



GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3 AS/Advanced

SUMMER 2014

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2014 examination in GCE MATHEMATICS C1-C4 & FP1-FP3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

	Page
C1	1
C2	6
C3	11
C4	16
FP1	21
FP2	26
FP3	30

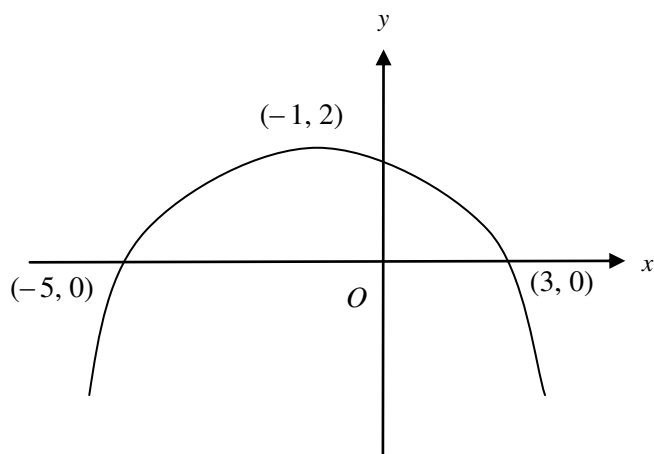
C1

1. (a) (i) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1
 Gradient of $AB = -\frac{1}{2}$ (or equivalent) A1
- (ii) A correct method for finding the equation of AB using the candidate's value for the gradient of AB . M1
 Equation of AB : $y - 3 = -\frac{1}{2}(x - 12)$ (or equivalent) A1
 (f.t. the candidate's value for the gradient of AB)
- (b) (i) Use of gradient $L \times \text{gradient } AB = -1$ M1
 Equation of L : $y = 2x - 1$ A1
 (f.t. the candidate's value for the gradient of AB)
- (ii) A correct method for finding the coordinates of D M1
 $D(4, 7)$ (convincing) A1
- (iii) A correct method for finding the length of $AD(BD)$ M1
 $AD = \sqrt{45}$ A1
 $BD = \sqrt{80}$ A1
- (c) (i) A correct method for finding the coordinates of E M1
 $E(8, 15)$ A1
- (ii) $ACBE$ is a kite (c.a.o.) B1
2. (a) $\frac{3\sqrt{3} + 1}{5\sqrt{3} - 7} = \frac{(3\sqrt{3} + 1)(5\sqrt{3} + 7)}{(5\sqrt{3} - 7)(5\sqrt{3} + 7)}$ M1
 Numerator: $45 + 21\sqrt{3} + 5\sqrt{3} + 7$ A1
 Denominator: $75 - 49$ A1
 $\frac{3\sqrt{3} + 1}{5\sqrt{3} - 7} = 2 + \sqrt{3}$ (c.a.o.) A1
- Special case**
 If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5\sqrt{3} - 7$
- (b) $\sqrt{12} \times \sqrt{24} = 12\sqrt{2}$ B1
 $\frac{\sqrt{150}}{\sqrt{3}} = 5\sqrt{2}$ B1
 $\frac{36}{\sqrt{2}} = 18\sqrt{2}$ B1
 $(\sqrt{12} \times \sqrt{24}) + \frac{\sqrt{150}}{\sqrt{3}} - \frac{36}{\sqrt{2}} = -\sqrt{2}$ (c.a.o.) B1

3. (a) $\frac{dy}{dx} = 2x - 8$
 (an attempt to differentiate, at least one non-zero term correct) M1
 An attempt to substitute $x = 6$ in candidate's expression for $\frac{dy}{dx}$ m1
 Value of $\frac{dy}{dx}$ at $P = 4$ (c.a.o.) A1
 Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1
 Equation of normal to C at P : $y - 2 = -\frac{1}{4}(x - 6)$ (or equivalent) (f.t. candidate's value for $\frac{dy}{dx}$ provided M1 and both m1's awarded) A1
- (b) Putting candidate's expression for $\frac{dy}{dx} = 2$ M1
 x -coordinate of $Q = 5$ A1
 y -coordinate of $Q = -1$ A1
 $c = -11$ A1
 (f.t. candidate's expression for $\frac{dy}{dx}$ and at most one error in the enumeration of the coordinates of Q for all three A marks provided both M1's are awarded)
4. (a) $(1 + x)^6 = 1 + 6x + 15x^2 + 20x^3 + \dots$
 All terms correct B2
If B2 not awarded, award B1 for three correct terms
- (b) An attempt to substitute $x = 0.1$ in the expansion of part (a) (f.t. candidate's coefficients from part (a)) M1
 $1.1^6 \approx 1 + 6 \times 0.1 + 15 \times 0.01 + 20 \times 0.001$
 (At least three terms correct, f.t. candidate's coefficients from part (a)) A1
 $1.1^6 \approx 1.77$ (c.a.o.) A1
5. (a) $a = 4$ B1
 $b = -1$ B1
 $c = 7$ B1
- (b) An attempt to substitute 1 for x in an appropriate quadratic expression (f.t. candidate's value for b) M1
 Greatest value of $\frac{1}{4x^2 - 8x + 29} = \frac{1}{25}$ (c.a.o.) A1

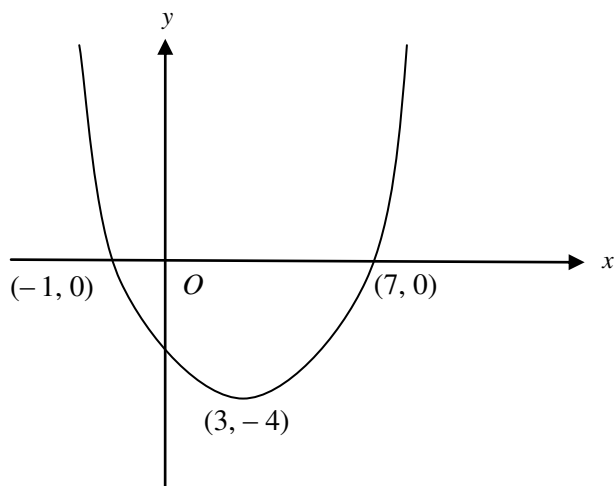
6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1
 $b^2 - 4ac = (2k)^2 - 4 \times (k - 1) \times (7k - 4)$ A1
 Putting $b^2 - 4ac < 0$ m1
 $6k^2 - 11k + 4 > 0$ (convincing) A1
 Finding critical values $k = 1/2, k = 4/3$ B1
 A statement (mathematical or otherwise) to the effect that
 $k < 1/2$ or $k > 4/3$ (or equivalent) (f.t. candidate's derived critical values) B2
 Deduct 1 mark for each of the following errors
 the use of non-strict inequalities
 the use of the word 'and' instead of the word 'or'
7. (a) $y + \delta y = -3(x + \delta x)^2 + 8(x + \delta x) - 7$ B1
 Subtracting y from above to find δy M1
 $\delta y = -6x\delta x - 3(\delta x)^2 + 8\delta x$ A1
 Dividing by δx and letting $\delta x \rightarrow 0$ M1
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = -6x + 8$ (c.a.o.) A1
- (b) $\frac{dy}{dx} = 9 \times \frac{5}{4} \times x^{1/4} - 8 \times -\frac{1}{3} \times x^{-4/3}$ B1, B1
8. **Either:** showing that $f(2) = 0$
Or: trying to find $f(r)$ for at least two values of r M1
 $f(2) = 0 \Rightarrow x - 2$ is a factor A1
 $f(x) = (x - 2)(6x^2 + ax + b)$ with one of a, b correct M1
 $f(x) = (x - 2)(6x^2 - x - 2)$ A1
 $f(x) = (x - 2)(3x - 2)(2x + 1)$ (f.t. only $6x^2 + x - 2$ in above line) A1
 $x = 2, 2/3, -1/2$ (f.t. for factors $3x \pm 2, 2x \pm 1$) A1
Special case
 Candidates who, after having found $x - 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are awarded B1 for final 4 marks

9. (a) (i)



Concave down curve with y-coordinate of maximum = 2 B1
 x-coordinate of maximum = -1 B1
 Both points of intersection with x-axis B1

(ii)



