C3

1. (*a*)

0	1		
0·25	1·064494459		
0·5	1·284025417		
0·75	1·755054657	(5 values correct)	B2
1	2·718281828	(3 or 4 values correct)	B1

Correct formula with $h = 0{\cdot}25$ M1

$$I \approx \frac{0{\cdot}25}{3} \times \{1 + 2{\cdot}718281828 + 4(1{\cdot}064494459 + 1{\cdot}755054657) + 2(1{\cdot}284025417)\}$$

$I \approx 17{\cdot}56452913 \times 0{\cdot}25 \div 3$

$I \approx 1{\cdot}463710761$

$I \approx 1{\cdot}4637$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$$\int_0^1 e^{x^2+3}\,dx = e^3 \times \int_0^1 e^{x^2}\,dx$$ M1

$$\int_0^1 e^{x^2+3}\,dx = 29{\cdot}399$$ (f.t. candidate's answer to (*a*)) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $\phi = 360° - \theta$ or $\phi = -\theta$ and noting that $\cos\theta = \cos\phi$ B1

$\sin\theta \neq \sin\phi$ (including correct evaluations) B1

(*b*) $13\tan^2\theta = 5(1 + \tan^2\theta) + 6\tan\theta.$

(correct use of $\sec^2\theta = 1 + \tan^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\tan\theta$, either by using the quadratic formula or by getting the expression into the form $(a\tan\theta + b)(c\tan\theta + d)$, with $a \times c$ = candidate's coefficient of $\tan^2\theta$ and $b \times d$ = candidate's constant m1

$8\tan^2\theta - 6\tan\theta - 5 = 0 \Rightarrow (4\tan\theta - 5)(2\tan\theta + 1) = 0$

$\Rightarrow \tan\theta = \frac{5}{4}, \tan\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 51{\cdot}34°, 231{\cdot}34°$ B1

$\theta = 153{\cdot}43°, 333{\cdot}43°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\tan\theta$ = +, –, f.t. for 3 marks, $\tan\theta$ = –, –, f.t. for 2 marks
$\tan\theta$ = +, +, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(-3x - 2) = -3$ B1

$\frac{d}{dx}(-4x^2y) = -4x^2\frac{dy}{dx} - 8xy$ B1

$\frac{d}{dx}(2y^3) = 6y^2\frac{dy}{dx}$ B1

$x = 3, y = 1 \Rightarrow \frac{dy}{dx} = \frac{6}{42} = \frac{1}{7}$ (c.a.o.) B1

(*b*) (i) Differentiating $\sin at$ and $\cos at$ with respect to t, at least one correct M1

candidate's x-derivative = $a\cos at$,
candidate's y-derivative = $-a\sin at$ (both values) A1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = -\tan at$ (c.a.o.) A1

(ii) $\frac{d}{dt}\left[\frac{dy}{dx}\right] = -a\sec^2 at$ (f.t. candidate's expression for $\frac{dy}{dx}$) B1

Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dx}\right] \div$ candidate's x-derivative M1

$\frac{d^2y}{dx^2} = -\sec^3 at$ (c.a.o.) A1

4. $f(x) = \cos x - 5x + 2$

An attempt to check values or signs of $f(x)$ at $x = 0$, $x = \pi/4$ M1

$f(0) = 3 > 0$, $f(\pi/4) = -1{\cdot}22 < 0$

Change of sign $\Rightarrow f(x) = 0$ has root in $(0, \pi/4)$ A1

$x_0 = 0{\cdot}6$

$x_1 = 0{\cdot}565067123$ B1

$x_2 = 0{\cdot}568910532$

$x_3 = 0{\cdot}568497677$

$x_4 = 0{\cdot}568542145 = 0{\cdot}56854$ (x_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(x)$ at $x = 0{\cdot}568535$, $x = 0{\cdot}568545$ M1

$f(0{\cdot}568535) = 1{\cdot}563 \times 10^{-5} > 0$, $f(0{\cdot}568545) = -3{\cdot}975 \times 10^{-5} < 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}56854$ correct to five decimal places A1

Note: 'change of sign' must appear at least once

5. (*a*) $\frac{dy}{dx} = \frac{a + bx}{7 + 2x - 3x^2}$ (including $a = 1$, $b = 0$) M1

$\frac{dy}{dx} = \frac{2 - 6x}{7 + 2x - 3x^2}$ A1

(*b*) $\frac{dy}{dx} = e^{\tan x} \times f(x)$ ($f(x) \neq 1, 0$) M1

$\frac{dy}{dx} = e^{\tan x} \times \sec^2 x$ A1

(*c*) $\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$ ($f(x), g(x) \neq 1, 0$) M1

$\frac{dy}{dx} = 5x^2 \times f(x) + \sin^{-1} x \times g(x)$

(either $f(x) = \frac{1}{\sqrt{1 - x^2}}$ or $g(x) = 10x$) A1

$\frac{dy}{dx} = 5x^2 \times \frac{1}{\sqrt{1 - x^2}} + 10x \times \sin^{-1} x$ A1

6. (*a*) (i) $\int 3e^{2-x/4}\,dx = k \times 3e^{2-x/4} + c$ $(k = 1, -\frac{1}{4}, 4, -4)$ M1

