



# **GCE MARKING SCHEME**

## **MATHEMATICS - C1-C4 & FP1-FP3 AS/Advanced**

**SUMMER 2012**

## **INTRODUCTION**

The marking schemes which follow were those used by WJEC for the Summer 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

# C1

1. (a) Gradient of  $AB = \frac{\text{increase in } y}{\text{increase in } x}$  M1  
 Gradient of  $AB = -\frac{4}{3}$  (or equivalent) A1
- (b) A correct method for finding  $C$  M1  
 $C(-1, 3)$  A1
- (c) Use of  $m_{AB} \times m_L = -1$  to find gradient of  $L$  M1  
 A correct method for finding the equation of  $L$  using candidate's coordinates for  $C$  and candidate's gradient for  $L$ . M1  
 Equation of  $L$ :  $y - 3 = \frac{3}{4}[x - (-1)]$  (or equivalent)  
 (f.t. candidate's coordinates for  $C$  and candidate's gradient for  $AB$ ) A1  
 Equation of  $L$ :  $3x - 4y + 15 = 0$  (convincing, c.a.o.) A1
- (d) (i) Substituting  $x = 7, y = k$  in equation of  $L$  M1  
 $k = 9$  A1  
 (ii) A correct method for finding the length of  $CA(DA)$  M1  
 $CA = 5$  (f.t. candidate's coordinates for  $C$ ) A1  
 $DA = \sqrt{125}$  A1  
 (iii)  $\sin ADC = \frac{CA}{DA} = \frac{5}{\sqrt{125}}$   
 (f.t. candidate's derived values for  $CA$  and  $DA$ ) M1  
 $\sin ADC = \frac{CA}{DA} = \frac{1}{\sqrt{5}}$  (c.a.o.) A1

2. (a)  $\frac{10}{7 + 2\sqrt{11}} = \frac{10(7 - 2\sqrt{11})}{(7 + 2\sqrt{11})(7 - 2\sqrt{11})}$  M1  
Denominator:  $49 - 44$  A1  
 $\frac{10}{7 + 2\sqrt{11}} = \frac{10(7 - 2\sqrt{11})}{5} = 2(7 - 2\sqrt{11}) = 14 - 4\sqrt{11}$  (c.a.o.) A1

**Special case**

If M1 not gained, allow B1 for correctly simplified denominator following multiplication of top and bottom by  $7 + 2\sqrt{11}$

(b)  $(4\sqrt{3})^2 = 48$  B1  
 $\sqrt{8} \times \sqrt{50} = 20$  B1  
 $\frac{5\sqrt{63}}{\sqrt{7}} = 15$  B1  
 $(4\sqrt{3})^2 - (\sqrt{8} \times \sqrt{50}) - \frac{5\sqrt{63}}{\sqrt{7}} = 13$  (c.a.o.) B1

3. (a)  $\frac{dy}{dx} = 4x - 11$  (an attempt to differentiate, at least one non-zero term correct) M1  
An attempt to substitute  $x = 2$  in candidate's expression for  $\frac{dy}{dx}$  m1  
Use of candidate's numerical value for  $\frac{dy}{dx}$  as gradient of tangent at  $P$  m1  
Equation of tangent at  $P$ :  $y - (-1) = -3(x - 2)$  (or equivalent) (c.a.o.) A1

(b) Gradient of tangent at  $Q = 9$  B1  
An attempt to equate candidate's expression for  $\frac{dy}{dx}$  and candidate's derived value for gradient of tangent at  $Q$  M1  
 $4x - 11 = 9 \Rightarrow x = 5$   
(f.t. one error in candidate's expression for  $\frac{dy}{dx}$ ) A1

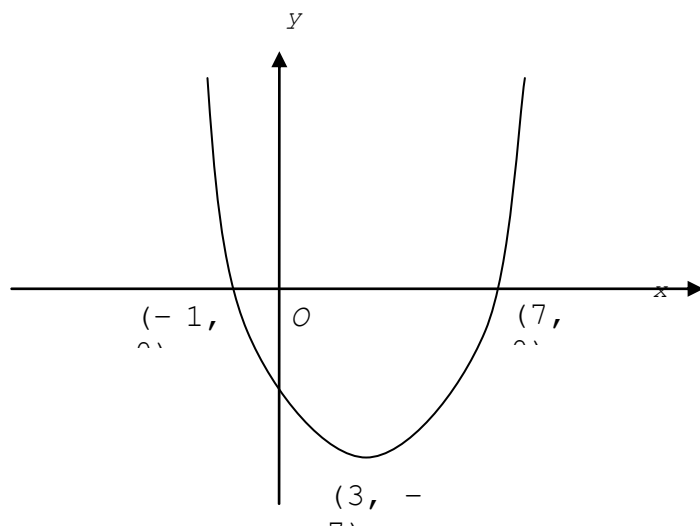
4.  $(1 - 2x)^6 = 1 - 12x + 60x^2 - 160x^3 + \dots$  B1 B1 B1 B1  
(– 1 for further incorrect simplification)

5. (a)  $a = 3$  B1  
 $b = -2$  B1  
 $c = 17$  B1

(b) Stationary value = 17 (f.t. candidate's value for  $c$ ) B1  
This is a minimum B1

6. (a) An expression for  $b^2 - 4ac$ , with at least two of  $a, b, c$  correct M1  
 $b^2 - 4ac = (2k - 1)^2 - 4(k^2 - k + 2)$  A1  
 $b^2 - 4ac = -7$  (c.a.o.) A1  
candidate's value for  $b^2 - 4ac < 0 \Rightarrow$  no real roots) A1
- (b) Finding critical values  $x = -6, x = \frac{2}{3}$  B1  
A statement (mathematical or otherwise) to the effect that  
 $x < -6$  or  $\frac{2}{3} < x$  (or equivalent)  
(f.t. critical values  $\pm 6, \pm \frac{2}{3}$  only) B2  
Deduct 1 mark for each of the following errors  
the use of  $\leq$  rather than  $<$   
the use of the word 'and' instead of the word 'or'
7. (a)  $y + \delta y = 3(x + \delta x)^2 - 7(x + \delta x) + 5$  B1  
Subtracting  $y$  from above to find  $\delta y$  M1  
 $\delta y = 6x\delta x + 3(\delta x)^2 - 7\delta x$  A1  
Dividing by  $\delta x$  and letting  $\delta x \rightarrow 0$  M1  
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 6x - 7$  (c.a.o.) A1
- (b) Required derivative  $= \frac{2}{3} \times \frac{1}{4} \times x^{-3/4} + 12 \times (-3) \times x^{-4}$  B1, B1
8. (a) Attempting to find  $f(r) = 0$  for some value of  $r$  M1  
 $f(2) = 0 \Rightarrow x - 2$  is a factor A1  
 $f(x) = (x - 2)(6x^2 + ax + b)$  with one of  $a, b$  correct M1  
 $f(x) = (x - 2)(6x^2 - 7x - 3)$  A1  
 $f(x) = (x - 2)(3x + 1)(2x - 3)$  (f.t. only  $6x^2 + 7x - 3$  in above line) A1  
 $x = 2, -\frac{1}{3}, \frac{3}{2}$  (f.t. for factors  $3x \pm 1, 2x \pm 3$ ) A1  
**Special case**  
Candidates who, after having found  $x - 2$  as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks
- (b) Use of  $g(a) = 11$  M1  
 $a^3 - 53 = 11 \Rightarrow a = 4$  A1

9. (a)



Concave up curve and y-coordinate of minimum =  $-7$

x-coordinate of minimum =  $3$

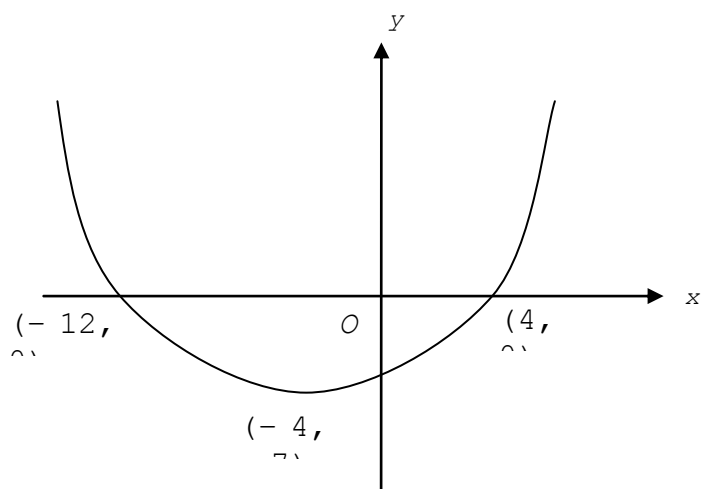
Both points of intersection with x-axis

B1

B1

B1

(b)



Concave up curve and y-coordinate of minimum =  $-7$

x-coordinate of minimum =  $-4$

Both points of intersection with x-axis

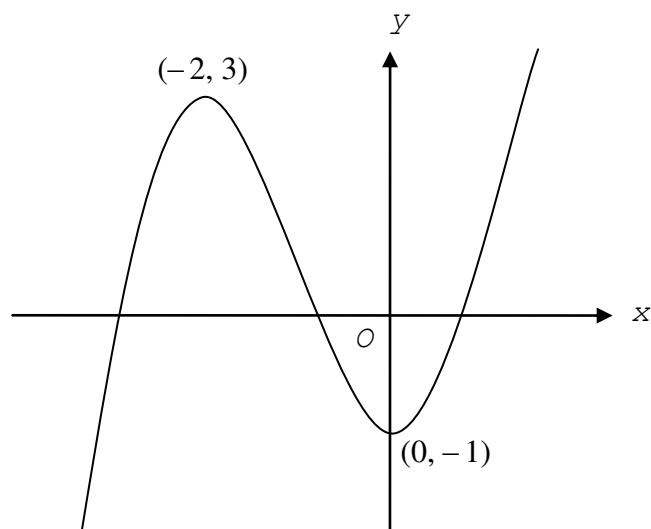
B1

B1

B1

10. (a)  $\frac{dy}{dx} = 3x^2 + 6x$  B1  
 $\frac{dy}{dx}$   
 Putting derived  $\frac{dy}{dx} = 0$  M1  
 $\frac{dy}{dx}$   
 $x = 0, -2$  (both correct) (f.t. candidate's  $\frac{dy}{dx}$ ) A1  
 $\frac{dy}{dx}$   
 Stationary points are  $(0, -1)$  and  $(-2, 3)$  (both correct) (c.a.o) A1  
 A correct method for finding nature of stationary points yielding  
**either**  $(0, -1)$  is a minimum point  
**or**  $(-2, 3)$  is a maximum point (f.t. candidate's derived values) M1  
 Correct conclusion for other point (f.t. candidate's derived values) A1

(b)



- Graph in shape of a positive cubic with two turning points M1  
 Correct marking of both stationary points  
 (f.t. candidate's derived maximum and minimum points) A1
- (c) One positive root (f.t. the number of times the candidate's curve crosses the positive  $x$ -axis) B1

## C2

1.	1	0.5		
	1.25	0.53935989		
	1.5	0.603022689		
	1.75	0.718421208	(5 values correct)	B2
	2	1	(3 or 4 values correct)	B1
	Correct formula with $h = 0.25$			M1
	$I \approx \frac{0.25}{2} \times \{0.5 + 1 + 2(0.53935989 + 0.603022689 + 0.718421208)\}$			
	$I \approx 5.221607574 \div 8$			
	$I \approx 0.652700946$			
	$I \approx 0.6527$			(f.t. one slip) A1

**Special case** for candidates who put  $h = 0.2$

	1	0.5		
	1.2	0.52999894		
	1.4	0.573539334		
	1.6	0.640184399		
	1.8	0.753778361		
	2	1	(all values correct)	B1
	Correct formula with $h = 0.2$			M1
	$I \approx \frac{0.2}{2} \times \{0.5 + 1 + 2(0.52999894 + 0.573539334 + 0.640184399 + 0.753778361)\}$			
	$I \approx 6.495002069 \div 10$			
	$I \approx 0.6495002069$			
	$I \approx 0.6495$			(f.t. one slip) A1

**Note:** Answer only with no working earns 0 marks



2. (a)  $10 \cos^2 \theta + 3 \cos \theta = 4(1 - \cos^2 \theta) - 2$  (correct use of  $\sin^2 \theta = 1 - \cos^2 \theta$ ) M1  
 An attempt to collect terms, form and solve quadratic equation in  $\cos \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \cos \theta + b)(c \cos \theta + d)$ , with  $a \times c =$  candidate's coefficient of  $\cos^2 \theta$  and  $b \times d =$  candidate's constant m1  
 $14 \cos^2 \theta + 3 \cos \theta - 2 = 0 \Rightarrow (2 \cos \theta + 1)(7 \cos \theta - 2) = 0$   
 $\Rightarrow \cos \theta = \frac{2}{7}, \cos \theta = -\frac{1}{2}$  (c.a.o.) A1  
 $\theta = 73.40^\circ, 286.60^\circ$  B1  
 $\theta = 120^\circ, 240^\circ$  B1 B1  
 Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.  
 $\cos \theta = +, -, \text{f.t. for 3 marks, } \cos \theta = -, -, \text{f.t. for 2 marks}$   
 $\cos \theta = +, +, \text{f.t. for 1 mark}$
- (b)  $3x - 21^\circ = -54^\circ, 234^\circ, 306^\circ, 594$  (one value) B1  
 $x = 85^\circ, 109^\circ$  B1 B1  
 Note: Subtract (from final two marks) 1 mark for each additional root in range, ignore roots outside range.
- (c) Use of  $\frac{\sin \phi}{\cos \phi} = \tan \phi$  M1  
 $\tan \phi = 0.2$  A1  
 $\phi = 11.31^\circ, 191.31^\circ$  (f.t.  $\tan \phi = a$ ) B1
3. (a)  $11^2 = 5^2 + x^2 - 2 \times 5 \times x \times \frac{2}{5}$  (correct use of cos rule) M1  
 An attempt to collect terms, form and solve quadratic equation in  $x$ , either by using the quadratic formula or by getting the expression into the form  $(x + b)(x + d)$ , with  $b \times d =$  candidate's constant m1  
 $x^2 - 4x - 96 = 0 \Rightarrow x = 12$  (c.a.o.) A1
- (b)  $\frac{\sin XZY}{32} = \frac{\sin 19^\circ}{15}$   
 (substituting the correct values in the correct places in the sin rule) M1  
 $XZY = 44^\circ, 136^\circ$  (at least one value) A1  
 Use of angle sum of a triangle =  $180^\circ$  M1  
 $YXZ = 117^\circ, 25^\circ$  (both values)  
 (f.t. candidate's values for XZY provided both M's awarded) A1