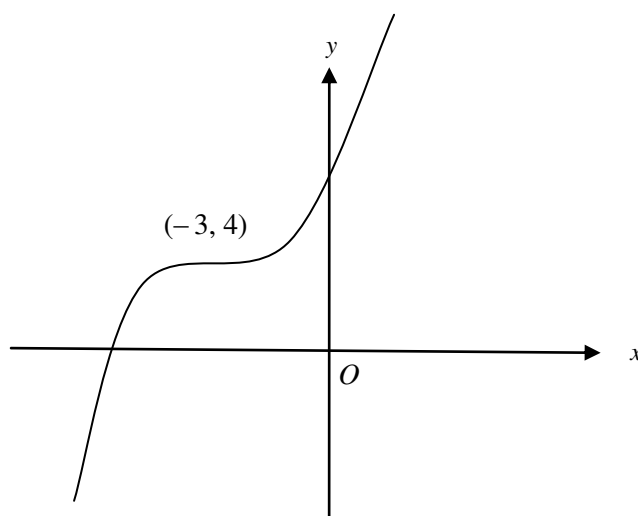
Concave up curve with x-coordinate of minimum = 3 B1
 y-coordinate of minimum = -4 B1
 Both points of intersection with x-axis B1

(b) $x = 3$ (c.a.o.) B1

10. (a) $\frac{dy}{dx} = 3x^2 + 18x + 27$ B1
 Putting derived $\frac{dy}{dx} = 0$ M1
 $3(x + 3)^2 = 0 \Rightarrow x = -3$ (c.a.o) A1
 $x = -3 \Rightarrow y = 4$ (c.a.o) A1

(b) **Either:**
 An attempt to consider value of $\frac{dy}{dx}$ at $x = -3^-$ and $x = -3^+$ M1
 $\frac{dy}{dx}$ has same sign at $x = -3^-$ and $x = -3^+ \Rightarrow (-3, 4)$ is a point of inflection A1
Or:
 An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = -3$, $x = -3^-$ and $x = -3^+$ M1
 $\frac{d^2y}{dx^2} = 0$ at $x = -3$ and $\frac{d^2y}{dx^2}$ has different signs at $x = -3^-$ and $x = -3^+$
 $\Rightarrow (-3, 4)$ is a point of inflection A1
Or:
 An attempt to find the value of y at $x = -3^-$ and $x = -3^+$ M1
 Value of y at $x = -3^- < 4$ and value of y at $x = -3^+ > 4 \Rightarrow (-3, 4)$ is a point of inflection A1
Or:
 An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = -3$ M1
 $\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = -3 \Rightarrow (-3, 4)$ is a point of inflection A1

(c)



G1

C2

1. (a)
- | | | |
|-----|-------------|-----------------------|
| 1 | 0.301029995 | |
| 1.5 | 0.544068044 | |
| 2 | 0.698970004 | |
| 2.5 | 0.812913356 | |
| 3 | 0.903089987 | (5 values correct) B2 |
- (If B2 not awarded, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0.5$ M1

$$I \approx \frac{0.5}{2} \times \{0.301029995 + 0.903089987 + 2(0.544068044 + 0.698970004 + 0.812913356)\}$$

$$I \approx 5.31602279 \times 0.5 \div 2$$

$$I \approx 1.329005698$$

$$I \approx 1.329 \quad (\text{f.t. one slip}) \quad \text{A1}$$

Note: Answer only with no working earns 0 marks

Special case for candidates who put $h = 0.4$

- | | | |
|-----|-------------|-------------------------|
| 1 | 0.301029995 | |
| 1.4 | 0.505149978 | |
| 1.8 | 0.643452676 | |
| 2.2 | 0.748188027 | |
| 2.6 | 0.832508912 | |
| 3 | 0.903089987 | (all values correct) B1 |

Correct formula with $h = 0.4$ M1

$$I \approx \frac{0.4}{2} \times \{0.301029995 + 0.903089987 + 2(0.505149978 + 0.643452676 + 0.748188027 + 0.832508912)\}$$

$$I \approx 6.662719168 \times 0.4 \div 2$$

$$I \approx 1.332543834$$

$$I \approx 1.333 \quad (\text{f.t. one slip}) \quad \text{A1}$$

Note: Answer only with no working earns 0 marks

- (b)
- $$\int_1^3 \log_{10}(3x-1)^2 dx \approx 2.658 \quad (\text{f.t. candidate's answer to (a)}) \quad \text{B1}$$

2. (a) $4 \cos^2 \theta + 1 = 4(1 - \cos^2 \theta) - 2 \cos \theta$ (correct use of $\sin^2 \theta = 1 - \cos^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation in $\cos \theta$, either by using the quadratic formula or by getting the expression into the form $(a \cos \theta + b)(c \cos \theta + d)$, with $a \times c =$ candidate's coefficient of $\cos^2 \theta$ and $b \times d =$ candidate's constant m1
 $8 \cos^2 \theta + 2 \cos \theta - 3 = 0 \Rightarrow (2 \cos \theta - 1)(4 \cos \theta + 3) = 0$
 $\Rightarrow \cos \theta = \frac{1}{2}, \cos \theta = -\frac{3}{4}$ (c.a.o.) A1
 $\theta = 60^\circ, 300^\circ$ B1
 $\theta = 138.59^\circ, 221.41^\circ$ B1 B1
 Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
 $\cos \theta = +, -, \text{f.t. for 3 marks, } \cos \theta = -, -, \text{f.t. for 2 marks}$
 $\cos \theta = +, +, \text{f.t. for 1 mark}$
- (b) $\alpha + 40^\circ = 45^\circ, 135^\circ, \Rightarrow \alpha = 5^\circ, 95^\circ$ (at least one value of α) B1
 $\alpha - 35^\circ = 60^\circ, 120^\circ, \Rightarrow \alpha = 95^\circ, 155^\circ$ (at least one value of α) B1
 $\alpha = 95^\circ$ (c.a.o.) B1
- (c) Correct use of $\frac{\sin \phi}{\cos \phi} = \tan \phi$ (o.e.) M1
 $\tan \phi = \frac{10}{7}$ A1
 $\phi = 55^\circ, 235^\circ$ (f.t. $\tan \phi = a$) B1
3. (a) $\frac{y}{4/5} = \frac{x}{8/17}$ (o.e.) (correct use of sine rule) M1
 $y = 1.7x$ (convincing) A1
- (b) $10 \cdot 5^2 = x^2 + y^2 - 2 \times x \times y \times (-^{13}/_{85})$ (correct use of the cosine rule) M1
 Substituting $1.7x$ for y in candidate's equation of form
 $10 \cdot 5^2 = x^2 + y^2 \pm 2 \times x \times y \times ^{13}/_{85}$ M1
 $10 \cdot 5^2 = x^2 + 2.89x^2 + 0.52x^2$ (o.e.) A1
 $x = 5$
 (f.t. candidate's equation for x^2 provided both M's awarded) A1

4. (a) $S_n = a + [a + d] + \dots + [a + (n - 1)d]$ (at least 3 terms, one at each end) B1
 $S_n = [a + (n - 1)d] + [a + (n - 2)d] + \dots + a$
 In order to make further progress, the two expressions for S_n must contain at least three pairs of terms, including the first pair, the last pair and one other pair of terms
 Either:
 $2S_n = [a + a + (n - 1)d] + [a + a + (n - 1)d] + \dots + [a + a + (n - 1)d]$
 Or:
 $2S_n = [a + a + (n - 1)d] \quad n \text{ times} \quad \text{M1}$
 $2S_n = n[2a + (n - 1)d]$
 $S_n = \frac{n[2a + (n - 1)d]}{2} \quad (\text{convincing}) \quad \text{A1}$
- (b) $\frac{n[2 \times 3 + (n - 1) \times 2]}{2} = 360 \quad \text{M1}$
 Rewriting above equation in a form ready to be solved
 $2n^2 + 4n - 720 = 0$ or $n^2 + 2n - 360 = 0$ or $n(n + 2) = 360 \quad \text{A1}$
 $n = 18 \quad (\text{c.a.o.}) \quad \text{A1}$
- (c) $a + 9d = 7 \times (a + 2d) \quad \text{B1}$
 $a + 7d + a + 8d = 80 \quad \text{B1}$
 An attempt to solve the candidate's linear equations simultaneously by eliminating one unknown M1
 $a = -5, d = 6$ (both values) (c.a.o.) A1
5. (a) $ar + ar^2 = -216 \quad \text{B1}$
 $ar^4 + ar^5 = 8 \quad \text{B1}$
 A correct method for solving the candidate's equations simultaneously e.g. multiplying the first equation by r^3 and subtracting
 or eliminating a and $(1 + r) \quad \text{M1}$
 $-216r^3 = 8 \quad (\text{o.e.}) \quad \text{A1}$
 $r = -\frac{1}{3} \quad (\text{convincing}) \quad \text{A1}$
- (b) $a \times (-\frac{1}{3}) \times (1 - \frac{1}{3}) = -216 \Rightarrow a = 972 \quad \text{B1}$
 $S_\infty = \frac{972}{1 - (-\frac{1}{3})} \quad (\text{correct use of formula for } S_\infty, \text{ f.t. candidate's derived value for } a) \quad \text{M1}$
 $S_\infty = 729 \quad (\text{f.t. candidate's derived value for } a) \quad \text{A1}$

