WJEC
CBAC

GCE MARKING SCHEME

MATHEMATICS

AS/Advanced

SUMMER 2011

S2

1. (a) (i) $z = \dfrac{30-28}{2} = 1.0$ M1A1

Prob = 0.1587 cao A1

[Award full marks for answer only]

(ii) Distribution of $\bar{X}$ is N(28, 4/5) M1A1

[Award M1A0 for N and 1 correct parameter]

$z = \dfrac{30-28}{\sqrt{4/5}} = 2.24$ m1A1

Prob = 0.987 cao A1

[Award m0A0A0 for answer only]

(b) Let A,B denote the times taken by Alan, Brenda.

Then $A - B$ is N(3,13). M1A1

[Award M1A0 for N and 1 correct parameter]

We require P($B > A$) = P($A - B < 0$)

$z = \dfrac{0-3}{\sqrt{13}} = -0.83$ [Accept +0.83] m1A1

Prob = 0.2033 cao A1

[Award m0A0A0 for answer only]

2. (a) $\bar{x} = \dfrac{1290}{60}$ (= 21.5) B1

SE of $\bar{X} = \dfrac{0.5}{\sqrt{60}}$ (= 0.0645…) B1

95% conf limits are

$21.5 \pm 1.96 \times 0.0645$ M1A1

[M1 correct form, A1 1.96]

giving [21.37, 21.63] cao A1

(b) We solve

$3.92 \times \dfrac{0.5}{\sqrt{n}} < 0.1$ M1A1

$n > 384.16$ A1

[Award M1A0A0 for 1.96 in place of 3.92]

Minimum sample size is 385. B1

[Award B1 for rounding up their n]

3. (a) $H_0 : \mu = 0.5;\ H_1 : \mu < 0.5$ B1

(b) Under H_0, mean = 15 B1

p-value = P($X \leq 12 | \mu = 15$) M1

= 0.2676 cao A1

Insufficient evidence to reject H_0. B1

[FT their p-value]

(c) X is now Po(100) which is approx N(100,100) si B1

$$z = \frac{80.5 - 100}{\sqrt{100}}$$ M1A1

[Award M1A0 for incorrect continuity correction]

$= -1.95$ A1

[80 gives $z = -2$, p = 0.02275;79.5 gives $z = -2.05$,p = 0.02018]

p-value = 0.0256 A1

Strong evidence to accept H_1. B1

[FT their p-value]

4. (a) $H_0 : \mu_x = \mu_y; H_1 : \mu_x \neq \mu_y$ B1

(b) $\bar{x} = \frac{114.8}{8} (= 14.35)$ B1

$\bar{y} = \frac{98.0}{7} = (14.0)$ B1

SE $(\bar{X} - \bar{Y}) = \sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{7}}$ (= 0.2587..) M1A1

$$z = \frac{14.35 - 14.0}{0.2587..} = 1.35$$ M1A1

Prob from tables = 0.0885 A1

p-value = 0.177 B1

Insufficient evidence to reject her belief (at the 5% level). B1

[FT their p-value, conclusion must refer to her belief]

5. (a) $f(u) = \frac{1}{b-a},\ a \leq u \leq b$, (= 0 otherwise) B1

$$E(U^2) = \frac{1}{(b-a)} \int_a^b u^2 du$$ (Limits not required here) M1

$$= \frac{1}{(b-a)} \left[\frac{u^3}{3} \right]_a^b$$ A1

$$= \frac{1}{(b-a)} \frac{(b^3 - a^3)}{3}$$ A1

$$= \frac{1}{(b-a)} \frac{(b-a)(a^2 + ab + b^2)}{3}$$ A1

$$= \frac{a^2 + ab + b^2}{3}$$

(b)(i) $E(X) = 3,\ \text{Var}(X) = 3$ B1B1

(ii) $Y = 12 - X$ B1

$E(XY) = E(12X - X^2)$ M1

$= 12 \times 3 - \frac{36}{3}$ A1

[FT their values from (i)]

$= 24$ A1

(iii) Let T denote the total length.

Then T is approx N(300,300). M1A1

[Award M1A0 for N and 1 correct parameter]

$z = \frac{280 - 300}{\sqrt{300}} = -1.15$ m1A1

Prob = 0.8749 A1

[Award m1A0A1 for use of continuity correction giving $z = -1.13$, p = 0.8708 or z = nm – 1.18, p = 0.8810]

6. (a)(i) X is B(20,0.3) si B1

$P(\text{Accept } H_1 \mid H_0 \text{ true}) = P(X \geq 9) \mid p = 0.3)$ M1

$= 0.1133$ A1

(ii) X is B(20,0.6) B1

$P(\text{Accept } H_0 \mid H_1 \text{ true} = P(X \leq 8 \mid p = 0.6)$ M1

The number of tails, T, is B(20,0.4) m1

Required prob = $P(T \geq 12 \mid p = 0.4)$ A1

$= 0.0565$ A1

(b)(i) Y is B(80,0.3) which is approx N(24,16.80) B1

$P(\text{Accept } H_1 \mid H_0 \text{ true}) = P(Y \geq 36 \mid H_0)$ M1

$z = \frac{35.5 - 24}{\sqrt{16.8}} = 2.81$ m1

Required prob = 0.00248 A1

[Award m1A0 for incorrect continuity correction]

(ii) Y is B(80,0.6) which is approx N(48,19.2) B1

$P(\text{Accept } H_0 \mid H_1 \text{ true}) = P(Y \leq 35 \mid H_0)$ M1

$z = \frac{35.5 - 48}{\sqrt{19.2}} = 2.85$ m1

Required prob = 0.00219 A1

[Award m1A0 for incorrect continuity correction]