



GCE AS/A level

981/01

MATHEMATICS M2
Mechanics 2

A.M. MONDAY, 13 June 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9.8 ms^{-2} .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves along the x -axis and its velocity $v \text{ ms}^{-1}$ at time $t \text{ s}$ is given by

$$v = 12 \sin 3t - 8 \cos 2t.$$

- (a) Find an expression for the acceleration of the particle at time $t \text{ s}$. [3]
- (b) Given that at time $t = 0$ the particle is at the origin O , find an expression for the displacement of the particle from O at time $t \text{ s}$. [5]

2. A particle of mass 0.5 kg is attached to one end of a light inextensible string of length 0.6 m . The other end of the string is fixed at a point O on a smooth horizontal surface. The particle moves on the surface in a circle with centre O , so that the string is taut and the angular velocity of the particle about O is $5 \text{ radians per second}$.

- (a) Calculate the speed of the particle. [2]
- (b) Find the tension in the string. [2]

3. A particle P , of mass 2 kg , is moving under the action of a force $\mathbf{F} \text{ N}$ so that its velocity $\mathbf{v} \text{ ms}^{-1}$ at time $t \text{ s}$ is given by

$$\mathbf{v} = 2\mathbf{i} + 6t\mathbf{j} + 4t^3\mathbf{k}.$$

- (a) Find an expression for \mathbf{F} at time $t \text{ s}$. [3]
- (b) Determine the value of $\mathbf{F} \cdot \mathbf{v}$ when $t = 1$ and state the units of your answer. [4]

4. A car of mass 800 kg is travelling against a constant resistance to motion of 540 N .

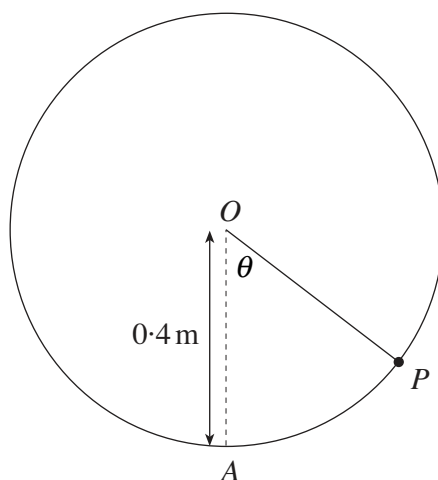
- (a) Find the power of the engine when the car is travelling on a level racing track at a constant speed of 60 ms^{-1} . [4]
- (b) With the engine working at 32.4 kW and the resistance to motion unchanged, the car ascends a hill inclined at an angle α to the horizontal where $\sin \alpha = \frac{1}{16}$.

Find the acceleration of the car when its velocity is 15 ms^{-1} . [5]

5. A light elastic string, of natural length 1.6 m and modulus of elasticity 80 N , has one end attached to a fixed point A and the other end attached to a particle P , of mass 4 kg . Initially, P is held at a point 0.5 m vertically below the point A . The particle P is released from rest and allowed to fall.

- (a) Calculate the tension in the string when the length of the string is 2 m . [2]
- (b) Determine the speed of P when the length of the string is 2 m . [8]

6. A stone is thrown from the top of a vertical cliff, 100 m above sea level. The initial velocity of the stone is 6.5 ms^{-1} at an angle α above the horizontal, where $\tan \alpha = \frac{5}{12}$.
- (a) Find the time taken for the stone to reach the sea. Give your answer correct to two decimal places. [5]
- (b) Calculate the horizontal distance from the bottom of the cliff to the point where the stone hits the sea. [2]
- (c) Calculate the magnitude and direction of the velocity with which the stone hits the sea. [7]
7. At time t , the position vectors relative to a fixed origin O , of two particles A and B are given by $\mathbf{OA} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} + t(2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k})$ and $\mathbf{OB} = 5\mathbf{i} - 8\mathbf{j} + 10\mathbf{k} + t(3\mathbf{i} - 6\mathbf{j} + 7\mathbf{k})$.
- (a) Find the speed of particle A . [3]
- (b) Show that the distance AB at time t is given by $AB^2 = 5t^2 - 30t + 211$. Determine the time at which the particles A and B are closest together. [7]
8. The diagram shows a particle P , of mass 3 kg, attached by a light inextensible string of length 0.4 m to a fixed point O . Initially, P is projected from the point A , which is vertically below O , with a horizontal speed of 4 ms^{-1} .



- (a) The speed of P when OP makes an angle θ with OA is $v \text{ ms}^{-1}$. Show that $v^2 = 8.16 + 7.84 \cos \theta$. [4]
- (b) Find an expression, in terms of θ , for the tension in the string when OP makes an angle θ with OA . [4]
- (c) Determine whether or not P describes complete circles. [3]
- (d) Would your conclusion to (c) be different if the string was replaced by a light rigid rod? Justify your answer. [2]