6. (a) $5 \times \frac{x^{1/4}}{1/4} - 7 \times \frac{x^{3/2}}{3/2} + c$ B1, B1
(-1 if no constant term present)

- (b) (i) $16 - x^2 = x + 10$ M1
An attempt to rewrite and solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b =$ candidate's constant m1
 $(x - 2)(x + 3) = 0 \Rightarrow x = 2, -3$ (both values, c.a.o.) A1
 $y = 12, y = 7$ (both values, f.t. candidate's x -values) A1
- (ii) Use of integration to find the area under the curve M1
 $\int 16 dx = 16x, \int x^2 dx = (1/3)x^3$, (correct integration) B1
Correct method of substitution of candidate's limits m1
 $[16x - (1/3)x^3]_{-3}^2 = (32 - 8/3) - (-48 - (-9)) = 205/3$
Use of a correct method to find the area of the trapezium (f.t. candidate's coordinates for A, B) M1
Use of candidate's values for x_A and x_B as limits and trying to find total area by subtracting area of trapezium from area under curve m1
Shaded area $= 205/3 - 95/2 = 125/6$ (c.a.o.) A1

7. (a) **Either:**
 $(5x/4 - 2) \log_{10} 3 = \log_{10} 7$
(taking logs on both sides and using the power law) M1
 $\frac{5x}{4} = \frac{(\log_{10} 7 + 2 \log_{10} 3)}{\log_{10} 3}$ A1
 $x = 3.017$ (f.t. one slip, see below) A1
- Or:**
 $5x/4 - 2 = \log_3 7$ (rewriting as a log equation) M1
 $5x/4 = \log_3 7 + 2$ A1
 $x = 3.017$ (f.t. one slip, see below) A1
Note: an answer of $x = -0.183$ from $\frac{5x}{4} = \frac{(\log_{10} 7 - 2 \log_{10} 3)}{\log_{10} 3}$
earns M1 A0 A1
an answer of $x = 0.183$ from $\frac{5x}{4} = \frac{(2 \log_{10} 3 - \log_{10} 7)}{\log_{10} 3}$
earns M1 A0 A1

Note: Answer only with no working earns 0 marks

- (b) (i) $b = a^5$ (relationship between log and power) B1
(ii) $a = b^{1/5}$ (the laws of indices) B1
 $\log_b a = 1/5$ (relationship between log and power) B1

8. (a) (i) A correct method for finding the length of AB M1
 $AB = 20$ A1
Sum of radii = distance between centres,
 \therefore circles touch A1
- (ii) Gradient $AP(BP)(AB) = \frac{\text{inc in } y}{\text{inc in } x}$ M1
Gradient $AP = \frac{9-5}{-2-1} = -\frac{4}{3}$ (o.e) A1
Use of $m_{\text{tan}} \times m_{\text{rad}} = -1$ M1
Equation of common tangent is:
 $y - 5 = \frac{3}{4}(x - 1)$ (o.e)
4
(f.t. one slip provided both M's are awarded) A1
- (b) **Either:**
An attempt to rewrite the equation of C with l.h.s. in the form
 $(x - a)^2 + (y - b)^2$ M1
 $(x + 2)^2 + (y - 3)^2 = -7$ A1
Impossible, since r.h.s. must be positive ($= r^2$) A1
Or:
 $g = 2, f = -3, c = 20$ and an attempt to use $r^2 = g^2 + f^2 - c$ M1
 $r^2 = -7$ A1
Impossible, since r^2 must be positive A1
9. (a) (i) Area of sector $POQ = \frac{1}{2} \times r^2 \times 0.9$ B1
(ii) Length of $PS = r \times \tan(0.9)$ B1
(iii) Area of triangle $POS = \frac{1}{2} \times r \times r \times \tan(0.9)$
(f.t. candidate's expression in r for the length of PS) B1
- (b) $\frac{1}{2} \times r \times r \times \tan(0.9) - \frac{1}{2} \times r^2 \times 0.9 = 95.22$
(f.t. candidate's expressions for area of sector and area of triangle,
at least one correct) M1
 $r^2 = \frac{2 \times 95.22}{(1.26 - 0.9)}$ (o.e.) (c.a.o.) A1
 $r = 23$ (f.t. one numerical slip) A1

C3

1. (a)

0	2.197224577
0.75	2.314217179
1.5	2.524262696
2.25	2.861499826
3	3.335254744

 (5 values correct) B2
(If B2 not awarded, award B1 for either 3 or 4 values correct)
Correct formula with $h = 0.75$ M1
$$I \approx \frac{0.75}{3} \times \{2.197224577 + 3.335254744 + 4(2.314217179 + 2.861499826) + 2(2.524262696)\}$$
$$I \approx 31.28387273 \times 0.75 \div 3$$
$$I \approx 7.820968183$$
$$I \approx 7.82 \quad \text{(f.t. one slip)} \quad \text{A1}$$

Note: Answer only with no working shown earns 0 marks

- (b)
$$\int_0^3 \ln(16 + 2e^x) dx = \int_0^3 \ln(8 + e^x) dx + \int_0^3 \ln 2 dx \quad \text{M1}$$
$$\int_0^3 \ln(16 + 2e^x) dx = 7.82 + 2.08 = 9.90 \quad \text{(f.t. candidate's answer to (a))} \quad \text{A1}$$

Note: Answer only with no working shown earns 0 marks

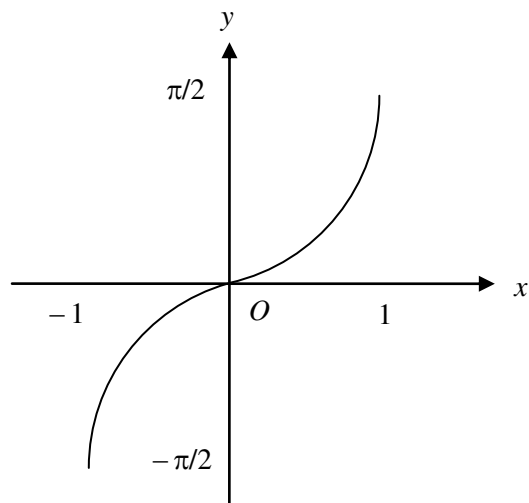
2. $8(\sec^2 \theta - 1) - 5 \sec^2 \theta = 7 + 4 \sec \theta$. (correct use of $\tan^2 \theta = \sec^2 \theta - 1$) M1
An attempt to collect terms, form and solve quadratic equation in $\sec \theta$, either by using the quadratic formula or by getting the expression into the form $(a \sec \theta + b)(c \sec \theta + d)$, with $a \times c =$ candidate's coefficient of $\sec^2 \theta$ and $b \times d =$ candidate's constant m1
 $3 \sec^2 \theta - 4 \sec \theta - 15 = 0 \Rightarrow (3 \sec \theta + 5)(\sec \theta - 3) = 0$
 $\Rightarrow \sec \theta = -\frac{5}{3}, \sec \theta = 3$
 $\Rightarrow \cos \theta = -\frac{3}{5}, \cos \theta = \frac{1}{3} \quad \text{(c.a.o.)} \quad \text{A1}$
 $\theta = 126.87^\circ, 233.13^\circ \quad \text{B1 B1}$
 $\theta = 70.53^\circ, 289.47^\circ \quad \text{B1}$

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
 $\cos \theta = +, -, \text{f.t. for 3 marks, } \cos \theta = -, -, \text{f.t. for 2 marks}$
 $\cos \theta = +, +, \text{f.t. for 1 mark}$