4. (a)  $S_n = a + [a + d] + \dots + [a + (n - 1)d]$  (at least 3 terms, one at each end) B1  
 $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots + a$   
 Either:  
 $2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \dots + [a + a + (n - 1)d]$   
 Or:  
 $2S_n = [a + a + (n - 1)d] \quad n \text{ times} \quad \text{M1}$   
 $2S_n = n[2a + (n - 1)d]$   
 $S_n = \frac{n}{2}[2a + (n - 1)d] \quad (\text{convincing}) \quad \text{A1}$
- (b)  $a + 2d + a + 3d + a + 9d = 79 \quad \text{B1}$   
 $a + 5d + a + 6d = 61 \quad \text{B1}$   
 An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1  
 $a = 3, d = 5 \text{ (both values)} \quad (\text{c.a.o.}) \quad \text{A1}$
- (c)  $a = 15, d = -2 \quad \text{B1}$   
 $S_n = \frac{n}{2}[2 \times 15 + (n - 1)(-2)] \quad (\text{f.t. candidate's } d) \quad \text{M1}$   
 $S_n = n(16 - n) \quad (\text{c.a.o.}) \quad \text{A1}$
5. (a)  $a + ar = 72 \quad \text{B1}$   
 $a + ar^2 = 120 \quad \text{B1}$   
 An attempt to solve candidate's equations simultaneously by correctly eliminating  $a$  M1  
 $3r^2 - 5r - 2 = 0 \quad (\text{convincing}) \quad \text{A1}$
- (b) An attempt to solve quadratic equation in  $r$ , either by using the quadratic formula or by getting the expression into the form  $(ar + b)(cr + d)$ , with  $a \times c = 3$  and  $b \times d = -2$  M1  
 $(3r + 1)(r - 2) = 0 \Rightarrow r = -\frac{1}{3} \quad \text{A1}$   
 $a \times (1 - \frac{1}{3}) = 72 \Rightarrow a = 108 \quad (\text{f.t. candidate's derived value for } r) \quad \text{B1}$   
 $S_\infty = \frac{108}{1 - (-\frac{1}{3})} \quad (\text{correct use of formula for } S_\infty, \text{ f.t. candidate's derived values for } r \text{ and } a) \quad \text{M1}$   
 $S_\infty = 81 \quad (\text{c.a.o.}) \quad \text{A1}$

6. (a)  $3 \times \frac{x^{3/2}}{3/2} - 2 \times \frac{x^{-2/3}}{-2/3} + c$  B1 B1  
(-1 if no constant term present)
- (b) (i)  $36 - x^2 = 5x$  M1  
An attempt to rewrite and solve quadratic equation  
in  $x$ , either by using the quadratic formula or by getting the  
expression into the form  $(x + a)(x + b)$ , with  $a \times b = -36$  m1  
 $(x - 4)(x + 9) = 0 \Rightarrow A(4, 20)$  (c.a.o.) A1  
 $B(6, 0)$  B1
- (ii) Area of triangle = 40 (f.t. candidate's coordinates for A) B1  
Area under curve =  $\int_4^6 (36 - x^2) dx$  (use of integration) M1  
 $\int 36 dx = 36x$  and  $\int x^2 dx = \frac{x^3}{3}$  B1  
Area under curve =  $[(216 - 216/3) - (144 - 64/3)]$   
(substitution of candidate's limits) m1  
=  $64/3$   
Use of candidate's,  $x_A$ ,  $x_B$  as limits and trying to find total area  
by adding area of triangle and area under curve m1  
Total area =  $40 + 64/3 = 184/3$  (c.a.o.) A1

7. (a) Let  $p = \log_a x$   
 Then  $x = a^p$  (relationship between log and power) B1  
 $x^n = a^{pn}$  (the laws of indices) B1  
 $\therefore \log_a x^n = pn$  (relationship between log and power)  
 $\therefore \log_a x^n = pn = n \log_a x$  (convincing) B1

- (b) **Either:**  
 $(x/2 - 3) \log_{10} 9 = \log_{10} 6$   
 (taking logs on both sides and using the power law) M1  
 $x = \frac{2(\log_{10} 6 + 3 \log_{10} 9)}{\log_{10} 9}$  A1

$x = 7.631$  (f.t. one slip, see below) A1

**Or:**

$x/2 - 3 = \log_9 6$  (rewriting as a log equation) M1

$x = 2(\log_9 6 + 3)$  A1

$x = 7.631$  (f.t. one slip, see below) A1

Note: an answer of  $x = -4.369$  from  $x = \frac{2(\log_{10} 6 - 3 \log_{10} 9)}{\log_{10} 9}$

earns M1 A0 A1

an answer of  $x = 3.815$  from  $x = \frac{\log_{10} 6 + 3 \log_{10} 9}{\log_{10} 9}$

earns M1 A0 A1

an answer of  $x = 1.908$  from  $x = \frac{(\log_{10} 6 + 3 \log_{10} 9)}{2 \log_{10} 9}$

earns M1 A0 A1

an answer of  $x = 4.631$  from  $x = \frac{2 \log_{10} 6 + 3 \log_{10} 9}{\log_{10} 9}$

earns M1 A0 A1

**Note: Answer only with no working earns 0 marks**

- (c)  $\log_a (x - 2) + \log_a (4x + 1) = \log_a [(x - 2)(4x + 1)]$  (addition law) B1  
 $2 \log_a (2x - 3) = \log_a (2x - 3)^2$  (power law) B1  
 $(x - 2)(4x + 1) = (2x - 3)^2$  (removing logs) M1  
 $x = 2.2$  (c.a.o.) A1

**Note: Answer only with no working earns 0 marks**

8. (a)  $A(2, -3)$  B1  
 A correct method for finding the radius M1  
 Radius =  $\sqrt{12}$  A1

- (b)  $AT^2 = 61$  (f.t. candidate's coordinates for A) B1  
 Use of  $RT^2 = AT^2 - AR^2$  M1  
 $RT = 7$  (f.t. candidate's radius and coordinates for A) A1

9. Area of sector  $POQ = \frac{1}{2} \times r^2 \times 1.12$  B1
- Area of triangle  $POQ = \frac{1}{2} \times r^2 \times \sin(1.12)$  B1
- $10.35 = \frac{1}{2} \times r^2 \times 1.12 - \frac{1}{2} \times r^2 \times \sin(1.12)$
- (f.t. candidate's expressions for area of sector and area of triangle) M1
- $r^2 = \frac{2 \times 10.35}{(1.12 - 0.9)}$  (o.e.) (c.a.o.) A1
- $r = 9.7$  (f.t. one numerical slip) A1

### C3

1. (a)
- |  |   |             |                         |    |    |
|--|---|-------------|-------------------------|----|----|
|  | 0   | 1           |                         |    |    |
|  | 0.25  | 1.064494459 |                         |    |    |
|  | 0.5   | 1.284025417 |                         |    |    |
|  | 0.75  | 1.755054657 | (5 values correct)      | B2 |    |
|  | 1   | 2.718281828 | (3 or 4 values correct) | B1 |    |
|  | Correct formula with $h = 0.25$   |             |                         |    | M1 |
|  | $I \approx \frac{0.25}{3} \times \{1 + 2 \cdot 718281828 + 4(1.064494459 + 1.755054657) + 2(1.284025417)\}$ |             |                         |    |    |
|  | $I \approx 17.56452913 \times 0.25 \div 3$  |             |                         |    |    |
|  | $I \approx 1.463710761$   |             |                         |    |    |
|  | $I \approx 1.4637$  |             |                         |    |    |
|  |   |             | (f.t. one slip)         | A1 |    |

**Note: Answer only with no working shown earns 0 marks**

- (b)
- |  |  |                                  |  |  |    |
|--|--|----------------------------------|--|--|----|
|  | $\int_0^1 e^{x^2+3} dx = e^3 \times \int_0^1 e^{x^2} dx$ |                                  |  |  | M1 |
|  | $\int_0^1 e^{x^2+3} dx = 29.399$                         | (f.t. candidate's answer to (a)) |  |  | A1 |

**Note: Answer only with no working shown earns 0 marks**

2. (a)  $\phi = 360^\circ - \theta$  or  $\phi = -\theta$  and noting that  $\cos \theta = \cos \phi$  B1  
 $\sin \theta \neq \sin \phi$  (including correct evaluations) B1
- (b)  $13 \tan^2 \theta = 5(1 + \tan^2 \theta) + 6 \tan \theta$ .  
 (correct use of  $\sec^2 \theta = 1 + \tan^2 \theta$ ) M1  
 An attempt to collect terms, form and solve quadratic equation  
 in  $\tan \theta$ , either by using the quadratic formula or by getting the  
 expression into the form  $(a \tan \theta + b)(c \tan \theta + d)$ ,  
 with  $a \times c =$  candidate's coefficient of  $\tan^2 \theta$  and  
 $b \times d =$  candidate's constant m1  
 $8 \tan^2 \theta - 6 \tan \theta - 5 = 0 \Rightarrow (4 \tan \theta - 5)(2 \tan \theta + 1) = 0$   
 $\Rightarrow \tan \theta = \frac{5}{4}, \tan \theta = -\frac{1}{2}$  (c.a.o.) A1
- $\theta = 51.34^\circ, 231.34^\circ$  B1  
 $\theta = 153.43^\circ, 333.43^\circ$  B1 B1  
 Note: Subtract 1 mark for each additional root in range for each  
 branch, ignore roots outside range.  
 $\tan \theta = +, -, \text{f.t. for 3 marks, } \tan \theta = -, -, \text{f.t. for 2 marks}$   
 $\tan \theta = +, +, \text{f.t. for 1 mark}$
3. (a)  $\frac{d}{dx}(x^3) = 3x^2$   $\frac{d}{dx}(-3x - 2) = -3$  B1  
 $\frac{d}{dx}(-4x^2y) = -4x^2 \frac{dy}{dx} - 8xy$  B1  
 $\frac{d}{dx}(2y^3) = 6y^2 \frac{dy}{dx}$  B1  
 $x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$  (c.a.o.) B1
- (b) (i) Differentiating  $\sin at$  and  $\cos at$  with respect to  $t$ , at least one  
 correct M1  
 candidate's  $x$ -derivative  $= a \cos at$ ,  
 candidate's  $y$ -derivative  $= -a \sin at$  (both values) A1  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = -\tan at$  (c.a.o.) A1
- (ii)  $\frac{d}{dt} \left[ \frac{dy}{dx} \right] = -a \sec^2 at$  (f.t. candidate's expression for  $\frac{dy}{dx}$ ) B1  
 Use of  $\frac{d^2y}{dx^2} = \frac{d}{dt} \left[ \frac{dy}{dx} \right] \div \text{candidate's } x\text{-derivative}$  M1  
 $\frac{d^2y}{dx^2} = -\sec^3 at$  (c.a.o.) A1

4.  $f(x) = \cos x - 5x + 2$   
 An attempt to check values or signs of  $f(x)$  at  $x = 0, x = \pi/4$  M1  
 $f(0) = 3 > 0, f(\pi/4) = -1.22 < 0$   
 Change of sign  $\Rightarrow f(x) = 0$  has root in  $(0, \pi/4)$  A1  
 $x_0 = 0.6$   
 $x_1 = 0.565067123$  B1  
 $x_2 = 0.568910532$   
 $x_3 = 0.568497677$   
 $x_4 = 0.568542145 = 0.56854$  ( $x_4$  correct to 5 decimal places) B1  
 An attempt to check values or signs of  $f(x)$  at  $x = 0.568535, x = 0.568545$  M1  
 $f(0.568535) = 1.563 \times 10^{-5} > 0, f(0.568545) = -3.975 \times 10^{-5} < 0$  A1  
 Change of sign  $\Rightarrow \alpha = 0.56854$  correct to five decimal places A1  
**Note: ‘change of sign’ must appear at least once**

5. (a)  $\frac{dy}{dx} = \frac{a + bx}{7 + 2x - 3x^2}$  (including  $a = 1, b = 0$ ) M1  
 $\frac{dy}{dx} = \frac{2 - 6x}{7 + 2x - 3x^2}$  A1
- (b)  $\frac{dy}{dx} = e^{\tan x} \times f(x)$  ( $f(x) \neq 1, 0$ ) M1  
 $\frac{dy}{dx} = e^{\tan x} \times \sec^2 x$  A1
- (c)  $\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$  ( $f(x), g(x) \neq 1, 0$ ) M1  
 $\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$   
 (either  $f(x) = \frac{1}{\sqrt{1-x^2}}$  or  $g(x) = 10x$ ) A1  
 $\frac{dy}{dx} = 5x^2 \times \frac{1}{\sqrt{1-x^2}} + 10x \times \sin^{-1} x$  A1



