GCE AS/A level

0981/01

MATHEMATICS – M2
Mechanics

A.M. THURSDAY, 6 June 2013

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9 \cdot 8 \, \text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. An object of mass $8\,\text{kg}$ slides in a straight line from point A to point B on a rough horizontal floor. At A, the speed of the object is $7\,\text{ms}^{-1}$. It is brought to rest at B by a constant frictional force between the object and the floor. The distance AB is $15\,\text{m}$.

   (a) Calculate the loss in kinetic energy. [2]

   (b) Determine the coefficient of friction between the object and the floor. [4]

2. A particle P, of mass $2\,\text{kg}$, is moving so that at time $t\,$s its velocity $\mathbf{v}\,\text{ms}^{-1}$ is given by $\mathbf{v} = (13t - 3)\mathbf{i} + (2 + 3t^2)\mathbf{j}$. At time $t = 0\,$s, the position vector of the particle is $(2\mathbf{i} + 7\mathbf{j})$ m.

   (a) Find the position vector $\mathbf{r}$ of P at time $t\,$s. [5]

   (b) Determine the acceleration $\mathbf{a}$ of P at time $t\,$s. [2]

   (c) Calculate the values of t when the velocity of P is perpendicular to the vector $\mathbf{i} - 2\mathbf{j}$. [5]

3. A person throws a ball from a point A to hit a vertical pole, which is placed at a horizontal distance of $9\,\text{m}$ from A. The point A is $1\,\text{m}$ above the horizontal ground. The ball is projected with initial speed $15\,\text{ms}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \dfrac{3}{4}$.

   (a) Given that the ball hits the pole at a point B,

   (i) find the time taken for the ball to reach B,

   (ii) determine the height of B above the ground. [7]

   (b) Given that the ball misses the pole and hits the ground, calculate the speed with which it hits the ground. [5]

4. The diagram shows a hollow cone, of base radius $3\,$m and height $4\,$m, which is fixed with its axis vertical and vertex V downwards. A particle P, of mass $M\,$kg, moves in the horizontal circle with centre C on the smooth inner surface of the cone with constant speed $\sqrt{\dfrac{8g}{3}}\ $ms^{-1}, where $g\,$ms^{-2} is the acceleration due to gravity.
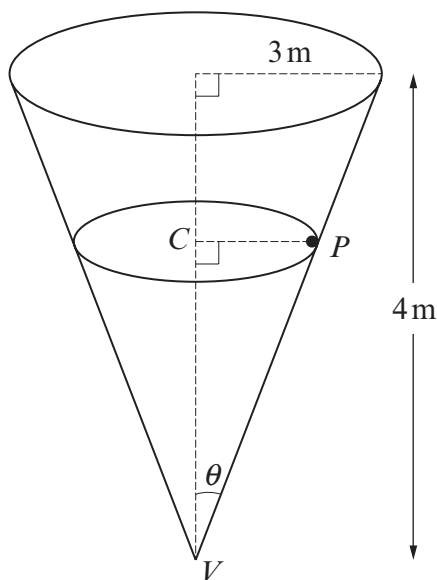
    (a)   Show that the normal reaction of the surface of the cone on the particle is $\dfrac{5Mg}{3}\,$N.   [4]

    (b)   Calculate the length of CP and hence determine the height of C above V.   [5]

5. A particle moves along a straight horizontal line. Its velocity $v\,$ms^{-1} at time $t\,$s, is given by
$$v = 2t(t - 6).$$

    (a)   Find the set of values of t for which the velocity of the particle is negative.   [2]

    (b)   Find the total distance travelled by the particle in the interval $0 \leqslant t \leqslant 9$.   [5]

6. A car of mass $1500\,$kg is towing a trailer of mass $500\,$kg by means of a rigid tow bar up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \dfrac{1}{14}$.

The resistance to motion acting on the car is $170\,$N and that acting on the trailer is $30\,$N. The car's engine is working at a constant rate of $60\,$kW. When the car and the trailer are moving at a speed of $20\,$ms^{-1},

    (a)   calculate the tractive force acting on the car,   [2]

    (b)   show that the acceleration of the car and the trailer is $0{\cdot}7\,$ms^{-2},   [4]

    (c)   determine the tension in the tow bar.   [4]

TURN OVER

7. The end *A* of a light elastic string *AB*, of natural length 1·2 m and modulus of elasticity 360 N, is fixed. A particle *P*, of mass 2 kg, is attached to the end *B*. Initially, *P* is held at rest at a point which is 0·7 m vertically below *A*. It is then released and allowed to fall.

    (a) Find the greatest extension of the string in the subsequent motion. Give your answer correct to 2 decimal places. [7]

    (b) Calculate the velocity of the particle when it is 1·2 m below *A*. [4]

8. The diagram shows a particle of mass 3 kg at a point *P* on the smooth outer surface *AB* of a sphere centre *O* and radius 4 m. The points *O*, *A*, *P* and *B* are in the same vertical plane.
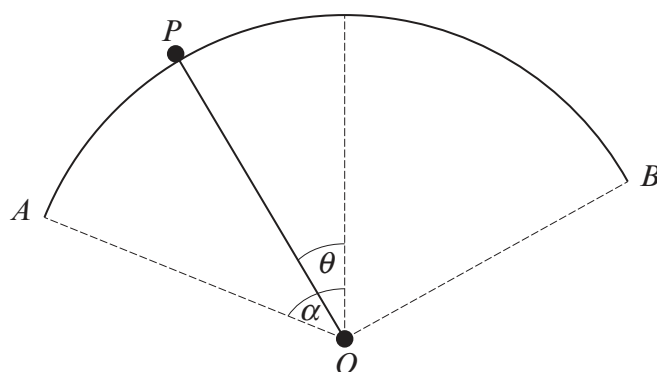
Initially, the particle is held at rest at the point *A*, where *OA* makes an angle α with the upwards vertical and $\cos \alpha = 0\cdot8$. The particle is then projected with velocity $5\,\text{ms}^{-1}$ in a direction which is perpendicular to *OA*, so that the particle moves along the arc *AB*. When the particle is at *P*, *OP* makes an angle θ with the upwards vertical.

    (a) Find, in terms of θ, the speed of the particle at *P*. [4]

    (b) Determine, in terms of θ, the reaction between the particle and the sphere at *P*. [4]