3. (a) $\frac{d(y^4)}{dx} = 4y^3 \frac{dy}{dx}$ B1
 $\frac{d(8xy^2)}{dx} = (8x)(2y)\frac{dy}{dx} + 8y^2$ B1
 $\frac{d(2x^2)}{dx} = 4x, \frac{d(9)}{dx} = 0$ B1
 $\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$ (convincing) (c.a.o.) B1
- (b) $\frac{dy}{dx} = 0 \Rightarrow x = 2y^2$ B1
Substitute $2y^2$ for x in equation of C M1
 $9y^4 + 9 = 0$ (o.e.) (c.a.o.) A1
 $9y^4 + 9 > 0$ for any real y (o.e.) and thus no such point exists A1
4. candidate's x -derivative $= 2e^t$ B1
candidate's y -derivative $= -8e^{-t} + 3e^t$ B1
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{-8e^{-t} + 3e^t}{2e^t}$ (o.e.) (c.a.o.) A1
Putting candidate's $\frac{dy}{dx} = -1$, rearranging and obtaining either an equation in
both e^t and e^{-t} , or an equation in e^{2t} , or an equation in e^{-2t} . M1
Either $e^{2t} = \frac{8}{5}$ or $e^{-2t} = \frac{5}{8}$
(f.t. one numerical slip in candidate's derived expression for $\frac{dy}{dx}$) A1
 $t = 0.235$ (c.a.o.) A1
5. (a) $\frac{d[\ln(3x^2 - 2x - 1)]}{dx} = \frac{ax + b}{3x^2 - 2x - 1}$ (including $a = 0, b = 1$) M1
 $\frac{d[\ln(3x^2 - 2x - 1)]}{dx} = \frac{6x - 2}{3x^2 - 2x - 1}$ A1
 $6x - 2 = 8x(3x^2 - 2x - 1)$ (o.e.) (f.t. candidate's a, b) A1
 $12x^3 - 8x^2 - 7x + 1 = 0$ (convincing) A1
- (b) $x_0 = -0.6$
 $x_1 = -0.578232165$ (x_1 correct, at least 4 places after the point) B1
 $x_2 = -0.582586354$
 $x_3 = -0.581770386$
 $x_4 = -0.581925366 = -0.5819$ (x_4 correct to 4 decimal places) B1
Let $g(x) = 12x^3 - 8x^2 - 7x + 1$
An attempt to check values or signs of $g(x)$ at $x = -0.58185$,
 $x = -0.58195$ M1
 $g(-0.58185) = 7.35 \times 10^{-4}, g(-0.58195) = -7.15 \times 10^{-4}$ A1
Change of sign $\Rightarrow \alpha = -0.5819$ correct to four decimal places A1

6. (a) (i) $\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times f(x) \quad (f(x) \neq 1)$ M1
 $\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times (-20x^4)$
 $\frac{dy}{dx} = 5x^4 \times (9 - 4x^5)^{-5/4}$ A1
- (ii) $\frac{dy}{dx} = \frac{(7 - x^3) \times f(x) - (3 + 2x^3) \times g(x)}{(7 - x^3)^2} \quad (f(x), g(x) \neq 1)$ M1
 $\frac{dy}{dx} = \frac{(7 - x^3) \times 6x^2 - (3 + 2x^3) \times (-3x^2)}{(7 - x^3)^2}$ A1
 $\frac{dy}{dx} = \frac{51x^2}{(7 - x^3)^2} \quad (\text{c.a.o.})$ A1

(b) (i)



- (ii) $x = \sin y \Rightarrow \frac{dx}{dy} = \cos y$ B1
 $\frac{dx}{dy} = \pm \sqrt{1 - \sin^2 y}$ B1
The +ive sign is chosen because the graph shows the gradient to be positive E1
 $\frac{dx}{dy} = \sqrt{1 - x^2}$ B1
 $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$ B1

7. (a) (i) $\int \cos(2-5x) dx = k \times \sin(2-5x) + c$ (k = 1, $\frac{1}{5}$, -5, $-\frac{1}{5}$) M1
 $\int \cos(2-5x) dx = -\frac{1}{5} \times \sin(2-5x) + c$ A1
- (ii) $\int \frac{4}{e^{3x-2}} dx = k \times 4 \times e^{2-3x} + c$ (k = 1, -3, $\frac{1}{3}$ - $\frac{1}{3}$) M1
 $\int \frac{4}{e^{3x-2}} dx = -\frac{4}{3} \times e^{2-3x} + c$ A1
- (iii) $\int \frac{5}{\frac{1}{6}x-3} dx = k \times 5 \times \ln|\frac{1}{6}x-3| + c$ (k = 1, $\frac{1}{6}$, 6) M1
 $\int \frac{5}{\frac{1}{6}x-3} dx = 30 \times \ln|\frac{1}{6}x-3| + c$ A1

Note: The omission of the constant of integration is only penalised once.

- (b) $\int (4x+1)^{1/2} dx = k \times \frac{(4x+1)^{3/2}}{3/2}$ (k = 1, 4, $\frac{1}{4}$) M1
 $\int_2^6 (4x+1)^{1/2} dx = \left[\frac{1}{4} \times \frac{(4x+1)^{3/2}}{3/2} \right]_2^6$ A1

A correct method for substitution of limits in an expression of the form $m \times (4x+1)^{3/2}$ M1

$$\int_2^6 (4x+1)^{1/2} dx = \frac{125}{6} - \frac{27}{6} = \frac{98}{6} = 16.33$$

(f.t. only for solutions of $\frac{392}{6}$ and $\frac{1568}{6}$ from $k = 1, 4$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (a) Choice of a, b , with one positive and one negative and one side correctly evaluated M1
Both sides of identity evaluated correctly A1
- (b) Trying to solve $3x-2=7x$ M1
Trying to solve $3x-2=-7x$ M1
 $x = -0.5, x = 0.2$ (both values) (c.a.o.) A1

Alternative mark scheme

$$(3x-2)^2 = 7^2 \times x^2 \quad \text{(squaring both sides)} \quad \text{M1}$$

$$40x^2 + 12x - 4 = 0 \quad \text{(o.e.)} \quad \text{(c.a.o.)} \quad \text{A1}$$

$$x = -0.5, x = 0.2 \quad \text{(both values, f.t. one slip in quadratic)} \quad \text{A1}$$

9. (a) $f(x) = (x - 4)^2 - 9$ B1
- (b) $y = (x - 4)^2 - 9$ and an attempt to isolate x
 (f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) M1
- $x = (\pm)\sqrt{y + 9} + 4$
 (f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
- $x = -\sqrt{y + 9} + 4$ (o.e.) (c.a.o.) A1
- $f^{-1}(x) = -\sqrt{x + 9} + 4$ (o.e.)
- (f.t. only incorrect choice of sign in front of the $\sqrt{}$ sign and candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
10. (a) $R(g) = [2k - 4, \infty)$ B1
- (b) (i) $2k - 4 \geq -2$ M1
 $k \geq 1$ (\Rightarrow least value of k is 1)
 (f.t. candidate's $R(g)$ provided it is of form $[a, \infty)$ A1
- (ii) $fg(x) = (kx - 4)^2 + k(kx - 4) - 8$ B1
- (iii) $(3k - 4)^2 + k(3k - 4) - 8 = 0$
 (substituting 3 for x in candidate's expression for $fg(x)$ and putting equal to 0) M1
- Either $12k^2 - 28k + 8 = 0$ or $6k^2 - 14k + 4 = 0$
 or $3k^2 - 7k + 2 = 0$ (c.a.o.) A1
- $k = \frac{1}{3}, 2$ (f.t. candidate's quadratic in k) A1
- $k = 2$ (c.a.o.) A1

C4

$$1. \quad 9x^2 - 5x \times 2y \frac{dy}{dx} - 5y^2 + 8y^3 \frac{dy}{dx} = 0 \quad \left[\begin{array}{l} -5x \times 2y \frac{dy}{dx} - 5y^2 \\ \frac{9x^2 + 8y^3 \frac{dy}{dx}}{dx} \end{array} \right] \quad \text{B1}$$

Either $\frac{dy}{dx} = \frac{9x^2 - 5y^2}{10xy - 8y^3}$ **or** $\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 2$ in candidate's expression **and** the use of $\text{grad}_{\text{normal}} \times \text{grad}_{\text{tangent}} = -1$ M1

Equation of normal: $y - 2 = -4(x - 1)$ $\left[\begin{array}{l} \text{f.t. candidate's value for } \frac{dy}{dx} \\ dx \end{array} \right] \quad \text{A1}$

$$2. \quad (a) \quad f(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-4)} \quad (\text{correct form}) \quad \text{M1}$$

$$5x^2 + 7x + 17 \equiv A(x-4) + B(x+1)(x-4) + C(x+1)^2$$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3, C = 5, B = 0$ (all three coefficients correct) A2
(If A2 not awarded, award A1 for either 1 or 2 correct coefficients)

$$(b) \quad \frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \frac{2}{(x+1)^2}$$

$$\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{-1}{(x+1)^2} + \frac{5}{(x-4)}$$