6. (a) (i)  $\int 3e^{2-x/4} dx = k \times 3e^{2-x/4} + c \quad (k = 1, -1/4, 4, -4) \quad \text{M1}$   
 $\int 3e^{2-x/4} dx = -4 \times 3e^{2-x/4} + c \quad \text{A1}$
- (ii)  $\int \frac{9}{(2x-3)^6} dx = \frac{k \times 9 \times (2x-3)^{-5}}{-5} + c \quad (k = 1, 2, 1/2) \quad \text{M1}$   
 $\int \frac{9}{(2x-3)^6} dx = \frac{9 \times (2x-3)^{-5}}{-5 \times 2} + c \quad \text{A1}$
- (iii)  $\int \frac{7}{3x+1} dx = k \times 7 \times \ln|3x+1| + c \quad (k = 1, 3, 1/3) \quad \text{M1}$   
 $\int \frac{7}{3x+1} dx = 7/3 \times \ln|3x+1| + c \quad \text{A1}$

**Note:** The omission of the constant of integration is only penalised once.

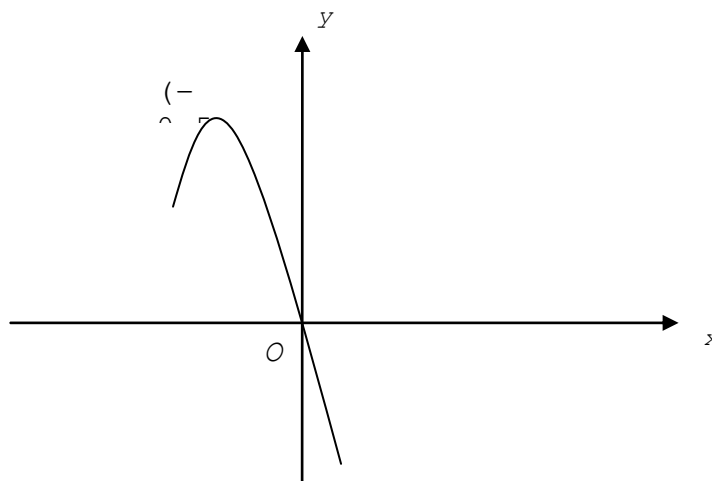
- (b)  $\int \sin 2x dx = k \times \cos 2x \quad (k = -1, -2, 1/2, -1/2) \quad \text{M1}$   
 $\int \sin 2x dx = -\frac{1}{2} \times \cos 2x \quad \text{A1}$   
 $k \times (\cos 2a - \cos 0) = 1/4$   
(f.t. candidate's value for  $k$ )  $\text{M1}$   
 $\cos 2a = 1/2 \quad \text{(c.a.o.)} \quad \text{A1}$   
 $a = \pi/6 \quad \text{(f.t. } \cos 2a = b \text{ provided both M's are awarded)} \quad \text{A1}$

7. (a)  $9|x-3| = 6 \quad \text{B1}$   
 $x-3 = \pm 2/3 \quad \text{(f.t. candidate's } a|x-3| = b, \text{ with at least one of } a, b \text{ correct)} \quad \text{B1}$   
 $x = 11/3, 7/3 \quad \text{(f.t. candidate's } a|x-3| = b, \text{ with at least one of } a, b \text{ correct)} \quad \text{B1}$
- (b) Trying to solve either  $5x-2 \leq 3$  or  $5x-2 \geq -3 \quad \text{M1}$   
 $5x-2 \leq 3 \Rightarrow x \leq 1$   
 $5x-2 \geq -3 \Rightarrow x \geq -1/5 \quad \text{(both inequalities)} \quad \text{A1}$   
Required range:  $-1/5 \leq x \leq 1 \quad \text{(f.t. one slip)} \quad \text{A1}$

**Alternative mark scheme**

- $(5x-2)^2 \leq 9 \quad \text{(forming and trying to solve quadratic)} \quad \text{M1}$   
Critical points  $x = -1/5$  and  $x = 1 \quad \text{A1}$   
Required range:  $-1/5 \leq x \leq 1 \quad \text{(f.t. one slip)} \quad \text{A1}$

8.



Concave down curve passing through the origin with maximum point in the second quadrant

B1

$x$ -coordinate of stationary point =  $-0.5$

B1

$y$ -coordinate of stationary point =  $8$

B1

9. (a) (i)  $f'(x) = \frac{(x^2 + 5) \times f(x) - (x^2 + 3) \times g(x)}{(x^2 + 5)^2}$  ( $f(x), g(x) \neq 1$ ) M1

$$f'(x) = \frac{(x^2 + 5) \times 2x - (x^2 + 3) \times 2x}{(x^2 + 5)^2}$$

A1

$$f'(x) = \frac{4x}{(x^2 + 5)^2}$$

(c.a.o.) A1

$f'(x) < 0$  since numerator is negative and denominator is positive

B1

(ii)  $R(f) = (\sqrt[3]{5}, 1)$  B1 B1

(b) (i)  $x^2 = \frac{3-5y}{y-1}$  (o.e.) (condone any incorrect signs) M1

$$x = (\pm) \sqrt{\frac{3-5y}{y-1}}$$

(f.t. at most one incorrect sign) A1

$$x = - \sqrt{\frac{3-5y}{y-1}}$$

(f.t. at most one incorrect sign) A1

$$f^{-1}(x) = - \sqrt{\frac{3-5x}{x-1}}$$

(c.a.o.) A1

(ii)  $R(f^{-1}) = (-\infty, 0), D(f^{-1}) = (\sqrt[3]{5}, 1),$   
(both intervals, f.t. candidate's  $R(f)$ ) B1

10.  $gg(x) = (3(g(x))^2 + 7)^{1/2}$  or  $gg(x) = g((3x^2 + 7)^{1/2})$  M1  
 $gg(x) = (3(3x^2 + 7) + 7)^{1/2}$  A1  
 An attempt to solve the equation by squaring both sides M1  
 $gg(x) = 8 \Rightarrow 9x^2 = 36$  (o.e.) (c.a.o.) A1  
 $x = \pm 2$  (c.a.o.) A1

## C4

1. (a)  $f(x) \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$  (correct form) M1
- $11 + x - x^2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$   
 (correct clearing of fractions and genuine attempt to find coefficients) m1
- $A = 1, C = 3, B = -2$  (2 correct coefficients) A1  
 (third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(b)  $f'(x) = -\frac{1}{(x+1)^2} + \frac{2}{(x-2)^2} - \frac{6}{(x-2)^3}$  (o.e.)

(f.t. candidate's values for A, B, C)  
 (at least one of the first two terms) B1  
 (third term) B1

$f'(0) = 1/4$  (c.a.o.) B1

2.  $3y^2 \frac{dy}{dx} - 8x - 3x \frac{dy}{dx} - 3y = 0$   $\left[ \begin{array}{l} 3y^2 \frac{dy}{dx} - 8x \\ -3x \frac{dy}{dx} - 3y \end{array} \right]$  B1
- Either**  $\frac{dy}{dx} = \frac{3y+8x}{3y^2-3x}$  **or**  $\frac{dy}{dx} = \frac{1}{3}$  (o.e.) (c.a.o.) B1

Equation of tangent:  $y - (-3) = \frac{1}{3}(x - 2)$   $\left[ \begin{array}{l} \text{f.t. candidate's value for } \frac{dy}{dx} \end{array} \right]$  B1

3. (a)  $4(1 - 2 \sin^2 \theta) = 1 - 2 \sin \theta$  (correct use of  $\cos 2\theta = 1 - 2 \sin^2 \theta$ ) M1  
 An attempt to collect terms, form and solve quadratic equation in  $\sin \theta$ , either by using the quadratic formula or by getting the expression into the form  $(a \sin \theta + b)(c \sin \theta + d)$ , with  $a \times c =$  candidate's coefficient of  $\sin^2 \theta$  and  $b \times d =$  candidate's constant m1  
 $8 \sin^2 \theta - 2 \sin \theta - 3 = 0 \Rightarrow (4 \sin \theta - 3)(2 \sin \theta + 1) = 0$   
 $\Rightarrow \sin \theta = \frac{3}{4}, \sin \theta = -\frac{1}{2}$  (c.a.o.) A1  
 $\theta = 48.59^\circ, 131.41^\circ$  B1  
 $\theta = 210^\circ, 330^\circ$  B1 B1  
 Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.  
 $\sin \theta = +, -, \text{f.t. for 3 marks, } \sin \theta = -, -, \text{f.t. for 2 marks}$   
 $\sin \theta = +, +, \text{f.t. for 1 mark}$
- (b) (i)  $R = 17$  B1  
 Correctly expanding  $\sin(x + \alpha)$  and using either  $17 \cos \alpha = 8$  or  $17 \sin \alpha = 15$  or  $\tan \alpha = \frac{15}{8}$  to find  $\alpha$   
 (f.t. candidate's value for  $R$ ) M1  
 $\alpha = 61.93^\circ$  (c.a.o.) A1  
 (ii)  $\sin(x + \alpha) = \frac{11}{17}$  (f.t. candidate's value for  $R$ ) B1  
 $x + 61.93^\circ = 40.32^\circ, 139.68^\circ, 400.32^\circ,$   
 (at least one value on R.H.S.,  
 f.t. candidate's values for  $\alpha$  and  $R$ ) B1  
 $x = 77.75^\circ, 338.39^\circ$  (c.a.o.) B1  
 (iii) Greatest possible value for  $k$  is 17 since greatest possible value for  $\sin$  is 1 (f.t. candidate's value for  $R$ ) E1

4. Volume =  $\pi \int_3^4 \left( \sqrt{x + \frac{5}{\sqrt{x}}} \right)^2 dx$  B1
- $\left( \sqrt{x + \frac{5}{\sqrt{x}}} \right)^2 = \left( x + 10 + \frac{25}{x} \right)$  B1
- $\int \left( ax + b + \frac{c}{x} \right) dx = \frac{ax^2}{2} + bx + c \ln x$ , where  $c \neq 0$  and at least one of  $a, b \neq 0$  B1
- Correct substitution of correct limits in candidate's integrated expression M1  
 of form  $\frac{ax^2}{2} + bx + c \ln x$ , where  $c \neq 0$  and at least one of  $a, b \neq 0$
- Volume =  $65(0.0059 \dots)$  (c.a.o.) A1

5.  $\left[ \frac{1+x}{3} \right]^{-1/2} = 1 - \frac{x}{6} + \frac{x^2}{24}$   $\left[ \frac{1-x}{6} \right]$  B1  
 $\left[ \frac{x^2}{24} \right]$  B1
- $|x| < 3$  or  $-3 < x < 3$  B1  
 $\left[ \frac{16}{15} \right]^{-1/2} \approx 1 - \frac{1}{30} + \frac{1}{600}$  (f.t. candidate's coefficients) B1  
 $\sqrt{15} \approx \frac{581}{150}$  (c.a.o.) B1
6. (a) candidate's  $x$ -derivative  $= 2t$   
candidate's  $y$ -derivative  $= 2$  (at least one term correct)  
and use of  
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$  M1  
 $\frac{dy}{dx} = \frac{1}{t}$  (o.e.) (c.a.o.) A1  
Use of  $\text{grad}_{\text{normal}} \times \text{grad}_{\text{tangent}} = -1$  m1  
Equation of normal at  $P$ :  $y - 2p = -p(x - p^2)$   
(f.t. candidate's expression for  $\frac{dy}{dx}$ ) m1  
 $y + px = p^3 + 2p$  (convincing) (c.a.o.) A1
- (b) (i) Substituting  $x = 9, y = 6$  in equation of normal M1  
 $p^3 - 7p - 6 = 0$  (convincing) A1  
(ii) A correct method for solving  $p^3 - 7p - 6 = 0$  M1  
 $p = -1$  A1  
 $p = -2$  A1  
 $P$  is either  $(1, -2)$  or  $(4, -4)$  (c.a.o.) A1

7. (a)  $u = x \Rightarrow du = dx$  (o.e.) B1  
 $dv = e^{-2x} dx \Rightarrow v = -\frac{1}{2}e^{-2x}$  (o.e.) B1  
 $\int x e^{-2x} dx = x \times -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x} dx$  M1  
 $\int x e^{-2x} dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + c$  (c.a.o.) A1
- (b)  $\int \frac{1}{x(1+3\ln x)} dx = \int \frac{k}{u} du$  ( $k = 1/3$  or 3) M1  
 $\int \frac{a}{u} du = a \ln u$  B1  
 $\int_1^e \frac{1}{x(1+3\ln x)} dx = k [\ln u]_1^4$  or  $k [\ln(1+3\ln x)]_1^e$  B1  
 $\int_1^e \frac{1}{x(1+3\ln x)} dx = 0.4621$  (c.a.o.) A1
8. (a)  $\frac{dV}{dt} = -kV^3$  (where  $k > 0$ ) B1
- (b)  $\int \frac{dV}{V^3} = - \int k dt$  (o.e.) M1  
 $-\frac{V^{-2}}{2} = -kt + c$  A1  
 $c = -\frac{1}{7200}$  (c.a.o.) A1  
 $V^2 = \frac{3600}{7200kt + 1} = \frac{3600}{at + 1}$  (convincing)  
where  $a = 7200k$  A1
- (c) Substituting  $t = 2$  and  $V = 50$  in expression for  $V^2$  M1  
 $a = 0.22$  A1  
Substituting  $V = 27$  in expression for  $V^2$  with candidate's value for  $a$  M1  
 $t = 17.9$  (c.a.o.) A1

