C4

1. (*a*) $f(x) \equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$ (correct form) M1

$11 + x - x^2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 1, C = 3, B = -2$ (2 correct coefficients) A1

(third coefficient, f.t. one slip in enumeration of other 2 coefficients) A1

(*b*) $f'(x) = -\frac{1}{(x+1)^2} + \frac{2}{(x-2)^2} - \frac{6}{(x-2)^3}$ (o.e.)

(f.t. candidate's values for *A*, *B*, *C*)

(at least one of the first two terms) B1

(third term) B1

$f'(0) = {}^1/_4$ (c.a.o.) B1

2. $3y^2\frac{dy}{dx} - 8x - 3x\frac{dy}{dx} - 3y = 0$ $\left[3y^2\frac{dy}{dx} - 8x\right]$ B1

$\left[-3x\frac{dy}{dx} - 3y\right]$ B1

Either $\frac{dy}{dx} = \frac{3y + 8x}{3y^2 - 3x}$ **or** $\frac{dy}{dx} = \frac{1}{3}$ (o.e.) (c.a.o.) B1

Equation of tangent: $y - (-3) = \frac{1}{3}(x - 2)$ $\left[\text{f.t. candidate's value for } \frac{dy}{dx}\right]$ B1

3. (*a*) $4(1 - 2\sin^2\theta) = 1 - 2\sin\theta$. (correct use of $\cos 2\theta = 1 - 2\sin^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\sin\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sin\theta + b)(c\sin\theta + d)$, with $a \times c$ = candidate's coefficient of $\sin^2\theta$ and $b \times d$ = candidate's constant m1

$8\sin^2\theta - 2\sin\theta - 3 = 0 \Rightarrow (4\sin\theta - 3)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{3}{4}, \quad \sin\theta = -\frac{1}{2}$ (c.a.o.) A1

$\theta = 48{\cdot}59°, 131{\cdot}41°$ B1

$\theta = 210°, 330°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\sin\theta = +, -$, f.t. for 3 marks, $\sin\theta = -, -$, f.t. for 2 marks
$\sin\theta = +, +$, f.t. for 1 mark

(*b*) (i) $R = 17$ B1

Correctly expanding $\sin(x + \alpha)$ and using either $17\cos\alpha = 8$ **or** $17\sin\alpha = 15$ **or** $\tan\alpha = \frac{15}{8}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 61{\cdot}93°$ (c.a.o) A1

(ii) $\sin(x + \alpha) = \frac{11}{17}$ (f.t. candidate's value for R) B1

$x + 61{\cdot}93° = 40{\cdot}32°, 139{\cdot}68°, 400{\cdot}32°$,
(at least one value on R.H.S.,
f.t. candidate's values for α and R) B1

$x = 77{\cdot}75°, 338{\cdot}39°$ (c.a.o.) B1

(iii) Greatest possible value for k is 17 since greatest possible value for sin is 1 (f.t. candidate's value for R) E1

4.

$$\text{Volume} = \pi\int_3^4 \left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 dx$$ B1

$$\left(\sqrt{x} + \frac{5}{\sqrt{x}}\right)^2 = \left(x + 10 + \frac{25}{x}\right)$$ B1

$$\int\left(ax + b + \frac{c}{x}\right) dx = \frac{ax^2}{2} + bx + c\ln x, \text{ where } c \neq 0 \text{ and at least one of } a, b \neq 0$$ B1

Correct substitution of correct limits in candidate's integrated expression of form $\frac{ax^2}{2} + bx + c\ln x$, where $c \neq 0$ and at least one of $a, b \neq 0$ M1

Volume = 65(·0059. . .) (c.a.o.) A1

5. $\left(1+\frac{x}{3}\right)^{-1/2} = 1 - \frac{x}{6} + \frac{x^2}{24}$ $\left(1-\frac{x}{6}\right)$ B1