(f.t. candidates values for A, B, C) A1

3. (a) $\frac{2 \tan x}{1 - \tan^2 x} = 3 \cot x$ (correct use of formula for $\tan 2x$) M1
- $\frac{2 \tan x}{1 - \tan^2 x} = \frac{3}{\tan x}$ (correct use of $\cot x = \frac{1}{\tan x}$) M1
- $\tan^2 x = \frac{3}{5}$ (o.e.) A1
- $x = 37.76^\circ, 142.24^\circ$ (both values)
- (f.t. $a \tan^2 x = b$ provided both M1's are awarded) A1
- (b) (i) $R = 29$ B1
- Correctly expanding $\sin(\theta - \alpha)$ and using either $29 \cos \alpha = 21$
or $29 \sin \alpha = 20$ **or** $\tan \alpha = \frac{20}{21}$ to find α
- (f.t. candidate's value for R) M1
- $\alpha = 43.6^\circ$ (c.a.o.) A1
- (ii) Greatest value of $\frac{1}{21 \sin \theta - 20 \cos \theta + 31} = \frac{1}{29 \times (\pm 1) + 31}$
- (f.t. candidate's value for R) M1
- Greatest value = $\frac{1}{2}$ (f.t. candidate's value for R) A1
- Corresponding value for $\theta = 313.6^\circ$ (o.e.)
- (f.t. candidate's value for α) A1
4. Volume = $\pi \int_0^{\pi/4} (3 + 2 \sin x)^2 dx$ B1
- Correct use of $\sin^2 x = \frac{(1 - \cos 2x)}{2}$ M1
- Integrand = $(9 + 2 + 12 \sin x - 2 \cos 2x)$ (c.a.o.) A1
- $\int (a + b \sin x + c \cos 2x) dx = (ax - b \cos x + \frac{c}{2} \sin 2x)$ (a $\neq 0$, b $\neq 0$, c $\neq 0$) B1
- Correct substitution of correct limits in candidate's integrated expression
of form $(ax - b \cos x + \frac{c}{2} \sin 2x)$ (a $\neq 0$, c $\neq 0$) M1
- Volume = 35 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. $(1 - 2x)^{1/2} = 1 + (1/2) \times (-2x) + \frac{(1/2) \times (1/2 - 1) \times (-2x)^2}{1 \times 2} + \dots$
 (–1 each incorrect term) B2
- $\frac{1}{1 + 4x} = 1 + (-1) \times (4x) + \frac{(-1) \times (-2) \times (4x)^2}{1 \times 2} + \dots$
 (–1 each incorrect term) B2
- $6\sqrt{1 - 2x} - \frac{1}{1 + 4x} = 5 - 2x - 19x^2 + \dots$
 (–1 each incorrect term) B2
- Expansion valid for $|x| < 1/4$ (o.e.) B1
6. (a) candidate's x -derivative = 2
 candidate's y -derivative = $15t^2$ (at least one term correct)
 and use of
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{15t^2}{2}$ (o.e.) (c.a.o.) A1
 Equation of tangent at P : $y - 5p^3 = \frac{15p^2}{2}(x - 2p)$
 (f.t. candidate's expression for $\frac{dy}{dx}$) m1
 $2y = 15p^2x - 20p^3$ (convincing) A1
- (b) Substituting $p = 1, x = 2q, y = 5q^3$ in equation of tangent M1
 $q^3 - 3q + 2 = 0$ (convincing) A1
 Putting $f(q) = q^3 - 3q + 2$
Either $f(q) = (q - 1)(q^2 + q - 2)$ **or** $f(q) = (q + 2)(q^2 - 2q + 1)$ M1
Either $f(q) = (q - 1)(q - 1)(q + 2)$ **or** $q = 1, q = -2$ A1
 $q = -2$ A1

7. (a) $u = \ln 2x \Rightarrow du = 2 \times \frac{1}{2x} dx$ (o.e.) B1
- $dv = x^4 dx \Rightarrow v = \frac{1}{5} x^5$ (o.e.) B1
- $\int x^4 \ln 2x dx = \ln 2x \times \frac{1}{5} x^5 - \int \frac{1}{5} x^5 \times \frac{1}{x} dx$ (o.e.) M1
- $\int x^4 \ln 2x dx = \ln 2x \times \frac{1}{5} x^5 - \frac{1}{25} x^5 + c$ (c.a.o.) A1
- (b) $\int \sqrt[3]{(10 \cos x - 1) \sin x} dx = \int k \times u^{1/2} du \quad (k = -1/10, 1/10 \text{ or } \pm 10)$ M1
- $\int a \times u^{1/2} du = a \times \frac{u^{3/2}}{3/2}$ B1
- $\int_0^{\pi/3} \sqrt[3]{(10 \cos x - 1) \sin x} dx = k \left[\frac{u^{3/2}}{3/2} \right]_9^4 \text{ or } k \left[\frac{(10 \cos x - 1)^{3/2}}{3/2} \right]_0^{\pi/3}$ B1
- $\int_0^{\pi/3} \sqrt[3]{(10 \cos x - 1) \sin x} dx = \frac{19}{15} = 1.27$ (c.a.o.) A1
8. (a) $\frac{dV}{dt} = kV$ B1
- (b) $\int \frac{dV}{V} = \int k dt$ M1
- $\ln V = kt + c$ A1
- $V = e^{kt+c} = Ae^{kt}$ (convincing) A1
- (c) (i) $292 = Ae^{2k}$
- $637 = Ae^{28k}$ (both values) B1
- Dividing to eliminate A M1
- $\frac{637}{292} = e^{26k}$ A1
- $k = \frac{1}{26} \ln \left[\frac{637}{292} \right] = 0.03$ A1
- (ii) $A = 275$ B1
- (iii) When $t = 0$, initial value of investment = £275
- (f.t. candidate's derived value for A) B1

9. (a) $\mathbf{p} \cdot \mathbf{q} = -18$ B1
 $|\mathbf{p}| = \sqrt{14}, |\mathbf{q}| = \sqrt{105}$ (at least one correct) B1
 Correctly substituting candidate's derived values in the formula
 $\mathbf{p} \cdot \mathbf{q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos \theta$ M1
 $\theta = 118^\circ$ (c.a.o.) A1
- (b) (i) Use of $\mathbf{CD} = \mathbf{CO} + \mathbf{OD}$ and the fact that $\mathbf{OC} = \frac{1}{2}\mathbf{b}$ and
 $\mathbf{OD} = 2\mathbf{a}$, leading to printed answer $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$ (convincing) B1
 Use of $\frac{1}{2}\mathbf{b} + \lambda\mathbf{CD}$ (o.e.) to find vector equation of CD M1
 Vector equation of CD : $\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}$ (convincing) A1
- (ii) **Either:**
 Either substituting $\frac{1}{3}$ for λ in the vector equation of CD
 or substituting 2 for μ in the vector equation of L M1
 At least one of these position vectors = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1
 Both position vectors = $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \Rightarrow$ this must be the position
 vector of the point of intersection E A1
Or:
 $2\lambda = \frac{\mu}{3}$
 $\frac{1}{2}(1 - \lambda) = \frac{1}{3}(\mu - 1)$
 (comparing candidate's coefficients of \mathbf{a} and \mathbf{b} and an attempt to solve) M1
 $\lambda = \frac{1}{3}$ or $\mu = 2$ A1
 $\mathbf{OE} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (convincing) A1
- (iii) **Either:** E lies on AB and is such that $AE : EB = 1 : 2$ (o.e.)
Or: E is the point of intersection of AB and CD B1
10. Squaring both sides we have
 $1 + 2 \sin \theta \cos \theta > 2$ B1
 $\sin 2\theta > 1$ B1
 Contradiction, since the sine of any angle ≤ 1 B1