9. (a) An attempt to evaluate  $\mathbf{a} \cdot \mathbf{b}$  M1  
 Correct evaluation of  $\mathbf{a} \cdot \mathbf{b}$  and  $\mathbf{a} \cdot \mathbf{b} \neq 0 \Rightarrow \mathbf{a}$  and  $\mathbf{b}$  not perpendicular A1
- (b) (i)  $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$  B1  
 (ii) Use of  $\mathbf{a} + \lambda\mathbf{AB}$ ,  $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ ,  $\mathbf{b} + \lambda\mathbf{AB}$  or  $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$  to find vector equation of  $AB$  M1  
 $\mathbf{r} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$  (o.e.)  
 (f.t. if candidate uses his/her expression for  $\mathbf{AB}$ ) A1
- (c)  $4 + 2\lambda = 2 - 2\mu$   
 $1 + \lambda = 6 + \mu$   
 $-6 + 2\lambda = p + 3\mu$  (o.e.)  
 (comparing coefficients, at least one equation correct) M1  
 (at least two equations correct) A1  
 Solving the first two of the equations simultaneously m1  
 (f.t. for all 3 marks if candidate uses his/her expression for  $\mathbf{AB}$ )  
 $\lambda = 2, \mu = -3$  (o.e.) (c.a.o.) A1  
 $p = 7$  from third equation  
 (f.t. candidates derived values for  $\lambda$  and  $\mu$ ) A1
10.  $a^2 = 5b^2 \Rightarrow (5k)^2 = 5b^2 \Rightarrow b^2 = 5k^2$  B1  
 $\therefore 5$  is a factor of  $b^2$  and hence 5 is a factor of  $b$  B1  
 $\therefore a$  and  $b$  have a common factor, which is a contradiction to the original assumption B1



**FP1**

Ques	Solution	Mark	Notes
1	$S_n = \sum_{r=1}^n r^3 - \sum_{r=1}^n r$ $= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)}{2}$ $= \frac{n(n+1)}{4} (n^2 + n - 2)$ $= \frac{n(n-1)(n+1)(n+2)}{4}$	<b>M1</b>  <b>A1A1</b>  <b>A1</b>  <b>A1</b>	
2(a)	$(1+2i)^2 = 1+4i+4i^2$ $= -3+4i$ $z = \frac{(-3+4i)(2-i)}{(2+i)(2-i)}$ $= \frac{-6+8i+3i-4i^2}{5}$ $= \frac{-2+11i}{5} \quad (-0.4+2.2i) \quad \text{cao}$	<b>M1</b> <b>A1</b>  <b>M1</b>  <b>A1A1</b>  <b>A1</b>	Award for 3 reasonable terms.   A1 numerator, A1 denominator FT 1 arithmetic slip from line 2
(b)	$r = \sqrt{5} \quad (2.24)$ $\tan^{-1}(-5.5) = -1.39 \quad (-79.6^\circ) \text{ or}$ $\tan^{-1}(5.5) = 1.39 \quad (79.6^\circ)$ $\theta = 1.75 \quad (100.3^\circ)$	<b>B1</b> <b>B1</b>  <b>B1</b>	FT on line above for $r$ . FT on line above for this B1  FT only if in 2 <sup>nd</sup> or 3 <sup>rd</sup> quad
3(a)	$\alpha + \beta = -\frac{1}{2}, \alpha\beta = 1$ $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$ $= \frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}$ $= \frac{(-1/2)^3 - 3 \times 1 \times (-1/2)}{1}$ $= \frac{11}{8}$	<b>B1</b>  <b>M1</b>  <b>M1A1</b>  <b>A1</b>	
(b)	<p>Consider</p> $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = 1$ <p>The required equation is</p> $x^2 - \frac{11}{8}x + 1 = 0 \quad (8x^2 - 11x + 8 = 0) \quad \text{cao}$	<b>M1A1</b>  <b>B1</b>	

<p><b>4(a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(b)</b></p>	<p>Cofactor matrix = <math>\begin{bmatrix} -13 &amp; 9 &amp; 1 \\ -18 &amp; 13 &amp; 1 \\ 14 &amp; -10 &amp; -1 \end{bmatrix}</math></p> <p>Adjugate matrix = <math>\begin{bmatrix} -13 &amp; -18 &amp; 14 \\ 9 &amp; 13 &amp; -10 \\ 1 &amp; 1 &amp; -1 \end{bmatrix}</math></p> <p>Determinant = <math>3(7 - 20) + 4(16 - 7) + 2(5 - 4)</math>  <math>= -1</math></p> <p>Inverse matrix = <math>\begin{bmatrix} 13 &amp; 18 &amp; -14 \\ -9 &amp; -13 &amp; 10 \\ -1 &amp; -1 &amp; 1 \end{bmatrix}</math></p> <p><math>\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 &amp; 18 &amp; -14 \\ -9 &amp; -13 &amp; 10 \\ -1 &amp; -1 &amp; 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}</math></p>	<p><b>M1</b> <b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b> <b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Award M1 if at least 5 cofactors are correct.</p> <p>No FT on cofactor matrix.</p> <p>FT the adjugate</p> <p>FT their inverse matrix.</p>
<p><b>5(a)</b></p> <p><b>(b)</b></p>	<p>By reduction to echelon form,</p> $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -2 & -10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ k-6 \end{bmatrix}$ <p>It follows now that <math>k - 6 = -2</math>  <math>k = 4</math></p> <p>Put <math>z = \alpha</math>.  Then <math>y = 1 - 5\alpha</math>  And <math>x = 7\alpha</math></p>	<p><b>M1</b></p> <p><b>A1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>A1</b></p>	
<p><b>6</b></p>	<p>Putting <math>n = 1</math>, the expression gives 3 which is divisible by 3 so the result is true for <math>n = 1</math>  Assume that the formula is true for <math>n = k</math>.  (<math>k^3 + 2k</math> is divisible by 3 or <math>k^3 + 2k = 3N</math>).  Consider (for <math>n = k + 1</math>)  <math>(k + 1)^3 + 2(k + 1)</math>  <math>= k^3 + 3k^2 + 3k + 1 + 2k + 2</math>  <math>= 3N - 2k + 3k^2 + 3k + 1 + 2k + 2</math>  <math>= 3(N + k^2 + k + 1)</math>  (This is divisible by 3), therefore true for <math>n = k \Rightarrow</math>  true for <math>n = k + 1</math> and since true for <math>n = 1</math>, the result is proved by induction.</p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>M1</b> <b>A1</b> <b>A1</b> <b>A1</b></p> <p><b>A1</b></p>	<p>Award this M1 only if it is clearly stated that this is an assumption</p> <p>Do not award the second M1 if this is stated as an assumption but the three A1s may be awarded if either of the M1s is awarded</p>

<p><b>7(a)</b></p>	<p>Ref matrix in <math>y = x =</math> <math>\begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p>Translation matrix = <math>\begin{bmatrix} 1 &amp; 0 &amp; -2 \\ 0 &amp; 1 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p>Ref matrix in <math>x</math>-axis = <math>\begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; -1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p><math>\mathbf{T} = \begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; -1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 1 &amp; 0 &amp; -2 \\ 0 &amp; 1 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} =</math></p> <p><math>\begin{bmatrix} 1 &amp; 0 &amp; -2 \\ 0 &amp; -1 &amp; -2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 0 &amp; 1 &amp; 0 \\ 1 &amp; 0 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math> or <math>\begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; -1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} 0 &amp; 1 &amp; -2 \\ 1 &amp; 0 &amp; 2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p><math>= \begin{bmatrix} 0 &amp; 1 &amp; -2 \\ -1 &amp; 0 &amp; -2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	
<p><b>(b)</b></p>	<p><math>\begin{bmatrix} 0 &amp; 1 &amp; -2 \\ -1 &amp; 0 &amp; -2 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}</math></p> <p><math>y - 2 = x, -x - 2 = y</math>  <math>x = -2, y = 0</math>    <b>cao</b></p>	<p><b>M1</b></p> <p><b>A1</b> <b>A1A1</b></p>	

<p><b>8(a)</b></p>	<p>Taking logs,  <math>\ln f(x) = x \ln x</math>  Differentiating,  <math>\frac{f'(x)}{f(x)} = \ln x + 1</math>  <math>f'(x) = x^x (\ln x + 1)</math></p> <p><b>(b)</b> At a stationary point, <math>f'(x) = 0</math>  <math>\ln x = -1</math>  <math>x = \frac{1}{e}; y = \left(\frac{1}{e}\right)^{1/e} (0.368, 0.692)</math></p> <p><b>(c)</b> Differentiating the expression in (a),  <math>f''(x) = x^x (\ln x + 1)(\ln x + 1) + x^x \times \frac{1}{x}</math>  <math>= x^{x-1} + x^x (1 + \ln x)^2</math>  <math>f''(1/e) = 1.88</math>  Since this is positive it is a minimum.</p>	<p><b>B1</b></p> <p><b>B1B1</b></p> <p><b>B1</b></p> <p><b>M1</b> <b>A1</b></p> <p><b>A1</b></p> <p><b>B1B1</b></p> <p><b>B1</b> <b>B1</b></p>	<p>B1 for LHS, B1 for RHS</p> <p>B1 each term</p> <p>Accept 'Since the first term is positive and the second term zero, it is a minimum'  FT the final B1 on the line above</p>
<p><b>9(a)</b></p> <p><b>(b)(i)</b></p> <p><b>(ii)</b></p> <p><b>(iii)</b></p>	<p><math>x + iy = \frac{1}{u + iv}</math>  <math>= \frac{u - iv}{u^2 + v^2}</math>  <math>x = \frac{u}{u^2 + v^2}</math>  <math>y = -\frac{v}{u^2 + v^2}</math></p> <p>We are given that  <math>-\frac{v}{u^2 + v^2} = \frac{mu}{u^2 + v^2} + 1</math>  <math>-v = mu + u^2 + v^2</math>  <math>u^2 + v^2 + mu + v = 0</math>  (This is the equation of a circle).  Completing the square or quoting the standard results,  Radius <math>= \frac{1}{2} \sqrt{m^2 + 1}</math>  Centre <math>\left(-\frac{1}{2}m, -\frac{1}{2}\right)</math></p> <p><math>v = -\frac{1}{2}</math></p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b> <b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>FT on their circle equation</p> <p>Accept <math>y = -\frac{1}{2}</math></p>

**FP2**

<b>Ques</b>	<b>Solution</b>	<b>Mark</b>	<b>Notes</b>
<b>1</b>	Putting $x = 2$ , $4a - 8 = 8 - 2b$ The two derivatives are $2ax$ and $3x^2 - b$ Putting $x = 2$ , $4a = 12 - b$ Solving, $a = 2$ , $b = 4$ cao	<b>M1A1</b> <b>M1</b> <b>A1</b>  <b>A1</b>	
<b>2</b>	$u = e^x \Rightarrow du = e^x dx$ , $[0,1] \rightarrow [1, e]$ $I = \int_1^e \frac{du/u}{u + 4/u}$ $= \int_1^e \frac{du}{u^2 + 4}$ $= \frac{1}{2} \left[ \tan^{-1}\left(\frac{u}{2}\right) \right]_1^e$ $= 0.236$	<b>B1</b> <b>B1</b>  <b>M1</b>   <b>A1</b>  <b>A1</b>  <b>A1</b>	
<b>3</b>	Put $t = \tan(x/2)$ $\frac{3 \times 2t}{1+t^2} = t$ $t(t^2 - 5) = 0$ $t = 0$ giving $x/2 = n\pi \rightarrow x = 2n\pi$ ( $360n^\circ$ ) $t = \sqrt{5}$ giving $x/2 = 1.15026.. + n\pi$ $\rightarrow x = 2.30 + 2n\pi$ ( $360n^\circ + 132^\circ$ ) $t = -\sqrt{5}$ giving $x/2 = -1.15026.. + n\pi$ $\rightarrow x = -2.30 + 2n\pi$ ( $360n^\circ - 132^\circ$ )	 <b>M1</b> <b>A1</b>  <b>M1A1</b> <b>M1</b> <b>A1</b> <b>M1</b>  <b>A1</b>	

<p><b>4(a)</b></p> <p><b>(b)</b></p>	<p>Let</p> $\frac{3x^2 - 4x + 1}{(x-2)(x^2+1)} \equiv \frac{A}{x-2} + \frac{Bx+C}{x^2+1}$ $= \frac{A(x^2+1) + (Bx+C)(x-2)}{(x-2)(x^2+1)}$ <p><math>x = 2</math> gives <math>A = 1</math></p> <p>Coeff of <math>x^2</math> gives <math>A + B = 3</math>, <math>B = 2</math></p> <p>Const term gives <math>A - 2C = 1</math>, <math>C = 0</math></p> $\int_3^4 f(x) dx = \int_3^4 \frac{1}{x-2} dx + \int_3^4 \frac{2x}{x^2+1} dx$ $= [\ln(x-2)]_3^4 + [\ln(x^2+1)]_3^4$ $= \ln 2 - \ln 1 + \ln 17 - \ln 10$ $= \ln\left(\frac{34}{10}\right) \text{ or } \ln\left(\frac{17}{5}\right)$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	
<p><b>5(a)</b></p> <p><b>(b)</b></p>	<p>Consider <math>f(-x) = (-x)^2 \sin(-x)</math></p> $= -x^2 \sin x = -f(x)$ <p><math>f</math> is therefore odd.</p> <p><math>\sin x</math> is odd and <math>x^n</math> is even if <math>n</math> is even and odd if <math>n</math> is odd.    si</p> <p>So <math>g</math> is even if <math>n</math> is odd</p> <p>and <math>g</math> is odd when <math>n</math> is even.</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Accept a specific value for <math>x</math>.</p> <p>For a valid attempt.</p> <p>Accept a specific value for <math>x</math>.</p>