$\left(\frac{x^2}{24}\right)$ B1

$|x| < 3$ or $-3 < x < 3$ B1

$\left(\frac{16}{15}\right)^{-1/2} \approx 1 - \frac{1}{30} + \frac{1}{600}$ (f.t. candidate's coefficients) B1

$\sqrt{15} \approx \frac{581}{150}$ (c.a.o.) B1

6. (*a*) candidate's x-derivative $= 2t$

candidate's y-derivative $= 2$ (at least one term correct)

and use of

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{1}{t}$ (o.e.) (c.a.o.) A1

Use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ m1

Equation of normal at P: $y - 2p = -p(x - p^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

$y + px = p^3 + 2p$ (convincing) (c.a.o.) A1

(*b*) (i) Substituting $x = 9$, $y = 6$ in equation of normal M1

$p^3 - 7p - 6 = 0$ (convincing) A1

(ii) A correct method for solving $p^3 - 7p - 6 = 0$ M1

$p = -1$ A1

$p = -2$ A1

P is either $(1, -2)$ or $(4, -4)$ (c.a.o.) A1

7. (*a*) $u = x \Rightarrow du = dx$ (o.e.) B1

$dv = e^{-2x}\,dx \Rightarrow v = -\frac{1}{2}e^{-2x}$ (o.e.) B1

$\int x e^{-2x}\,dx = x \times -\frac{1}{2}e^{-2x} - \int -\frac{1}{2}e^{-2x}\,dx$ M1

$\int x e^{-2x}\,dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + c$ (c.a.o.) A1

(*b*) $\int \frac{1}{x(1+3\ln x)}\,dx = \int \frac{k}{u}\,du$ ($k = {}^1/_3$ or 3) M1

$\int \frac{a}{u}\,du = a\ln u$ B1

$\int_1^e \frac{1}{x(1+3\ln x)}\,dx = k[\ln u]_1^4$ or $k[\ln(1+3\ln x)]_1^e$ B1

$\int_1^e \frac{1}{x(1+3\ln x)}\,dx = 0{\cdot}4621$ (c.a.o.) A1

8. (*a*) $\frac{dV}{dt} = -kV^3$ (where $k > 0$) B1

(*b*) $\int \frac{dV}{V^3} = -\int k\,dt$ (o.e.) M1

$-\frac{V^{-2}}{2} = -kt + c$ A1

$c = -\frac{1}{7200}$ (c.a.o.) A1

$V^2 = \frac{3600}{7200kt + 1} = \frac{3600}{at + 1}$ (convincing)

where $a = 7200k$ A1

(*c*) Substituting $t = 2$ and $V = 50$ in expression for V^2 M1

$a = 0{\cdot}22$ A1

Substituting $V = 27$ in expression for V^2 with candidate's value for a M1

$t = 17{\cdot}9$ (c.a.o) A1

9. (*a*) An attempt to evaluate **a.b** M1
Correct evaluation of **a.b** and $\mathbf{a.b} \neq 0 \Rightarrow$ **a** and **b** not perpendicular A1

(*b*) (i) $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ B1
(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1
$\mathbf{r} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda\,(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.)
(f.t. if candidate uses his/her expression for **AB**) A1

(*c*) $4 + 2\lambda = 2 - 2\mu$
$1 + \lambda = 6 + \mu$
$-6 + 2\lambda = p + 3\mu$ (o.e.)
(comparing coefficients, at least one equation correct) M1
(at least two equations correct) A1
Solving the first two of the equations simultaneously m1
(f.t. for all 3 marks if candidate uses his/her expression for **AB**)
$\lambda = 2,\ \mu = -3$ (o.e.) (c.a.o.) A1
$p = 7$ from third equation
(f.t. candidates derived values for λ and μ) A1

10. $a^2 = 5b^2 \Rightarrow (5k)^2 = 5b^2 \Rightarrow b^2 = 5k^2$ B1
$\therefore$ 5 is a factor of b^2 and hence 5 is a factor of b B1
$\therefore$ a and b have a common factor, which is a contradiction to the original assumption B1