FP1

Ques	Solution	Mark	Notes
1(a)	$f(x+h) - f(x) = \frac{1}{(x+h)^2} - \frac{1}{x^2}$ $= \frac{x^2 - (x+h)^2}{x^2(x+h)^2}$ $= \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2}$ $= \frac{-2xh - h^2}{x^2(x+h)^2}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2xh - h^2}{hx^2(x+h)^2} = -\frac{2}{x^3}$	<p>M1A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	
(b)	$\ln f(x) = x \ln \sec x$ $\frac{f'(x)}{f(x)} = \ln \sec x + \frac{x \sec x \tan x}{\sec x}$ $f'(x) = (\sec x)^x (\ln \sec x + x \tan x)$	<p>B1</p> <p>B1B1</p> <p>B1</p>	B1 each side
2(a)	$S_n = \sum_{r=1}^n r(r+3) = \sum_{r=1}^n r^2 + \sum_{r=1}^n 3r$ $= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}$ $= \frac{n(n+1)}{6} (2n+1+9)$ $= \frac{n(n+1)(n+5)}{3} \text{ or } \frac{n^3 + 6n^2 + 5n}{3} \text{ oe}$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	
(b)	$T_n = S_n - S_{n-1}$ $= n(n+3) - (n-1)(n+2)$ $= n^2 + 3n - (n^2 + n - 2)$ $= 2(n+1)$	<p>M1</p> <p>A1</p> <p>A1</p>	

Ques	Solution	Mark	Notes
3(a)	$x + 2y + 4z = 3$ $x - y + 2z = 4$ $4x - y + 10z = k$ <p>Attempting to use row operations,</p> $x + 2y + 4z = 3$ $3y + 2z = -1$ $9y + 6z = 12 - k$ <p>Since the 3rd equation is three times the 2nd equation, it follows that</p> $12 - k = -3 ; k = 15$	M1	
(b)	<p>Let $z = \alpha$</p> $y = -\frac{(1 + 2\alpha)}{3}$ $x = \frac{11 - 8\alpha}{3}$ <p>(or equivalent)</p>	A1 A1 M1 A1 M1 A1 A1	
4	<p>EITHER</p> $z = \frac{1+2i}{1-i} \times \frac{1+i}{1+i}$ $= \frac{1+2i+i+2i^2}{1-i+i-i^2}$ $= \frac{-1+3i}{2}$ $\text{Mod}(z) = \frac{\sqrt{10}}{2} (\sqrt{2.5}, 1.58)$ $\text{Arg}(z) = \tan^{-1}(-3) + \pi$ $= 1.89 (108^\circ)$ <p>OR</p> $\text{Mod}(1 + 2i) = \sqrt{5}$ $\text{Mod}(1 - i) = \sqrt{2}$ $\text{Mod}\left(\frac{1+2i}{1-i}\right) = \sqrt{\frac{5}{2}}$ $\text{Arg}(1 + 2i) = \tan^{-1} 2 = 1.107..$ $\text{Arg}(1 - i) = \tan^{-1}(-1) = -0.785..$ $\text{Arg}\left(\frac{1+2i}{1-i}\right) = 1.107.. + 0.785..$ $= 1.89 (108^\circ)$	M1 A1 A1 B1 M1A1 B1 B1 B1 B1 B1 B1	FT their z Award M1A0 for $\tan^{-1}(-3)$ (- 1.25 or - 72°) FT one incorrect mod FT one incorrect arg

Ques	Solution	Mark	Notes
5(a)	$\alpha + \beta + \gamma = -2, \beta\gamma + \gamma\alpha + \alpha\beta = 2, \alpha\beta\gamma = -3$ $\beta\gamma \times \gamma\alpha + \beta\gamma \times \alpha\beta + \gamma\alpha \times \alpha\beta = \alpha\beta\gamma(\alpha + \beta + \gamma)$ $= -3 \times -2 = 6$ $\beta\gamma \times \gamma\alpha \times \alpha\beta = (\alpha\beta\gamma)^2 = 9$ The required equation is $x^3 - 2x^2 + 6x - 9 = 0$	B1 M1 A1 M1A1 B1	FT their first line if one error FT previous values
(b)	$\alpha^2 + \beta^2 + \gamma^2$ $= (\alpha + \beta + \gamma)^2 - 2(\beta\gamma + \gamma\alpha + \alpha\beta)$ $= 4 - 2 \times 2 = 0$ (convincing) The equation has 1 real root Any valid reason, eg cubic equations have either 1 or 3 real roots and since $\alpha^2 + \beta^2 + \gamma^2 = 0$, not all roots are real	M1 A1 B1 B1	
6(a)	$\text{Det}(A) = \lambda(2 - \lambda) + 2 \times 4 + 3(-\lambda - 2)$ $= -\lambda^2 - \lambda + 2$ A is singular when $-\lambda^2 - \lambda + 2 = 0$ $\lambda = 1, -2$	M1 A1 M1 A1	
(b)(i)	$A = \begin{bmatrix} -1 & 2 & 3 \\ -1 & 1 & 1 \\ 2 & -1 & 2 \end{bmatrix}$ Cofactor matrix = $\begin{bmatrix} 3 & 4 & -1 \\ -7 & -8 & 3 \\ -1 & -2 & 1 \end{bmatrix}$ si	M1A1	Award M1 if at least 5 cofactors are correct
(ii)	Adjugate matrix = $\begin{bmatrix} 3 & -7 & -1 \\ 4 & -8 & -2 \\ -1 & 3 & 1 \end{bmatrix}$ Determinant = 2 Inverse matrix = $\frac{1}{2} \begin{bmatrix} 3 & -7 & -1 \\ 4 & -8 & -2 \\ -1 & 3 & 1 \end{bmatrix}$	A1 B1 B1	No FT on cofactor matrix FT the adjugate or determinant

Ques	Solution	Mark	Notes
7(a)	<p>Rotation matrix = $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$</p> <p>Translation matrix = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$</p> <p>Ref matrix in y-axis = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$</p> <p> $T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$ </p> <p> $\begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ </p> <p> $= \begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ </p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	
(b)	<p>EITHER</p> <p>The general point on the line is given by $(\lambda, 2\lambda + 1)$</p> <p>Consider</p> <p> $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda \\ 2\lambda + 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2\lambda - 2 \\ -\lambda + 2 \\ 1 \end{bmatrix}$ </p> <p>$x = -2\lambda - 2; y = -\lambda + 2$</p> <p>Eliminating λ,</p> <p>$x - 2y + 6 = 0$ oe</p> <p>OR</p> <p>Consider</p> <p> $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$ </p> <p>$-y - 1 = X, -x + 2 = Y$</p> <p>$y = -1 - X, x = 2 - Y$</p> <p>$y = 2x + 1$ leading to $x - 2y + 6 = 0$</p>	<p>M1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	

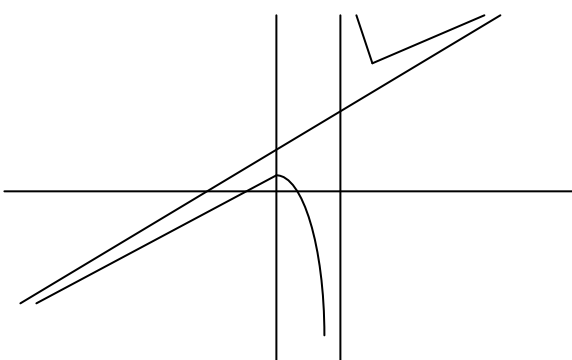
Ques	Solution	Mark	Notes
8	<p>Putting $n = 1$, the formula gives 1 which is the first term of the series so the result is true for $n = 1$.</p> <p>Assume formula is true for $n = k$, ie</p> $\left(\sum_{r=1}^k r \times 2^{r-1} = 1 + 2^k(k-1) \right)$ <p>Consider, for $n = k + 1$,</p> $\sum_{r=1}^{k+1} r \times 2^{r-1} = \sum_{r=1}^k r \times 2^{r-1} + 2^k(k+1)$ $= 1 + 2^k(k-1) + 2^k(k+1)$ $= 1 + 2^{k+1}k$ <p>Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$, the result is proved by induction.</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>Award the final A1 only if a correct conclusion is made and the proof is correctly laid out</p>
9(a)	$u + iv = (x + iy)(x - 1 + iy)$ $= x(x-1) - y^2 + i(xy + xy - y)$ <p>Equating real and imaginary parts,</p> $u = x(x-1) - y^2$ $v = y(2x-1)$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	
(b)	<p>Putting $y = -x$,</p> $u = x(x-1) - x^2 = -x$ $v = -x(2x-1)$ <p>Eliminating x,</p> $v = u(-2u-1) \quad \text{cao (oe)}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>FT their expressions from (a)</p>

