



GCE AS/A level

0975/01

MATHEMATICS C3

Pure Mathematics

A.M. FRIDAY, 1 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^1 e^{x^2} dx.$$

Show your working and give your answer correct to four decimal places. [4]

- (b) Use **your answer to part (a)** to deduce an approximate value for the integral

$$\int_0^1 e^{x^2+3} dx. \quad [2]$$

2. (a) Show, by counter-example, that the statement

$$\text{'If } \cos \theta = \cos \phi \text{ then } \sin \theta = \sin \phi \text{'}$$

is false. [2]

- (b) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$13 \tan^2 \theta = 5 \sec^2 \theta + 6 \tan \theta. \quad [6]$$

3. (a) The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

- (b) Given that

$$x = \sin at, y = \cos at,$$

where a is a constant, find and simplify

- (i) an expression for $\frac{dy}{dx}$ in terms of a and t ,

- (ii) an expression for $\frac{d^2y}{dx^2}$ in terms of a and t . [7]

4. Show that the equation

$$\cos x - 5x + 2 = 0$$

has a root α between 0 and $\frac{\pi}{4}$.

The recurrence relation

$$x_{n+1} = \frac{1}{5}(2 + \cos x_n)$$

with $x_0 = 0.6$ can be used to find α . Find and record the values of x_1, x_2, x_3, x_4 . Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

5. Differentiate each of the following with respect to x .

$$(a) \quad \ln(7 + 2x - 3x^2) \qquad (b) \quad e^{\tan x} \qquad (c) \quad 5x^2 \sin^{-1} x \qquad [2], [2], [3]$$

6. Find

$$(a) \quad (i) \quad \int 3e^{2-\frac{x}{4}} dx \qquad (ii) \quad \int \frac{9}{(2x-3)^6} dx \qquad (iii) \quad \int \frac{7}{3x+1} dx \qquad [6]$$

(b) Given that $0 < a < \frac{\pi}{2}$ and that

$$\int_0^a \sin 2x \, dx = \frac{1}{4},$$

find the value of the constant a . [5]

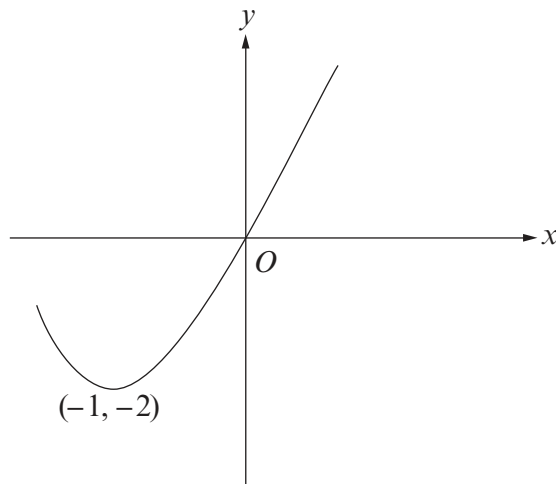
7. Solve the following.

$$(a) \quad 4|x-3| + 2 = 8 - 5|x-3| \qquad [3]$$

$$(b) \quad |5x-2| \leq 3 \qquad [3]$$

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8. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the origin and has a minimum point at $(-1, -2)$.



Sketch the graph of $y = -4f(2x)$, indicating the coordinates of the stationary point. [3]

9. The function f has domain $(-\infty, 0)$ and is defined by

$$f(x) = \frac{x^2 + 3}{x^2 + 5}.$$

- (a) (i) Show that $f'(x)$ is always negative.
 (ii) Write down the range of f . [6]
 (b) (i) Find an expression for $f^{-1}(x)$.
 (ii) Write down the range and domain of f^{-1} . [5]

10. The function g has domain $(-\infty, \infty)$ and is defined by

$$g(x) = \sqrt{3x^2 + 7}.$$

Solve the equation

$$gg(x) = 8. \quad [5]$$