<p><b>7(a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(iii)</b></p> <p><b>(b)(i)</b></p> <p><b>(ii)</b></p>	<p>Completing the square,  <math>(y - 1)^2 = 8x - 24</math>  The vertex is therefore (3,1)  In the usual notation, <math>a = 2</math> si  The focus is (5,1)  The equation of the directrix is <math>x = 1</math></p> <p>Substituting <math>y = mx</math>,  <math>m^2x^2 - 2mx - 8x + 25 = 0</math>  For coincident roots,  <math>(2m + 8)^2 = 100m^2</math>  <math>3m^2 - m - 2 = 0</math>  Solving using a valid method,  <math>m = 1, -2/3</math></p>	<p><b>M1</b>  <b>A1</b>  <b>A1</b>  <b>B1</b>  <b>B1</b>  <b>B1</b></p> <p><b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b></p>	<p>FT on 1 arithmetic slip</p>
<p><b>8(a)</b></p> <p><b>(b)(i)</b></p> <p><b>(ii)</b></p>	<p>The result is true for <math>n = 1</math> since it gives  <math>(\cos \theta + i \sin \theta)^1 = \cos 1\theta + i \sin 1\theta</math>  Let the result be true for <math>n = k</math>, ie  <math>(\cos \theta + i \sin \theta)^k = \cos k\theta + i \sin k\theta</math>  Consider  <math>(\cos \theta + i \sin \theta)^{k+1} = (\cos \theta + i \sin \theta)^k (\cos \theta + i \sin \theta)</math>  <math>= (\cos k\theta + i \sin k\theta)(\cos \theta + i \sin \theta)</math>  <math>\cos k\theta \cos \theta - \sin k\theta \sin \theta</math>  <math>+ i(\sin k\theta \cos \theta + \cos k\theta \sin \theta)</math>  <math>= \cos(k + 1)\theta + i \sin(k + 1)\theta</math>  True for <math>n = k \Rightarrow</math> true for <math>n = k + 1</math> and since true  for <math>n = 1</math> the result is proved by induction.</p> <p>Consider  <math>(w(\cos 2\pi/3 + i \sin 2\pi/3))^3</math>  <math>= w^3(\cos 2\pi + i \sin 2\pi)</math>  <math>= z \times 1 = z</math>  Showing that <math>(w(\cos 2\pi/3 + i \sin 2\pi/3))</math> is a cube  root of <math>z</math>.</p> <p>The real cube root of <math>-8</math> is <math>-2</math>.  The other cube roots are  <math>-2(\cos 2\pi/3 + i \sin 2\pi/3) = 1 - \sqrt{3}i</math>  <math>-2(\cos 4\pi/3 + i \sin 4\pi/3) = 1 + \sqrt{3}i</math></p>	<p><b>B1</b>  <b>M1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>  <b>A1</b>  <b>A1</b></p> <p><b>M1</b>  <b>A1</b>  <b>A1</b>  <b>A1</b></p> <p><b>B1</b>  <b>M1A1</b>  <b>A1</b></p>	



**FP3**

Ques	Solution	Mark	Notes
<b>1</b>	$\int_0^1 x \sinh x dx = \left[ x \cosh x \right]_0^1 - \int_0^1 \cosh x dx$ $= \cosh 1 - [\sinh x]_0^1$ $= \cosh 1 - \sinh 1$ $= \frac{e^1 + e^{-1}}{2} - \frac{e^1 - e^{-1}}{2}$ $= \frac{1}{e}$	<p>M1A1</p> <p>A1A1</p> <p>A1</p> <p>A1</p>	<p>Do not accept an argument which evaluates this as 0.367879... and shows that this is also the numerical value of 1/e.</p>
<b>2(a)</b>	<p>The equation can be rewritten as</p> $\sinh^2 x - \sinh x + 1 - k = 0$ <p>The condition for no real roots is</p> $1 - 4(1 - k) = 4k - 3 < 0$ $k < \frac{3}{4}$	<p>M1A1</p> <p>m1</p> <p>A1</p>	
<b>(b)</b>	$\sinh^2 x - \sinh x - 2 = 0$ $(\sinh x - 2)(\sinh x + 1) = 0$ $\sinh x = 2$ $x = \sinh^{-1} 2 = \ln(2 + \sqrt{5})$	<p>M1</p> <p>A1</p> <p>A1</p>	
<b>3</b>	<p>Let <math>f(x) = \tan^{-1} x</math></p> $p = f(1) = \frac{\pi}{4}$ $f'(x) = \frac{1}{1+x^2}; q = f'(1) = \frac{1}{2}$ $f''(x) = -\frac{2x}{(1+x^2)^2}; r = \frac{f''(1)}{2} = -\frac{1}{4}$ $f'''(x) = \frac{-2(1+x^2)^2 + 2(1+x^2).4x^2}{(1+x^2)^4}; s = \frac{f'''(1)}{6} = \frac{1}{12}$	<p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>M1A1</p>	

4(a)	<p>Consider</p> $y = r \sin \theta$ $= 2 \sin \theta \cos \theta - \sin^2 \theta$ $\frac{dy}{d\theta} = 2 \cos^2 \theta - 2 \sin^2 \theta - 2 \sin \theta \cos \theta$ $= 2 \cos 2\theta - \sin 2\theta$ <p>The tangent is parallel to the initial line where</p> $2 \cos 2\theta = \sin 2\theta$ $\tan 2\theta = 2$ $\theta = 0.554, r = 1.18$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	Accept 31.7°
	<p>(b)</p> <p>The curves intersect where</p> $2 \cos \theta - \sin \theta = 1 + \sin \theta$ $2 \cos \theta - 2 \sin \theta = 1$ <p>EITHER</p> <p>Putting <math>t = \tan(\theta/2)</math></p> $\frac{2(1-t^2)}{1+t^2} - \frac{4t}{1+t^2} = 1$ $3t^2 + 4t - 1 = 0$ $\tan(\theta/2) = \frac{-4 + \sqrt{28}}{6} \quad (0.21525..)$ $\theta = 0.424, r = 1.41$ <p>OR</p> <p>Putting</p> $2 \cos \theta - 2 \sin \theta = r \cos(\theta + \alpha)$ $\alpha = \pi/4$ $r = 2\sqrt{2}$ $\cos(\theta + \pi/4) = \frac{1}{2\sqrt{2}}$ $\theta = 0.424, r = 1.41$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p>	

5	<p>Putting <math>t = \tan(x/2)</math> gives <math>dx = \frac{2dt}{1+t^2}</math>  <math>(0, \pi/2) \rightarrow (0,1)</math></p> $I = \int_0^1 \frac{2dt/(1+t^2)}{4(1-t^2)/(1+t^2) + 3}$ $= 2 \int_0^1 \frac{dt}{7-t^2}$ $= 2 \times \frac{1}{2\sqrt{7}} \left[ \ln \left  \frac{\sqrt{7}+t}{\sqrt{7}-t} \right  \right]_0^1 \text{ or } \frac{2}{\sqrt{7}} \left[ \tanh^{-1} \left( \frac{t}{\sqrt{7}} \right) \right]_0^1$ $= \frac{1}{\sqrt{7}} \left( \ln \left( \frac{\sqrt{7}+1}{\sqrt{7}-1} \right) - \ln(1) \right)$ $\text{or } \frac{2}{\sqrt{7}} \left( \tanh^{-1} \left( \frac{1}{\sqrt{7}} \right) - \tanh^{-1}(0) \right)$ $= 0.301$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p>	
<p>6(a)</p> <p>(b)(i)</p> <p>(b)(ii)</p>	$I_n = \left[ \theta^n \sin \theta \right]_0^{\pi/2} - n \int_0^{\pi/2} \theta^{n-1} \sin \theta d\theta$ $= \left( \frac{\pi}{2} \right)^n - n \int_0^{\pi/2} \theta^{n-1} \sin \theta d\theta$ $= \left( \frac{\pi}{2} \right)^n + \left[ n \theta^{n-1} \cos \theta \right]_0^{\pi/2} - n(n-1) I_{n-2}$ $= \left( \frac{\pi}{2} \right)^n - n(n-1) I_{n-2}$ $I_0 = \int_0^{\pi/2} \cos \theta d\theta = \left[ \sin \theta \right]_0^{\pi/2} = 1$ $I_4 = \left( \frac{\pi}{2} \right)^4 - 12 I_2$ $= \left( \frac{\pi}{2} \right)^4 - 12 \left( \left( \frac{\pi}{2} \right)^2 - 2 I_0 \right)$ $= 0.479$ $\int_0^{\pi/2} \theta^5 \sin \theta d\theta = - \left[ \theta^5 \cos \theta \right]_0^{\pi/2} + 5 \int_0^{\pi/2} \theta^4 \cos \theta d\theta$ $= 5 I_4 = 2.4$	<p>M1A1</p> <p>A1</p> <p>M1A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1A1</p> <p>A1</p>	<p>.</p> <p>FT their answer from (b)(i)</p>

<p><b>7(a)</b></p> <p><b>(b)</b></p>	<p>The Newton-Raphson iteration is</p> $x_{n+1} = x_n - \frac{(x_n - 2 \tanh x_n)}{(1 - 2 \operatorname{sech}^2 x_n)}$ $= \frac{x_n - 2x_n \operatorname{sech}^2 x_n - x_n + 2 \tanh x_n}{1 - 2 \operatorname{sech}^2 x_n}$ $= \frac{-2x_n + 2 \sinh x_n \cosh x_n}{\cosh^2 x_n - 2}$ $= \frac{\sinh 2x_n - 2x_n}{\cosh^2 x_n - 2}$ <p><math>x_0 = 2</math>  <math>x_1 = 1.916216399</math>  <math>x_2 = 1.915008327</math></p> <p>Rounding to three decimal places gives 1.915  Let <math>f(x) = x - 2 \tanh x</math>  <math>f(1.9155) = 4.1 \times 10^{-4}</math>  <math>f(1.9145) = -4.2 \times 10^{-4}</math>  The change of sign shows <math>\alpha = 1.915</math> correct 3dp</p>	<p>M1A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>The values are required</p>
<p><b>8(a)</b></p> <p><b>(b)</b></p>	<p>The curve cuts the <math>x</math>-axis where <math>x = \cosh^{-1} 2 = \alpha</math></p> $\frac{dy}{dx} = -\sinh x$ $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 x = \cosh^2 x$ $\text{Arc length} = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ $= \int_{-\alpha}^{\alpha} \cosh x dx$ $= [\sinh x]_{-\alpha}^{\alpha}$ $= 2\sqrt{3} \quad (3.46) \text{ cao}$ <p>Curved surface area <math>= 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx</math></p> $= 2\pi \int_{-\alpha}^{\alpha} (2 - \cosh x) \cosh x dx$ $= 4\pi \int_{-\alpha}^{\alpha} \cosh x dx - \pi \int_{-\alpha}^{\alpha} (\cosh 2x + 1) dx$ $= \pi \left[ 4 \sinh x - \frac{1}{2} \sinh 2x - x \right]_{-\alpha}^{\alpha}$ $= 2\pi (4\sqrt{3} - 2\sqrt{3} - \cosh^{-1} 2)$ $= 13.5$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A2</p> <p>A1</p> <p>A1</p>	<p>Seen or implied</p> <p>Minus 1 each error</p>



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# **GCE MARKING SCHEME**

## **MATHEMATICS - M1-M3 & S1-S3 AS/Advanced**

**SUMMER 2012**

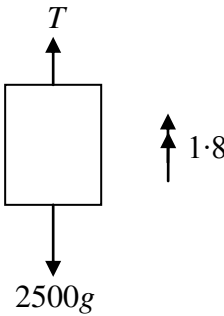
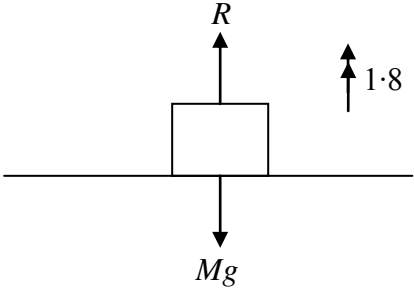
## **INTRODUCTION**

The marking schemes which follow were those used by WJEC for the Summer 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

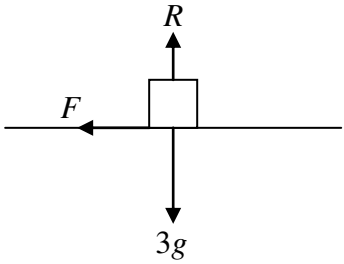
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

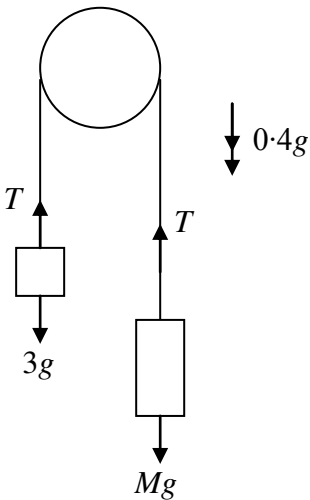
# M1

Q	Solution	Mark	Notes
1(a).	 <p>N2L dim correct equation attempted  <math>T - 2500g = 2500 \times a</math>  <math>T = 2500(9.8 + 1.8)</math>  <math>T = \underline{29000 \text{ (N)}}</math></p>	<p>M1  A1  A1</p>	<p>T, 2500g opposing  Any form correct equ.  cao</p>
1(b)	 <p>N2L attempted  <math>R - Mg = Ma</math>  <math>696 = M(9.8 + 1.8)</math>  <math>M = \underline{60 \text{ (kg)}}</math></p>	<p>M1  A1  A1</p>	<p>R, Mg opposing, no extra forces  Any form correct equ.  cao</p>