FP2

Ques	Solution	Mark	Notes
1(a) (b)	$f(-x) = \frac{((-x)^2 + 1)}{-x((-x)^2 + 2)} = -f(x)$ <p>Therefore f is odd.</p> <p>Let</p> $\frac{x^2 + 1}{x(x^2 + 2)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2} = \frac{A(x^2 + 2) + x(Bx + C)}{x(x^2 + 2)}$ $A = \frac{1}{2}; B = \frac{1}{2}; C = 0$ $\left(\frac{x^2 + 1}{x(x^2 + 2)} = \frac{1}{2x} + \frac{x}{2(x^2 + 2)} \right)$	M1A1 A1 M1 A1A1A1	
2	$u = \sin^2 x \Rightarrow du = 2 \sin x \cos x dx,$ $[0, \pi/2] \rightarrow [0, 1]$ $I = \int_0^1 \frac{du}{\sqrt{4 - u^2}}$ $= \left[\sin^{-1} \left(\frac{u}{2} \right) \right]_0^1$ $= \pi/6 \text{ cao}$	B1 B1 M1 A1 A1	FT a multiple of this
3(a) (b)	<p>Denoting the two functional expressions by f_1, f_2</p> $f_1(0) = 1, f_2(0) = 1$ <p>Therefore f is continuous when $x = 0$.</p> $f_1'(x) = 2e^{2x}, f_2'(x) = 2(1 + x)$ $f_1'(0) = 2, f_2'(0) = 2$ <p>Therefore f' is continuous when $x = 0$.</p>	M1A1 A1 M1 A1 A1	No FT No FT
4(a) (b)	$ z = 2, \arg(z) = \pi/3$ <p>Root 1 = $\sqrt[3]{2}(\cos \pi/9 + i \sin \pi/9) = 1.184 + 0.431i$ Root 2 = $\sqrt[3]{2}(\cos 7\pi/9 + i \sin 7\pi/9) = -0.965 + 0.810i$ Root 3 = $\sqrt[3]{2}(\cos 13\pi/9 + i \sin 13\pi/9) = -0.219 - 1.241i$</p>	B1B1 M1A1 M1A1 M1A1	Penalise lack of accuracy once only

Ques	Solution	Mark	Notes
5	<p>The equation can be rewritten</p> $2\sin 3\theta \cos 2\theta = \cos 2\theta$ $\cos 2\theta(2\sin 3\theta - 1) = 0$ <p>Either $\cos 2\theta = 0$,</p> $2\theta = 2n\pi \pm \frac{\pi}{2}$ $\theta = n\pi \pm \frac{\pi}{4}$ <p>Or $\sin 3\theta = 1/2$</p> $3\theta = n\pi + (-1)^n \frac{\pi}{6}$ <p>or $\theta = \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}$</p>	<p>M1A1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Accept equivalent answers</p> <p>Accept degrees throughout</p>
6	<p>Consider $\cos 6\theta + i\sin 6\theta = (\cos \theta + i\sin \theta)^6$</p> <p>Expanding and equating imaginary terms,</p> $i\sin 6\theta = 6\cos^5 \theta(i\sin \theta) + 20\cos^3 \theta(i\sin \theta)^3 + 6\cos \theta(i\sin \theta)^5$ $\sin 6\theta = 6\cos^5 \theta \sin \theta - 20\cos^3 \theta \sin^3 \theta + 6\cos \theta \sin^5 \theta$ $\frac{\sin 6\theta}{\sin \theta} = 6\cos^5 \theta - 20\cos^3 \theta(1 - \cos^2 \theta) + 6\cos \theta(1 - \cos^2 \theta)^2$ $= 32\cos^5 \theta - 32\cos^3 \theta + 6\cos \theta$ <p>Letting $\theta \rightarrow \pi$ in the right hand side,</p> <p>Limit $= -32 + 32 - 6 = -6$</p>	<p>M1</p> <p>m1 A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1 M1 A1</p>	<p>FT their expression in the line above</p>

Ques	Solution	Mark	Notes
7(a)(i)	The equation can be rewritten as $\frac{x^2}{9} + \frac{y^2}{4} = 1$ In the usual notation, $a = 3, b = 2$. $e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{5}}{3}$	M1 A1 A1	FT their a, b
(ii)	The foci are $(\pm ae, 0)$, ie $(\pm\sqrt{5}, 0)$ cao	A1	
(b)(i)	Substituting the x, y expressions, $4 \times 9 \cos^2 \theta + 9 \times 4 \sin^2 \theta = 36(\cos^2 \theta + \sin^2 \theta) = 36$ showing that P lies on the ellipse.	B1	
(ii)	EITHER $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{2\cos\theta}{3\sin\theta}$ OR $8x + 18y \frac{dy}{dx} = 0; \frac{dy}{dx} = -\frac{8x}{18y} = -\frac{2\cos\theta}{3\sin\theta}$ This equation of the tangent is $y - 2\sin\theta = -\frac{2\cos\theta}{3\sin\theta}(x - 3\cos\theta)$ $3y\sin\theta - 6\sin^2\theta = -2x\cos\theta + 6\cos^2\theta$ $3y\sin\theta + 2x\cos\theta = 6 \quad (\text{convincing})$	M1A1 M1 A1	
(iii)	Putting $y = 0$, R is the point $\left(\frac{3}{\cos\theta}, 0\right)$ Putting $x = 0$, S is the point $\left(0, \frac{2}{\sin\theta}\right)$ So M is the point $\left(\frac{3}{2\cos\theta}, \frac{1}{\sin\theta}\right)$ $x = \frac{3}{2\cos\theta}, y = \frac{1}{\sin\theta}$ Eliminating θ , $\cos\theta = \frac{3}{2x}; \sin\theta = \frac{1}{y}$ $\frac{9}{4x^2} + \frac{1}{y^2} = \cos^2\theta + \sin^2\theta = 1$	B1 B1 B1 M1 A1 A1	

Ques	Solution	Mark	Notes
8(a)	$(0,2) ; (-4,0) ; (2,0)$	B1	M1 any valid method
(b)(i)	$x = 4$	B1	
(ii)	$f(x) = x + 6 + \frac{16}{x-4}$	M1A1	
(c)	Oblique asymptote is $y = x + 6$.	A1	
	$f'(x) = 1 - \frac{16}{(x-4)^2}$ or $\frac{x^2 - 8x}{(x-4)^2}$	B1	
	At a stationary point, $f'(x) = 0$	M1	
	$(x-4)^2 = 16$ or $x^2 - 8x = 0$	A1	
	Stationary points are $(0,2) ; (8,18)$	A1	
(d)		G1 G1 G1	LH branch RH branch Asymptotes
(e)(i)	$f(-7) = -27/11 ; f(3) = -7$	M1	
(ii)	$f(S) = [-7, 2]$	A1	
	Solve		
	$\frac{(x+4)(x-2)}{x-4} = -7$	M1	
	$x^2 + 9x - 36 = 0$	A1	
	$x = -12, 3$	A1	
	$f^{-1}(S) = [-12, 3]$	A1	

FP3

Ques	Solution	Mark	Notes
1(a)	<p>Let $y = \sinh^{-1} x$ so that $x = \sinh y = \frac{e^y - e^{-y}}{2}$</p> $e^{2y} - 2xe^y - 1 = 0$ $e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$ $y = \ln(x + \sqrt{x^2 + 1})$ <p>rejecting the negative sign since $e^y > 0$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	
(b)	<p>Substituting for $\cosh 2x$,</p> $1 + 2\sinh^2 x = 2\sinh x + 5$ $\sinh^2 x - \sinh x - 2 = 0$ <p>Solving for $\sinh x$,</p> $\sinh x = -1, 2$ $x = \ln(-1 + \sqrt{2}); \ln(2 + \sqrt{5})$	<p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1</p>	
2(a)	<p>Consider</p> $\frac{d}{dx}(3-x)^{1/3} = \frac{-(3-x)^{-2/3}}{3}$ $= -0.2295\dots \text{ when } x = 1.25$ <p>The sequence converges because this is less than 1 in modulus.</p> <p>$x_0 = 1.25$</p> <p>$x_1 = 1.205071132$</p> <p>$x_2 = 1.215296967$</p> <p>$x_3 = 1.212984693$</p> <p>$x_4 = 1.213508318$</p> <p>$x_5 = 1.21338978$</p> <p>$x_6 = 1.213416617$</p> <p>$\alpha = 1.2134$ correct to 4 decimal places.</p>	<p>M1A1</p> <p>A1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>A1</p>	<p>Allow any x between 1.2 and 1.3 M1A0A1 if negative sign omitted</p> <p>FT the f' value if M1 awarded</p>