$\int 3e^{2-x/4}\,dx = -4 \times 3e^{2-x/4} + c$ A1

(ii) $\int \frac{9}{(2x-3)^6}\,dx = \frac{k \times 9 \times (2x-3)^{-5}}{-5} + c$ $(k = 1, 2, \frac{1}{2})$ M1

$\int \frac{9}{(2x-3)^6}\,dx = \frac{9 \times (2x-3)^{-5}}{-5 \times 2} + c$ A1

(iii) $\int \frac{7}{3x+1}\,dx = k \times 7 \times \ln|3x+1| + c$ $(k = 1, 3, \frac{1}{3})$ M1

$\int \frac{7}{3x+1}\,dx = \frac{7}{3} \times \ln|3x+1| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int \sin 2x\,dx = k \times \cos 2x$ $(k = -1, -2, \frac{1}{2}, -\frac{1}{2})$ M1

$\int \sin 2x\,dx = -\frac{1}{2} \times \cos 2x$ A1

$k \times (\cos 2a - \cos 0) = \frac{1}{4}$

(f.t. candidate's value for k) M1

$\cos 2a = \frac{1}{2}$ (c.a.o.) A1

$a = \pi/6$ (f.t. $\cos 2a = b$ provided both M's are awarded) A1

7. (*a*) $9|x-3| = 6$ B1

$x - 3 = \pm\frac{2}{3}$ (f.t. candidate's $a|x-3| = b$, with at least one of a, b correct) B1

$x = \frac{11}{3}, \frac{7}{3}$ (f.t. candidate's $a|x-3| = b$, with at least one of a, b correct) B1

(*b*) Trying to solve either $5x - 2 \le 3$ or $5x - 2 \ge -3$ M1

$5x - 2 \le 3 \Rightarrow x \le 1$

$5x - 2 \ge -3 \Rightarrow x \ge -\frac{1}{5}$ (both inequalities) A1

Required range: $-\frac{1}{5} \le x \le 1$ (f.t. one slip) A1

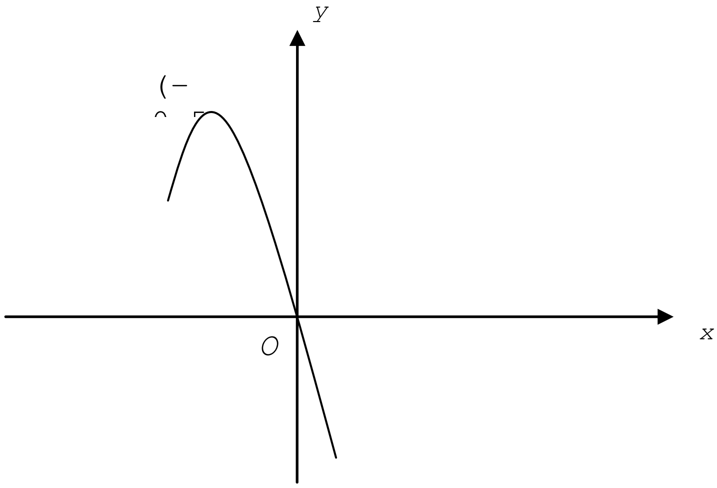
Alternative mark scheme

$(5x-2)^2 \le 9$ (forming and trying to solve quadratic) M1

Critical points $x = -\frac{1}{5}$ and $x = 1$ A1

Required range: $-\frac{1}{5} \le x \le 1$ (f.t. one slip) A1

8.



Concave down curve passing through the origin with maximum point in the second quadrant B1
x-coordinate of stationary point = – 0·5 B1
y-coordinate of stationary point = 8 B1

9. (*a*) (i) $f'(x) = \dfrac{(x^2+5)\times f(x) - (x^2+3)\times g(x)}{(x^2+5)^2}$ $(f(x), g(x) \neq 1)$ M1

$f'(x) = \dfrac{(x^2+5)\times 2x - (x^2+3)\times 2x}{(x^2+5)^2}$ A1

$f'(x) = \dfrac{4x}{(x^2+5)^2}$ (c.a.o.) A1

$f'(x) < 0$ since numerator is negative and denominator is positive B1

(ii) $R(f) = (^3/_5, 1)$ B1 B1

(*b*) (i) $x^2 = \dfrac{3-5y}{y-1}$ (o.e.) (condone any incorrect signs) M1

$x = (\pm)\left[\dfrac{3-5y}{y-1}\right]^{1/2}$ (f.t. at most one incorrect sign) A1

$x = -\left[\dfrac{3-5y}{y-1}\right]^{1/2}$ (f.t. at most one incorrect sign) A1

$f^{-1}(x) = -\left[\dfrac{3-5x}{x-1}\right]^{1/2}$ (c.a.o.) A1

(ii) $R(f^{-1}) = (-\infty, 0)$, $D(f^{-1}) = (^3/_5, 1)$,
(both intervals, f.t. candidate's $R(f)$) B1

10.	$gg(x) = (3(g(x))^2 + 7)^{1/2}$ or $gg(x) = g((3x^2 + 7)^{1/2})$			M1
	$gg(x) = (3(3x^2 + 7) + 7)^{1/2}$			A1
	An attempt to solve the equation by squaring both sides			M1
	$gg(x) = 8 \Rightarrow 9x^2 = 36$	(o.e.)	(c.a.o.)	A1
	$x = \pm 2$		(c.a.o.)	A1