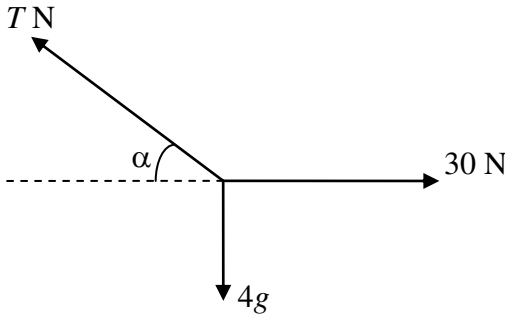


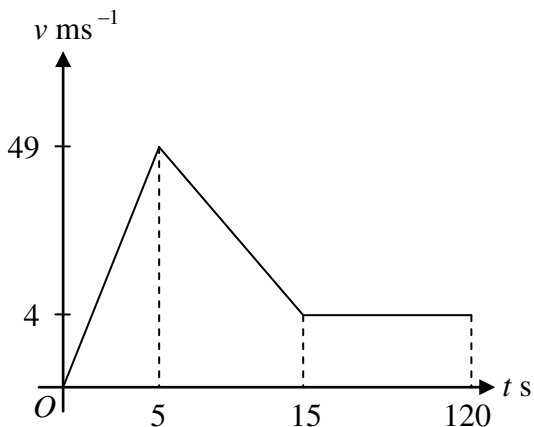
Q	Solution	Mark	Notes
2(a).	 <p>Resolve vertically <math>R = 3g</math></p> <p><math>F = \mu R = \frac{6}{49} \times 3 \times 9.8</math></p> <p><math>F = \underline{3.6 \text{ (N)}}</math></p> <p>N2L <math>F = ma</math></p> <p><math>\pm 3.6 = 3a</math></p> <p><math>a = \underline{-1.2 \text{ (ms}^{-2}\text{)}}</math></p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>May be implied</p> <p>used</p> <p>needs to see -</p>
2(b)	<p>Using <math>v^2 = u^2 + 2as</math> with <math>u=9</math>, <math>v=0</math>, <math>a=(-)1.2</math></p> <p><math>0 = 9^2 + 2 \times (-1.2) s</math></p> <p><math>s = \underline{33.75 \text{ (m)}}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>allow sign errors, oe</p> <p>allow -33.75</p>

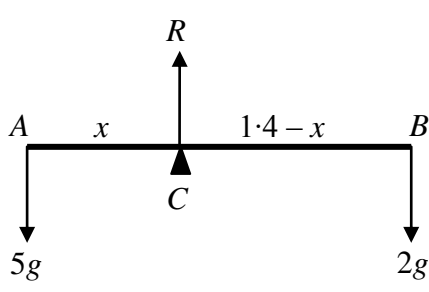


Q	Solution	Mark	Notes
4.	 <p>Apply N2L to B  <math>Mg - T = Ma</math></p> <p>Apply N2L to A  <math>T - 3g = 3a</math></p> <p>Adding</p> $Mg - 3g = 0.4g(M + 3)$ $M - 3 = 0.4M + 1.2$ $0.6M = 4.2$ $M = \underline{7}$ <p><math>T = 3 \times 9.8 + 3 \times 0.4 \times 9.8</math>  <math>T = \underline{41.16 \text{ (N)}}</math></p> <p><u>Alternative solution</u>  Apply N2L to A  <math>T - 3g = 3a</math>  <math>T = 3(9.8 + 0.4 \times 9.8)</math>  <math>T = \underline{41.16 \text{ (N)}}</math></p> <p>Apply N2L to B  <math>Mg - T = Ma</math>  <math>9.8M - 0.4 \times 9.8M = 41.16</math>  <math>5.88M = 41.16</math>  <math>M = \underline{7}</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1 m1 A1</p>	<p>dim correct equation</p> <p>dim correct equation</p> <p>correct method. dep on both M's</p> <p>cao</p> <p>cao</p> <p>dim. correct equation</p> <p>cao</p> <p>dim correct equation</p> <p>cao</p>



Q	Solution	Mark	Notes
6.	 <p>Resolve vertically  <math>T \sin \alpha = 4g</math></p> <p>Resolve horizontally  <math>T \cos \alpha = 30</math></p> <p>Dividing  <math>\tan \alpha = \frac{4 \times 9.8}{30}</math>  <math>\alpha = \underline{52.5(7)^\circ}</math></p> <p><math>T^2 = (4 \times 9.8)^2 + (30)^2</math>  <math>T = \underline{49.36 \text{ (N)}}</math></p>	<p>M1 A1</p> <p>M1 A1</p> <p>m1 A1</p> <p>m1 A1</p>	<p>dep on both M's cao</p> <p>cao</p>

Q	Solution	Mark	Notes
7(a)	Using $v = u + at$ with $u=0$ , $a=(\pm)9.8$ , $t=5$ $v = 0 + 9.8 \times 5$ $v = \underline{49 \text{ (ms}^{-1}\text{)}}$	M1 A1 A1	accept -49
7(b)		B1 B1 B1 B1	units, labels and correct shape starting (0,0) (0, 0) to (5, v) (5, v) to (15, 4) (15, 4) to (120, 4)
7(c)	Distance = Area under graph Distance = $0.5 \times 5 \times 49 + 0.5(4 + 49) \times 10$ + $105 \times 4$  Distance = $122.5 + 265 + 420$ Distance = <u>807.5 (m)</u>	M1 B1 A1	oe any one correct area, ft graph ft graph

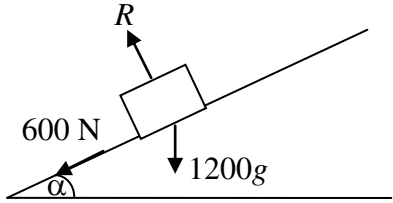
Q	Solution	Mark	Notes
8.			
8(a)	Resolve vertically $R = 5g + 2g$ $R = \underline{7g \text{ (N)}}$	M1  A1	
8(b)	Moments about C $5gx = 2g(1.4 - x)$ $5x = 2.8 - 2x$ $7x = 2.8$ $x = 0.4$ $AC = \underline{0.4 \text{ (m)}}$  <u>Alternative solution</u> Moments about A $7gx = 2g \times 1.4$ $x = \underline{0.4 \text{ (m)}}$	M1 A1 A1  A1  SC1	 dim correct equation, no extra forces rhs correct lhs correct  cao  dim correct equation rhs correct lhs correct cao No marks at all, one correct moment, sc1.

Q	Solution	Mark	Notes																				
9.																							
9(a)	<table border="1"> <thead> <tr> <th></th><th>Area</th><th>from AG</th><th>from AB</th></tr> </thead> <tbody> <tr> <td>(i)</td><td>24</td><td>1</td><td>6</td></tr> <tr> <td>(ii)</td><td>12</td><td>5</td><td>1</td></tr> <tr> <td>(iii)</td><td>18</td><td>5</td><td>4</td></tr> <tr> <td>Lamina</td><td>54</td><td>x</td><td>y</td></tr> </tbody> </table> <p>Moments about AG</p> $54x = 24 \times 1 + 12 \times 5 + 18 \times 5$ $x = \frac{29}{9} = 3.22$ <p>Moments about AB</p> $54y = 24 \times 6 + 12 \times 1 + 18 \times 4$ $y = \frac{38}{9} = 4.22$		Area	from AG	from AB	(i)	24	1	6	(ii)	12	5	1	(iii)	18	5	4	Lamina	54	x	y	<p>B1 correct distances</p> <p>B1 correct distances</p> <p>B1 correct distances</p> <p>B1 areas all correct</p> <p>M1</p> <p>A1 ft table if 2 or more B marks for distances gained.</p> <p>A1 cao</p> <p>M1</p> <p>A1 ft table</p> <p>A1 cao</p>	
	Area	from AG	from AB																				
(i)	24	1	6																				
(ii)	12	5	1																				
(iii)	18	5	4																				
Lamina	54	x	y																				
9(b)	$\theta = \tan^{-1}\left(\frac{x}{12 - y}\right)$ $= \tan^{-1}\left(\frac{29}{12 \times 9 - 38}\right)$ $\theta = \underline{22.5^\circ}$	<p>M1 correct triangle</p> <p>A1 correct equation, ft x, y</p> <p>A1 ft x and y</p>																					

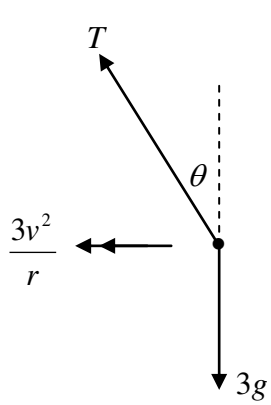


## M2

Q	Solution	Mark	Notes
1.	$s = \int_0^{\frac{\pi}{6}} 4 \cos 2t \, dt$ $s = [2 \sin 2t]$ $s = 2 \sin \frac{\pi}{3} - 0$ $s = \sqrt{3} = \underline{1.732}$	<p>M1</p> <p>A1</p> <p>A1</p>	<p>limits not required</p> <p>correct integration</p> <p>cao</p>
2(a)	<p>N2L <math>T = 7.5g</math></p> <p>Hooke's Law <math>T = \frac{245x}{5/3} (= 147x)</math></p> <p><math>7.5 \times 9.8 = 147x</math></p> <p><math>x = \underline{0.5}</math></p>	<p>B1</p> <p>M1</p> <p>A1</p>	<p></p> <p></p> <p>cao</p>
2(b)	<p>Elastic Energy <math>= \frac{1}{2} \times \frac{x^2}{l}</math></p> <p><math>EE = \frac{1}{2} \times \frac{245 \times 0.5^2}{5/3}</math></p> <p><math>EE = \underline{18.375 \text{ (J)}}</math></p>	<p>M1</p> <p>A1</p>	<p>used</p> <p>ft c's x value</p>
3(a).	<p><math>\underline{v} = \frac{dr}{dt}</math></p> <p><math>\underline{v} = (1 + 4t)\underline{i} + (3t - 2)\underline{j}</math></p> <p>we required <math>\underline{v} \cdot (-\underline{i} + 2\underline{j}) = 0</math></p> <p><math>-(1 + 4t) + 2(3t - 2) = 0</math></p> <p><math>-1 - 4t + 6t - 4 = 0</math></p> <p><math>2t = 5</math></p> <p><math>t = \underline{2.5}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>used</p> <p></p> <p></p> <p></p> <p>cao</p>
3(b)	<p><math>\underline{a} = \frac{dv}{dt}</math></p> <p><math>\underline{a} = 4\underline{i} + 3\underline{j}</math></p> <p><math>\underline{a}</math> is independent of t and constant.</p> <p><math> \underline{a}  = \sqrt{4^2 + 3^2} = \underline{5}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>used</p> <p>ft c's v provided constant</p> <p>ft constant <math>\underline{a} = x\underline{i} + y\underline{j}</math></p>

Q	Solution	Mark	Notes
4.			
4(a)	$T = \frac{P}{v} = \frac{75 \times 1000}{25}$ $T = 3000 \text{ N}$ <p>N2L up plane</p> $T - 1200g \sin \alpha - 600 = 1200a$ $1200a = 3000 - 1200 \times 9.8 \times 0.1 - 600$ $a = \underline{1.02 \text{ (ms}^{-2}\text{)}}$	M1	
4(b)	$T = \frac{90 \times 1000}{v}$ <p>N2L up plane</p> $T - 1200g \sin \alpha - 600 = 1200a$ $a = 0$ $\frac{90000}{v} = 1776$ $v = \underline{50.7 \text{ (ms}^{-1}\text{)}}$	M1 M1 m1 A1	dim correct, all forces A2 -1 each error cao
5.	<p>KE at A = <math>0.5 \times 0.1 \times v^2</math></p> <p>PE at A = <math>0.1 \times 9.8 \times 0.5</math></p> <p>PE at B = <math>0.1 \times 9.8 \times 1.4</math></p> <p>WD against resistance = <math>6 \times 1.2</math></p> <p>Work-energy principle</p> $0.05 v^2 = 7.2 + 0.1 \times 9.8 \times 0.9$ $v^2 = 161.64$ $v = \underline{12.7 \text{ (ms}^{-1}\text{)}}$	B1 M1 A1 B1 M1 A1 A1	both or difference all terms included correct equation cao

Q	Solution	Mark	Notes
6(a).	$u_H = V \cos \alpha (= 0.8V)$ $u_V = V \sin \alpha (= 0.6V)$	M1 A1	attempt to resolve both answers correct
6(b)	Consider horizontal motion $0.8V \times T = 12$ $VT = 15$	M1 A1	correctly obtained
6(c)	Consider vertical motion $s = ut + 0.5at^2$ with $s=(\pm)5.4$ , $u=0.6V$ , $t=T$ $a=(\pm)9.8$ $-5.4 = 0.6VT - 4.9T^2$ $-5.4 = 0.6 \times 15 - 4.9T^2$ $4.9T^2 = 14.4$ $T = \frac{12}{7}$ $\frac{12}{7}V = 15$ $V = \underline{8.75}$	M1 A1  A1  A1	   cao  cao
6(d)	Using $v = u + at$ with $u=5.25$ , $a=(\pm)9.8$ , $t = \frac{12}{7}$ $v = 5.25 - 9.8 \times \frac{12}{7}$ $v = -11.55$ $u_H = 0.8 \times 8.75 = 7$ Speed = $\sqrt{11.55^2 + 7^2}$ Speed = <u>13.5 (ms<sup>-1</sup>)</u>	M1  A1 B1 M1 A1	  si, cao

Q	Solution	Mark	Notes
7.			
7(a)	Resolve vertically $T \cos \theta = mg$ $\theta = \cos^{-1} \left( \frac{3 \times 9 \cdot 8}{88 \cdot 2} \right)$ $\theta = \underline{70.5^\circ}$	M1 A1  A1	   cao
7(b)	N2L towards centre $T \sin \theta = ma$ $a = r\omega^2$ $r = \frac{T \sin \theta}{m\omega^2}$ length of string = $l$ $l \sin \theta = r$ $l = \frac{r}{\sin \theta}$ $l = \frac{T}{m\omega^2} = \frac{88 \cdot 2}{3 \times 2 \cdot 8^2}$ $l = \underline{3.75 \text{ (m)}}$  <u>Alternative Solution</u> N2l towards centre $T \sin \theta = ma$ $a = r\omega^2$ $88.2 \sin \theta = 3 \times r \times 2.8^2$ $r = 3.53553 \text{ m}$ $AP = \frac{r}{\sin \theta}$ $AP = \underline{3.75 \text{ (m)}}$	M1 A1 m1   m1   A1   M1 A1 m1  m1 A1	   attempted  used     cao   attempted used   cao

