



GCE AS/A level

0976/01

MATHEMATICS – C4
Pure Mathematics

A.M. THURSDAY, 13 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{6 + x - 9x^2}{x^2(x + 2)}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) Using your result to part (a),

(i) find an expression for $f'(x)$,

(ii) verify that $f(x)$ has a stationary value when $x = 2$. [3]

2. Find the equation of the normal to the curve

$$x^3 - 2xy^2 + y^3 = 5$$

at the point $(2, 1)$. [5]

3. (a) Find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$8 \cos 2\theta + 6 = \cos^2 \theta + \cos \theta. [6]$$

(b) Express $\sqrt{15} \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Hence find all values of θ in the range $0^\circ \leq \theta \leq 360^\circ$ satisfying

$$\sqrt{15} \cos \theta - \sin \theta = 3. [6]$$

4. The region R is bounded by the curve $y = \sin 2x$, the x -axis and the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{2}$.

Find the volume generated when R is rotated through four right angles about the x -axis. Give your answer correct to three decimal places. [5]

5. (a) (i) Expand $(1 + 6x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2 .

(ii) State the range of values of x for which your expansion is valid. [3]

(b) Use your expansion in part (a) to find an approximate value for one root of the equation

$$2(1 + 6x)^{\frac{1}{3}} = 2x^2 - 15x. [2]$$

6. The curve C has the parametric equations

$$x = at, y = \frac{b}{t},$$

where a, b are positive constants.

The point P lies on C and has parameter p .

- (a) Show that the equation of the tangent to C at the point P is

$$ap^2y + bx - 2abp = 0. \quad [5]$$

- (b) The tangent to C at the point P meets the x -axis at the point A and the y -axis at the point B . Find the area of triangle AOB , where O denotes the origin. Give your answer in its simplest form. [3]

- (c) The point D has coordinates $(2a, b)$. Show that there is no point P on C such that the tangent to C at the point P passes through D . [3]

7. (a) Find $\int (3x - 1)\cos 2x \, dx$. [4]

- (b) Use the substitution $u = 2x + 1$ to evaluate

$$\int_0^1 \frac{x}{(2x + 1)^3} \, dx. \quad [5]$$

8. Part of the surface of a small lake is covered by green algae. The area of the lake covered by the algae at time t years is $A \text{ m}^2$. The rate of increase of A is directly proportional to $\sqrt{A}$.

- (a) Write down a differential equation satisfied by A . [1]

- (b) The area of the lake covered by the algae at time $t = 3$ is 64 m^2 and the area covered at time $t = 5.5$ is 196 m^2 . Find an expression for A in terms of t . [6]

TURN OVER

9. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= -\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}, \\ \mathbf{b} &= 7\mathbf{i} - \mathbf{j} + 5\mathbf{k},\end{aligned}$$

respectively.

- (a) Write down the vector $\mathbf{AB}$. [1]

- (b) The point C lies on the line AB and is such that $AC : CB = 3 : 1$.
Find the position vector of C . [2]

- (c) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} + \mathbf{j} + 11\mathbf{k} + \lambda (-4\mathbf{i} + \mathbf{j} + 3\mathbf{k}).$$

- (i) Find the vector equation of the line parallel to L which passes through A .

- (ii) **Verify** that B is in fact the foot of the perpendicular from A to L . [8]

10. Prove by contradiction the following proposition.

When x is real,

$$(5x - 3)^2 + 1 \geq (3x - 1)^2.$$

The first line of the proof is given below.

Assume that there is a real value of x such that

$$(5x - 3)^2 + 1 < (3x - 1)^2. \quad [3]$$