ (convincing) A1

5. (*a*) $A(-3, 0)$, $B(3, 0)$, $C(0, 3)$ B1

(*b*) (i)

$$\text{Volume} = \pi\int_{-3}^{3}(9 - x^2)\,dx$$

(f.t candidate's x-coordinates for A, B) M1

$$\int(9 - x^2)\,dx = 9x - \frac{x^3}{3}$$ B1

Volume = 36π (c.a.o.) A1

Note: Answer only with no working earns 0 marks

(ii) This is the volume of a sphere of radius 3 E1

6. $$(1 + 2x)^{1/2} = 1 + (1/2) \times (2x) + \frac{(1/2) \times (1/2 - 1) \times (2x)^2}{1 \times 2} + \ldots$$

(– 1 each incorrect term) B2

$$\frac{1}{(1 + 3x)^2} = 1 + (-2) \times (3x) + \frac{(-2) \times (-3) \times (3x)^2}{1 \times 2} + \ldots$$

(– 1 each incorrect term) B2

$$4(1 + 2x)^{1/2} - \frac{1}{(1 + 3x)^2} = 3 + 10x - 29x^2 + \ldots$$

(– 1 each incorrect term) B2

Expansion valid for $|x| < 1/3$ B1

7. (*a*) $\int x \sin 2x \, dx = x \times k \times \cos 2x - \int k \times \cos 2x \times g(x) \, dx$ $(k = \pm \frac{1}{2}, \pm 2 \text{ or } \pm 1)$ M1

$k = -\frac{1}{2}, g(x) = 1$ A1, A1

$\int x \sin 2x \, dx = -\frac{1}{2} \times x \times \cos 2x + \frac{1}{4} \times \sin 2x + c$ (c.a.o.) A1

(*b*) $\int \frac{x}{(5 - x^2)^3} \, dx = \int \frac{k}{u^3} \, du$ $(k = \pm \frac{1}{2} \text{ or } \pm 2)$ M1

$\int \frac{a}{u^3} \, du = -\frac{a}{2} u^{-2}$ B1

$\int_0^2 \frac{x}{(5 - x^2)^3} \, dx = -\frac{k}{2}\left[u^{-2}\right]_5^1$ or $-\frac{k}{2}\left[\frac{1}{(5 - x^2)^2}\right]_0^2$

(f.t. candidate's value for k, $k = \pm \frac{1}{2}$ or ± 2) A1

$\int_0^2 \frac{x}{(5 - x^2)^3} \, dx = \frac{6}{25}$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (*a*) $\frac{dN}{dt} = kN$ B1

(*b*) $\int \frac{dN}{N} = \int k \, dt$ M1

$\ln N = kt + c$ A1

$N = e^{kt + c} = Ae^{kt}$ (convincing) A1

(*c*) (i) $100 = Ae^{2k}$

$160 = Ae^{12k}$ (both values) B1

Dividing to eliminate A M1

$1{\cdot}6 = e^{10k}$ A1

$k = \frac{1}{10} \ln 1{\cdot}6 = 0{\cdot}047$ (convincing) A1

(ii) $A = 91({\cdot}0283)$ (o.e.) B1

When $t = 20$, $N = 91({\cdot}0283) \times e^{0{\cdot}94}$

(f.t. candidate's derived value for A) M1

$N = 233$ (c.a.o.) A1

9. (*a*) Use of $(5\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}).(4\mathbf{i} + 6\mathbf{j} + a\mathbf{k}) = 0$ M1

$5 \times 4 + (-8) \times 6 + 4 \times a = 0$ m1

$a = 7$ A1

(*b*) (i) $\mathbf{r} = 8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.) B1

(ii) $8 + 2\lambda = 4 - 2\mu$

$3 + \lambda = 7 + \mu$

$-7 + 2\lambda = 5 + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

$\lambda = 1$, $\mu = -3$ (o.e.) (c.a.o.) A1

Correct verification that values of λ and μ do not satisfy third equation B1

10. Assume that there is a real and positive value of x such that $4x + \frac{9}{x} < 12$

$4x^2 - 12x + 9 < 0$ B1

$(2x - 3)^2 < 0$ B1

This contradicts the fact that x is real and thus $4x + \frac{9}{x} \geq 12$ B1