Q	Solution	Mark	Notes
8(a)	$\underline{v} = \frac{1}{3} [(14\underline{i} - 5\underline{j}) - (8\underline{i} + 7\underline{j})]$ $\underline{v} = \frac{1}{3} (6\underline{i} - 12\underline{j})$ $\underline{v} = (2\underline{i} - 4\underline{j})$	M1  A1	
8(b)	$\underline{r}_S = (8\underline{i} + 7\underline{j}) + (2\underline{i} - 4\underline{j})t$ $\underline{r}_S = (8 + 2t)\underline{i} + (7 - 4t)\underline{j}$	M1 A1	
8(c)	$\underline{r}_B = (x\underline{i} + y\underline{j})(t - 10)$ $\underline{r}_B = x(t - 10)\underline{i} + y(t - 10)\underline{j}$	M1 A1	
	At t = 50 $\underline{r}_S = \underline{r}_B$	M1	
	$8 + 2t = x(t - 10)$	m1	
	$40x = 108$		
	$x = \underline{2.7}$	A1	cao
	$7 - 4 \times 50 = 40y$		
	$y = \underline{-4.825}$	A1	cao
	<u>Alternative solution</u>		
	At t = 50		
	$\underline{r}_S = 108\underline{i} - 193\underline{j}$		
	$\underline{r}_B = 40x\underline{i} + 40y\underline{j}$	M1 A1	
	$\underline{r}_S = \underline{r}_B$	M1	si
	$40x = 108$	m1	
	$x = \underline{2.7}$	A1	cao
	$40y = -193$		
	$y = \underline{-4.825}$	A1	cao



### M3

Q	Solution	Mark	Notes
1(a)	$N2L \frac{27000}{(t+3)^2} = 600a$ $\frac{45}{(t+3)^2} = \frac{dv}{dt}$ $v = -\frac{45}{(t+3)} (+ C)$ <p>When <math>t = 0, v = 0</math>  <math>C = 15</math></p> $v = 15 - \frac{45}{(t+3)}$ <p>As <math>t \rightarrow \infty, v \rightarrow 15</math></p>	<p>M1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>+/-, no additional terms</p> <p>use of dv/dt</p> <p>k/(t+3)</p> <p>completely correct</p> <p>use of initial conditions</p> <p>ft similar expression</p>
1(b)	$v = \frac{dx}{dt} = 15 - \frac{45}{(t+3)}$ $x = 15t - 45 \ln(t+3) (+ C)$ <p><math>t = 0, x = 0 \quad C = 45 \ln 3</math></p> $x = 15t + 45 \ln\left(\frac{3}{t+3}\right)$ <p>When <math>t = 6 \quad x = 90 + 45 \ln\left(\frac{3}{9}\right)</math></p> <p><math>x = 90 - 45 \ln(3)</math>  <math>x = \underline{40.56 \text{ (m)}}</math></p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>ft similar expressions</p> <p>ft</p> <p>cao</p>

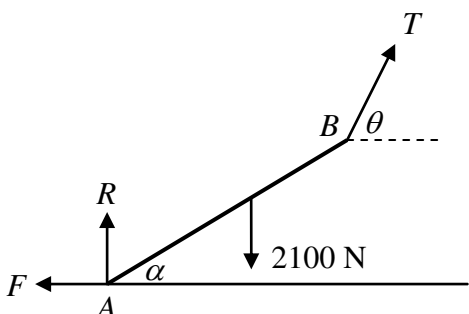
Q	Solution	Mark	Notes
2(a).	Using $v^2 = \omega^2(a^2 - x^2)$ $0.09 \times 3 = \omega^2(a^2 - 0.6^2)$ $0.04 \times 5 = \omega^2(a^2 - 0.8^2)$ $0.07 = 0.28\omega^2$ $\omega = 0.5$ $0.2 = 0.25(a^2 - 0.64)$ $a = 1.2$  Period = $\frac{2\pi}{\omega}$ Period = $\frac{4\pi}{1}$	M1 A1 A1 m1   A1  M1 A1	used      used
2(b)	$\ddot{x} = -\omega^2 x$ $ \ddot{x}  = 0.5^2 \times 0.6$ $ \ddot{x}  = \underline{0.15 \text{ (ms}^{-2}\text{)}}$	M1  A1	used
2(c)	$x = 1.2\sin(0.5t)$ At A, $0.6 = 1.2\sin(0.5t)$ $t = 2\sin^{-1}(0.5) = 1.0472$ At B, $0.8 = 1.2\sin(0.5t)$ $t = 2\sin^{-1}(0.667) = 1.4595$ Required t = $1.4595 - 1.0472$ Required t = <u>0.412 (s)</u>	M1 A1 A1 A1	used, accept cos or 2.0944 or 1.6821 cao
2(d)	$x = a\sin(\omega t)$ $x = 1.2\sin(0.5t)$ $x = 1.2\sin(0.5 \times 2\pi/3)$ $x = \underline{1.0392 \text{ (m)}}$	M1  A1	
2(e)	$v = a\omega\cos(\omega t)$ $v = 1.2 \times 0.5\cos(0.5t)$ $v = 0.6\cos(0.5t)$ When $t = 2\pi/3$ , $v = 0.6\cos(0.5 \times 2\pi/3)$ $v = 0.6\cos(\pi/3)$ $v = \underline{0.3 \text{ (ms}^{-1}\text{)}}$	M1 A1  A1	oe   cao



Q	Solution	Mark	Notes
3.	<p>Auxiliary equation  <math>2m^2 + 5m + 2 = 0</math>  <math>(2m + 1)(m + 2) = 0</math>  <math>m = -0.5, -2</math>  CF is <math>x = Ae^{-0.5t} + Be^{-2t}</math></p> <p>For PI, try <math>x = at + b</math>  <math>\frac{dx}{dt} = a</math>  <math>5a + 2(at + b) = 6t + 5</math>  Comparing coefficients  <math>2a = 6</math>  <math>a = 3</math>  <math>15 + 2b = 5</math>  <math>b = -5</math></p> <p>General solution is  <math>x = Ae^{-0.5t} + Be^{-2t} + 3t - 5</math></p> <p>When <math>t = 0, x = 3</math>  <math>3 = A + B - 5</math>  <math>A + B = 8</math></p> <p><math>\frac{dx}{dt} = -0.5Ae^{-0.5t} - 2Be^{-2t} + 3</math></p> <p>When <math>t = 0, \frac{dx}{dt} = 2</math>  <math>2 = -0.5A - 2B + 3</math>  <math>0.5A + 2B = 1</math>  <math>A + 4B = 2</math>  <math>A + B = 8</math>  <math>3B = -6</math>  <math>B = \underline{-2}</math>  <math>A = \underline{10}</math></p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>A1</p>	<p>cao</p> <p>cao</p> <p>ft solutions for m</p> <p>both answers cao</p> <p>ft CF and PI</p> <p>use of conditions in GS</p> <p>ft similar expressions</p> <p>cao</p> <p>cao</p>

Q	Solution	Mark	Notes
4(a)	<p>N2L <math>F = ma</math></p> $\frac{4}{2x+1} = 0.5v \frac{dv}{dx}$ $\int \frac{8}{2x+1} dx = \int v dv$ $4 \ln 2x+1  = \frac{1}{2} v^2 + C$ $v^2 = 8 \ln 2x+1  + C$ <p>When <math>x = 3, v = 4</math></p> $16 = 8 \ln 7 + C$ $C = 16 - 8 \ln 7$ $v^2 = 8 \ln \left  \frac{2x+1}{7} \right  + 16$ <p>When <math>x = 10</math> <math>v^2 = 8 \ln \left  \frac{2 \times 10 + 1}{7} \right  + 16</math></p> $v^2 = 8 \ln 3 + 16$ $v = \underline{4.98 \text{ (ms}^{-1}\text{)}}$	<p>M1</p> <p>m1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>used, no extra term</p> <p>use of vdv/dx</p> <p>separating variables</p> <p>kln(2x+1)</p> <p>all correct</p> <p>ft kln(2x+1) + C</p> <p>cao</p>
4(b)	<p><math>v = 6, 6^2 = 8 \ln \left  \frac{2x+1}{7} \right  + 16</math></p> $\ln \left  \frac{2x+1}{7} \right  = \frac{20}{8}$ $2x+1 = 7e^{5/2}$ $x = 0.5[7e^{5/2} - 1]$ $x = \underline{42.1 \text{ (m)}}$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>allow similar expressions</p> <p>correct inversion</p> <p>cao</p>

Q	Solution	Mark	Notes
5.	<p>Using <math>v = u + at</math> with <math>u=0</math>, <math>a=(\pm)9.8</math>, <math>t=2.5</math></p> <p><math>v = 9.8 \times 0.5</math></p> <p><math>v = 4.9 \text{ ms}^{-1}</math></p> <p>Impulse = Change in momentum</p> <p>For A <math>J = 5v</math></p> <p>For B <math>J = 2 \times 4.9 - 2v</math></p> <p>Solving <math>5v = 9.8 - 2v</math></p> <p><math>7v = 9.8</math></p> <p><math>v = \underline{1.4 \text{ (ms}^{-1}\text{)}}</math></p> <p><math>J = 5 \times 1.4</math></p> <p><math>J = \underline{7 \text{ (Ns)}}</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>used</p> <p>cao</p> <p>cao</p>

Q	Solution	Mark	Notes
6.(a)	 <p> <math>F = \mu R = \frac{3}{4} R</math> </p> <p>Moments about B</p> $R \times 2 \cos \alpha + F \times 2 \sin \alpha = 2100 \times 1 \cos \alpha$ $R \times 2 \times \frac{12}{13} + \frac{3}{4} R \times 2 \times \frac{5}{13} = 2100 \times \frac{12}{13}$ $24R + \frac{15}{2} R = 25200$ $R = \underline{800 \text{ (N)}}$	<p>M1</p> <p>M1</p> <p>A3</p> <p>A1</p>	<p>dim correct equation, 3 terms, perp distance -1 each error</p> <p>cao</p>
6(b)	<p>Resolve vertically</p> $T \sin \theta = 2100 - R$ $T \sin \theta = 1300$ <p>Resolve horizontally</p> $T \cos \theta = F$ $T \cos \theta = \frac{3}{4} \times 800$ $T \cos \theta = 600$ $T = \sqrt{1300^2 + 600^2}$ $T = \underline{1432 \text{ (N)}}$ $\theta = \tan^{-1} \left( \frac{1300}{600} \right)$ $\theta = \underline{65.2^\circ}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>oe</p> <p>cao</p> <p>oe</p> <p>cao</p>

Ques	Solution	Mark	Notes
<b>1(a)(i)</b>	$P(A \cup B) = P(A) + P(B)$ $= 0.8$	<b>M1</b> <b>A1</b>	Award M1 for using formula
<b>(ii)</b>	$P(A \cap B) = P(A)P(B) = 0.5 \times 0.3$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= 0.5 + 0.3 - 0.5 \times 0.3 = 0.65$	<b>B1</b> <b>M1</b> <b>A1</b>	Award M1 for using formula
<b>(b)</b>	$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.1$ $P(B A) = \frac{P(A \cap B)}{P(A)}$ $= 0.2$	<b>B1</b> <b>M1</b> <b>A1</b>	Award M1 for using formula
<b>2(a)</b>	$E(X^2) = \text{Var}(X) + [E(X)]^2$ $= 66$	<b>M1</b> <b>A1</b>	Award M1 for using formula
<b>(b)</b>	$E(Y) = 3E(X) + 4$ $= 28$ $\text{Var}(Y) = 3^2 \text{Var}(X)$ $= 18$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b>	Award M1 for using formula Award M1 for using formula Award M1 for using formula
<b>3(a)</b>	$P(\text{no white}) = \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \text{ or } \binom{4}{3} \div \binom{9}{3}$ $= \frac{1}{21}$	<b>M1</b> <b>A1</b>	
<b>(b)</b>	$P(2 \text{ white}) = \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \times 3 \text{ or } \binom{5}{2} \times \binom{4}{1} \div \binom{9}{3}$ $= \frac{10}{21}$	<b>M1</b> <b>A1</b>	M0 if 3 omitted.
<b>(c)</b>	EITHER $P(2 \text{ blue}) = \frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} \times 3 \text{ or } \binom{3}{2} \times \binom{6}{1} \div \binom{9}{3}$ $= \left( \frac{3}{14} \right)$ $P(2 \text{ the same}) = \frac{10}{21} + \frac{3}{14}$ $= \frac{29}{42} \text{ cao}$ OR $P(2 \text{ the same}) = \frac{5}{9} \times \frac{4}{8} \times \frac{1}{7} \times 3 + \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times 3$ $+ \frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times 3 + \frac{3}{9} \times \frac{2}{8} \times \frac{5}{7} \times 3$ $= \frac{29}{42} \text{ cao}$	<b>M1A1</b>     <b>A1</b>   <b>M1A1</b>  <b>A1</b>	M0 if 3 omitted      M0 if 3 omitted Accept $\binom{5}{2} \times \binom{1}{1} \div \binom{9}{3} + \binom{5}{2} \times \binom{3}{1} \div \binom{9}{3}$ $+ \binom{3}{2} \times \binom{1}{1} \div \binom{9}{3} + \binom{3}{2} \times \binom{5}{1} \div \binom{9}{3}$

4(a)(i)	$P(X = 4) = \binom{10}{4} \times 0.75^4 \times 0.25^6$ $= 0.0162$	M1 A1	Accept 0.9965 – 0.9803 or 0.0197 – 0.0035
(ii)	Let $Y$ denote the number of games won by Dave so that $Y$ is $B(10, 0.25)$ . si	M1 m1 A1	
(b)	<p>We require <math>P(Y \leq 4)</math>  <math>= 0.9219</math></p> <p>The number of games lasting less than 1 hr, <math>G</math>, is  <math>B(45, 0.08) \approx \text{Poi}(3.6)</math>. si  <math>P(G &gt; 6) = 0.0733</math></p>	B1 M1A1	Award M1A0 for use of adjacent row or column. FT their mean
5(a)	$P(\text{CB}) = \frac{6}{10} \times \frac{8}{100} + \frac{4}{10} \times \frac{3}{100}$ $= 0.06$	M1A1 A1	M1 Use of Law of Total Prob (Accept tree diagram)
(b)	$P(\text{F} \text{CB}) = \frac{12/1000}{0.06}$ $= 0.2 \quad \text{cao}$	B1B1 B1	FT denominator from (a) B1 num, B1 denom
6(a)	$\frac{1}{6}$	B1 M1A1	
(b)	$\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$		
(c)	$\frac{1}{6}, \frac{25}{216}$ and $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \left( \frac{625}{7776} \right)$	M1A1	Award M1A1 if only 3 <sup>rd</sup> term given.
(d)	$\text{Prob} = \frac{1/6}{1 - 25/36}$ $= \frac{6}{11}$	M1 A1	FT their answer to (a)
7(a)(i)	$P(X = 10) = \frac{e^{-12} \times 12^{10}}{10!}$ $= 0.105$	M1 A1	Working must be shown. Accept 0.3472 – 0.2424 or 0.7576 – 0.6528
(ii)	$P(X > 10) = 1 - 0.3472 = 0.6528$	M1A1	Award M1 for adjacent row/col
(b)	Using tables, we see that $P(X \leq 18) = 0.9626$ He needs to take 18 jars.	M1 A1	Award M1A0 for 17 or 19

<p><b>8(a)</b></p> <p><b>(b)</b></p> <p><b>(c)(i)</b></p> <p><b>(ii)</b></p>	$0 \leq \theta \leq 0.3$ $E(X) = 2(0.3 - \theta) + 3 \times 2\theta + 4(0.7 - \theta)$ $= 3.4$ <p><math>E(X)</math> is therefore independent of <math>\theta</math></p> $E(X^2) = 4(0.3 - \theta) + 9 \times 2\theta + 16(0.7 - \theta)$ $= 12.4 - 2\theta$ $\text{Var}(X) = 12.4 - 2\theta - 3.4^2$ $= 0.84 - 2\theta$ $0.84 - 2\theta = 0.8^2$ $\theta = 0.1 \text{ cao}$ <p>Possibilities are 3,3; 4,2 si</p> $P(\text{Sum} = 6) = 0.2 \times 0.6 \times 2 + 0.2 \times 0.2$ $= 0.28$	<p><b>B1B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Accept use of &lt;.</p> <p>Use of <math>\sum xp_x</math> with <math>\theta</math></p> <p>Need not be seen</p> <p>Must include <math>\theta</math></p> <p>FT their <math>E(X)</math> if possible</p> <p>Award M1A0 if 2 is missing in 1<sup>st</sup> term or present in 2<sup>nd</sup> term</p> <p>FT their value of <math>\theta</math> if sensible</p>
<p><b>9(a)(i)</b></p> <p><b>(ii)</b></p> <p><b>(b)(i)</b></p> <p><b>(ii)</b></p> <p><b>(iii)</b></p>	$E(X) = \frac{1}{10} \int_1^2 x(2x + 3x^2) dx$ $= \frac{1}{10} \left[ \frac{2x^3}{3} + \frac{3x^4}{4} \right]_1^2$ $= 1.59$ $E(X^2) = \frac{1}{10} \int_1^2 x^2(2x + 3x^2) dx$ $= \frac{1}{10} \left[ \frac{2x^4}{4} + \frac{3x^5}{5} \right]_1^2$ $= 2.61$ $\text{Var}(X) = 2.61 - 1.59^2 = 0.08$ $F(x) = \int_1^x \frac{1}{10} (2t + 3t^2) dt$ $= \frac{1}{10} [t^2 + t^3]_1^x$ $= \frac{1}{10} (x^2 + x^3 - 2) \text{ cao}$ $P(X \leq 1.4) = F(1.4)$ $= 0.27$ <p>The lower quartile is less than 1.4 since <math>F(1.4)</math> is more than 0.25.</p>	<p><b>M1A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>M1 for the integral of <math>xf(x)</math>, A1 for completely correct although limits may be left until 2<sup>nd</sup> line.</p> <p>For evaluating the integral</p> <p>Integral and limits</p> <p>Correct evaluation of integral</p> <p>FT their <math>E(X)</math></p> <p>Limits may be left until 2<sup>nd</sup> line</p> <p>FT their <math>F(x)</math> if possible</p> <p>FT their answer to (a)(ii)</p>

Ques	Solution	Mark	Notes
1(a)	$E(X^2) = \text{Var}(X) + [E(X)]^2$ $= 27$ <p>Similarly, <math>E(Y^2) = 39</math></p>	M1 A1 A1	Award M1 for <b>using</b> formula
(b)	$E(U) = E(X)E(Y)$ $= 30$ $E(X^2Y^2) = E(X^2)E(Y^2) = 27 \times 39$ $\text{Var}(U) = E(X^2Y^2) - [E(XY)]^2$ $= 27 \times 39 - 30^2 = 153$	M1 A1 B1 M1 A1	FT their $E(X^2), E(Y^2)$ but not their $E(X), E(Y)$ Award M1 for <b>using</b> formula
2(a)(i)	$z = \frac{4.5 - 4.4}{0.2} = 0.5$	M1A1	
(ii)	$P(X > 4.5) = 0.3085$ <p>95<sup>th</sup> percentile = <math>\mu + 1.645\sigma</math></p> $= 4.73$	A1 M1 A1	Award only for $\mu + z\sigma$
(b)(i)	$E(2Y - X) = 0.8$ $\text{Var}(2Y - X) = 4\text{Var}(Y) + \text{Var}(X)$ $= 0.13$	B1 M1 A1	
(ii)	$z = \frac{0 - 0.8}{\sqrt{0.13}} = -2.22 \quad (\text{Accept } \pm)$ <p>We require <math>P(2Y - X &lt; 0)</math></p> <p>Prob = 0.0132</p>	M1A1 M1 A1	FT their values from (b)(i)
(iii)	<p>Let total weight = S</p> $E(S) = 2 \times 4.4 + 3 \times 2.6 = 16.6$ $\text{Var}(S) = 2 \times 0.04 + 3 \times 0.0225 = 0.1475$ $z = \frac{16 - 16.6}{\sqrt{0.1475}} = -1.56$ <p>Prob = 0.9406</p>	B1 M1A1 m1A1 A1	
3(a)	$\bar{x} = \frac{69.9}{75} (= 0.932)$ $\text{SE of } \bar{X} = \frac{0.1}{\sqrt{75}} (= 0.011547\dots)$ <p>90% conf limits are</p> $0.932 \pm 1.645 \times 0.011547\dots$ <p>giving [0.913, 0.951]</p>	B1 B1 M1A1 A1	M1 correct form, A1 correct z. SE must have $\sqrt{75}$ in denom for M1.
(b)	If the method for finding the confidence interval is repeated a large number of times, then 90% of the intervals obtained will contain $\mu$ (or equivalent)	B1	Award B0 for any solution which suggests that the calculated interval contains $\mu$ with a probability of 0.9



<p><b>4(a)</b></p> <p><b>(b)(i)</b></p> <p><b>(ii)</b></p>	<p>The total number of errors, <math>X</math>, is <math>\text{Poi}(8)</math>  <math>P(X &lt; 5) = 1 - 0.9004 = 0.0996</math></p> <p><math>H_0 : \mu = 0.8; H_1 : \mu &lt; 0.8</math></p> <p>Under <math>H_0</math>, number of errors is <math>\text{Poi}(64) \approx N(64, 64)</math>.  <math display="block">z = \frac{60.5 - 64}{8}</math> <math display="block">= -0.4375</math></p> <p><math>p\text{-value} = 0.33</math>  Insufficient evidence to reject <math>H_0</math> / Accept <math>H_0</math></p>	<p><b>B1</b></p> <p><b>M1A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Award M1A0 for use of adjacent row/column</p> <p>Award M1A0A1A1 for incorrect or no continuity correction  No c/c gives <math>z = -0.5, p = 0.31</math>  Incorrect c/c gives <math>z = -0.5625, p = 0.29</math></p> <p>FT their p-value</p>
<p><b>5(a)</b></p> <p><b>(b)</b></p>	<p><math>H_0 : \mu_D = \mu_F; H_1 : \mu_D \neq \mu_F</math></p> <p><math>\bar{x}_D = \frac{890.4}{6} (=148.4); \bar{x}_F = \frac{879}{6} (=146.5)</math> si</p> <p>SE of difference of means = <math>\sqrt{\frac{1.5^2}{6} + \frac{1.5^2}{6}}</math> (0.866..)</p> <p>Test statistic = <math>\frac{148.4 - 146.5}{0.866..}</math>  <math>= 2.19</math></p> <p>Prob from tables = 0.01426  <math>p\text{-value} = 0.02852</math></p> <p><b>Strong</b> evidence that there is a difference in mean distances for the two players.  OR  <b>Strong</b> evidence that David's mean is larger than Frank's mean.</p>	<p><b>B1</b></p> <p><b>B1B1</b></p> <p><b>M1A1</b></p> <p><b>M1A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>FT arithmetic slip in evaluating means</p> <p>FT from previous line</p> <p>FT on their p-value</p>



Ques	Solution	Mark	Notes																																																	
1(a)	<p>The possibilities are</p> <table><tr><th>Numbers drawn</th><th>Mean</th><th>Median</th></tr><tr><td>1 2 3</td><td>2</td><td>2</td></tr><tr><td>1 2 4</td><td>7/3</td><td>2</td></tr><tr><td>1 2 5</td><td>8/3</td><td>2</td></tr><tr><td>1 3 4</td><td>8/3</td><td>3</td></tr><tr><td>1 3 5</td><td>3</td><td>3</td></tr><tr><td>1 4 5</td><td>10/3</td><td>4</td></tr><tr><td>2 3 4</td><td>3</td><td>3</td></tr><tr><td>2 3 5</td><td>10/3</td><td>3</td></tr><tr><td>2 4 5</td><td>11/3</td><td>4</td></tr><tr><td>3 4 5</td><td>4</td><td>4</td></tr></table> <p>The sampling distribution of the mean is</p> <table><tr><td><math>\bar{x}</math></td><td>2</td><td>7/3</td><td>8/3</td><td>3</td><td>10/3</td><td>11/3</td><td>4</td></tr><tr><td>Prob</td><td>1/10</td><td>1/10</td><td>2/10</td><td>2/10</td><td>2/10</td><td>1/10</td><td>1/10</td></tr></table>	Numbers drawn	Mean	Median	1 2 3	2	2	1 2 4	7/3	2	1 2 5	8/3	2	1 3 4	8/3	3	1 3 5	3	3	1 4 5	10/3	4	2 3 4	3	3	2 3 5	10/3	3	2 4 5	11/3	4	3 4 5	4	4	$\bar{x}$	2	7/3	8/3	3	10/3	11/3	4	Prob	1/10	1/10	2/10	2/10	2/10	1/10	1/10	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>B1 each column</p> <p>Special case – B2 if one combination is missing.</p> <p>No FT from earlier work.</p>
Numbers drawn	Mean	Median																																																		
1 2 3	2	2																																																		
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$\bar{x}$	2	7/3	8/3	3	10/3	11/3	4																																													
Prob	1/10	1/10	2/10	2/10	2/10	1/10	1/10																																													
(b)	<p>The sampling distribution of the median is</p> <table><tr><td>Median</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Prob</td><td>3/10</td><td>4/10</td><td>3/10</td></tr></table>	Median	2	3	4	Prob	3/10	4/10	3/10	<p><b>M1</b></p> <p><b>A1</b></p>																																										
Median	2	3	4																																																	
Prob	3/10	4/10	3/10																																																	
2(a)	<p>UE of <math>\mu = 99.03</math></p> <p><math>\Sigma x^2 = 98088.11</math></p> <p>UE of <math>\sigma^2 = \frac{98088.11}{9} - \frac{990.3^2}{9 \times 10}</math></p> <p><math>= 2.08 (2.0778...)</math></p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>No working need be seen</p> <p>Answer only no marks</p>																																																	
(b)(i)	<p><math>H_0 : \mu = 100; H_1 : \mu \neq 100</math></p>	<p><b>B1</b></p>																																																		
(ii)	<p><math>t = \frac{99.03 - 100}{\sqrt{2.0778../10}}</math></p> <p><math>= -2.13</math></p> <p>DF = 9    si</p> <p>Critical value = 2.262</p> <p>Insufficient evidence to reject the manager’s claim or Accept the manager’s claim</p> <p>Because <math>2.13 &lt; 2.262</math> or equivalent using the term ‘acceptance region’ or by means of a diagram</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	<p>M0 if treated as z</p> <p>FT their critical value but not their p-value obtained from using the normal distribution</p>																																																	





<p><b>(b)(i)</b></p>	$E(U) = cE(\bar{X}) \text{ (or } cE(X)) = c \times \frac{2a}{3}$ $E(U) = a \Rightarrow c = \frac{3}{2}$ $\text{Var}(U) = \frac{9}{4} \text{Var}(\bar{X})$ $= \frac{9}{4} \times \frac{a^2}{18n}$ $= \frac{a^2}{8n}$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Penalise the omission of E once in the question</p>
<p><b>(ii)</b></p>	$E(V) = dE(Y) = d \times \frac{2na}{2n+1}$ $E(V) = a \Rightarrow d = \frac{2n+1}{2n}$ $\text{Var}(V) = \left( \frac{2n+1}{2n} \right)^2 \text{Var}(Y)$ $= \left( \frac{2n+1}{2n} \right)^2 \times \left( \frac{na^2}{(n+1)(2n+1)^2} \right)$ $= \frac{a^2}{4n(n+1)}$	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	
<p><b>(iii)</b></p>	$\frac{\text{Var}(U)}{\text{Var}(V)} = \frac{a^2}{8n} \div \frac{a^2}{4n(n+1)}$ $= \frac{n+1}{2}$ <p>V is the better estimator Because (for <math>n &gt; 1</math>) it has the smaller variance</p>	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	



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