Ques	Solution	Mark	Notes
(b)	<p>The Newton-Raphson iteration is</p> $x_{n+1} = x_n - \frac{(x_n^3 + x_n - 3)}{3x_n^2 + 1} \text{ or } \frac{2x_n^3 + 3}{3x_n^2 + 1}$ <p>$x_0 = 1.25$</p> <p>$x_1 = 1.214285714$</p> <p>$x_2 = 1.213412176$</p> <p>$x_3 = 1.213411663$</p> <p>($x_4 = 1.213411663$)</p> <p>$\alpha = 1.213412$ correct to 6 decimal places</p>	<p>M1A1</p> <p>M1A1</p> <p>A1</p> <p>A1</p>	
3(a)	$\frac{d}{dx}(\operatorname{sech} x) = \frac{d}{dx}\left(\frac{1}{\cosh x}\right)$ $= -\frac{\sinh x}{\cosh^2 x} = -\operatorname{sech} x \tanh x$	B1	Convincing
(b)	<p>$f'(x) = \operatorname{sech}^2 x$</p> <p>$f''(x) = -2\operatorname{sech}^2 x \tanh x$</p> <p>$f'''(x) = 4\operatorname{sech}^2 x \tanh^2 x - 2\operatorname{sech}^4 x$</p> <p>$f(0) = 0, f'(0) = 1, f''(0) = 0, f'''(0) = -2$</p> <p>The Maclaurin series for $\tanh x$ is</p> $x - \frac{x^3}{3} + \dots$	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>M1A1</p>	<p>FT 1 slip</p>
(c)	$(1+x)\tanh x \approx x + x^2 - \frac{x^3}{3} - \frac{x^4}{3}$ $\int_0^{0.5} (1+x)\tanh x dx \approx \int_0^{0.5} \left(x + x^2 - \frac{x^3}{3} - \frac{x^4}{3}\right) dx$ $= \left[\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{12} - \frac{x^5}{15}\right]_0^{0.5}$ $= 0.159 \text{ cao}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>FT their series</p> <p>FT 1 slip</p>

Ques	Solution	Mark	Notes
4	$dx = \frac{2dt}{1+t^2}; [0, \pi/2] \rightarrow [0, 1]$ $I = \int_0^1 \frac{1}{2 - \left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2dt}{1+t^2}$ $= \int_0^1 \frac{2}{3t^2 + 1} dt$ $= \frac{2}{3} \int_0^1 \frac{1}{t^2 + 1/3} dt$ $= \frac{2\sqrt{3}}{3} \left[\tan^{-1}(t\sqrt{3}) \right]_0^1$ $= \frac{2\sqrt{3}\pi}{9} \quad (1.21) \text{ cao}$	B1B1 M1A1 A1 A1 A1 A1	
5(a)	$I_n = -\frac{1}{2} \int_0^1 x^{n-1} \frac{d}{dx} (e^{-x^2}) dx$ $= -\frac{1}{2} \left[x^{n-1} e^{-x^2} \right]_0^1 + \frac{n-1}{2} \int_0^1 x^{n-2} e^{-x^2} dx$ $= -\frac{e^{-1}}{2} + \left(\frac{n-1}{2} \right) I_{n-2}$	M1 A1A1	
(b)	$I_1 = \int_0^1 x e^{-x^2} dx = -\frac{1}{2} \left[e^{-x^2} \right]_0^1$ $= \frac{1}{2} (1 - e^{-1})$ $I_5 = -\frac{e^{-1}}{2} + 2I_3$ $= -\frac{e^{-1}}{2} + 2 \left(-\frac{e^{-1}}{2} + I_1 \right)$ $= 1 - 2.5e^{-1}$	M1A1 A1 M1 M1 A1	M1A1A1 for evaluating I_1 at any stage

Ques	Solution	Mark	Notes
7(a)	$x = a \sinh \theta \rightarrow dx = a \cosh \theta d\theta$ $I = \int \sqrt{a^2(1 + \sinh^2 \theta)} a \cosh \theta d\theta$ $= a^2 \int \cosh^2 \theta d\theta$ $= \frac{a^2}{2} \int (1 + \cosh 2\theta) d\theta$ $= \frac{a^2}{2} (\theta + \sinh \theta \cosh \theta)$ $= \frac{a^2}{2} \left(\sinh^{-1} \left(\frac{x}{a} \right) + \frac{x \sqrt{x^2 + a^2}}{a^2} \right) (+ C)$	B1 M1 A1 A1 A1	FT line above Answer given
(b)	$\frac{dy}{dx} = 2x$ $L = \int \sqrt{1 + \left(\frac{dy}{dx} \right)^2} dx$ $= \int_0^1 \sqrt{1 + 4x^2} dx$ $= 2 \int_0^1 \sqrt{x^2 + 1/4} dx$ $= \frac{2}{8} \left[\sinh^{-1} 2x + 4x \sqrt{x^2 + 1/4} \right]_0^1$ $= 1.48$	B1 M1 A1 A1 A1 A1	



WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk



GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3 AS/Advanced

SUMMER 2014

INTRODUCTION

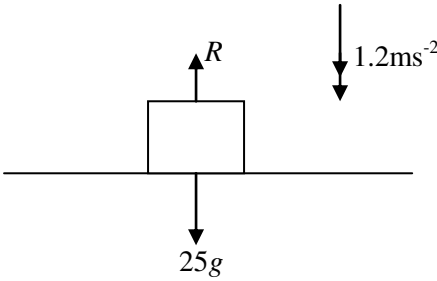
The marking schemes which follow were those used by WJEC for the Summer 2014 examination in GCE MATHEMATICS - M1-M3 & S1-S3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

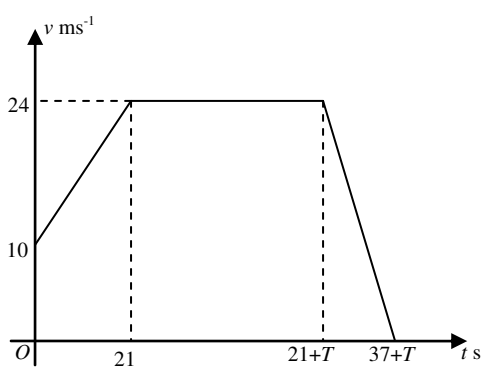
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

	Page
M1	1
M2	9
M3	16
S1	24
S2	28
S3	31

M1

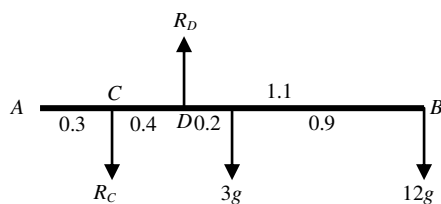
Q	Solution	Mark	Notes
1(a)			
	Apply N2L to crate	M1	R and $25g$ opposing. Dim. Correct
	$25g - R = 25 \times 1.2$	A1	correct equation
	$R = \underline{215 \text{ (N)}}$	A1	Any form
1(b)	$R = 25g = \underline{245 \text{ (N)}}$	B1	

Q	Solution	Mark	Notes
2(a)	Use of $v = u + at$ with $u=10, v=24, t=21$ $24 = 10 + 21a$ $a = \frac{2}{3} (\text{ms}^{-2})$	M1 A1 A1	oe accept anything derived from $\frac{2}{3}$ rounded correctly
2(b)	$s = \frac{1}{2}(u + v)t$ with $v=0, u=24, t=16$ $s = \frac{1}{2} \times 24 \times 16$ $s = \underline{192 \text{ (m)}}$	M1 A1 A1	oe
2(c)		B1 B1 B1 B1	(0, 10) to (21, 24) (21, 24) to (21+T, 24) (21+T, 24) to (37+T, 0) all labels, units and shape.
2(d)	Area under graph = 15000 $0.5(10+24)21 + 24T + 192 = 15000$ $24T = 14451$ $T = \underline{602(.125)}$	M1 A1 B1 A1	used ft (b) $0.5(10+24)21$ or $24T$ Ft graph Accept 600 from correct working. Cao.

Q	Solution	Mark	Notes
3(a)	Resolve perpendicular to plane $R = mg \cos \alpha$ $F = \mu mg \cos \alpha$ $F = 0.6 \times 7 \times 9.8 \times \frac{4}{5}$ $F = \underline{32.9(28 \text{ N})}$	M1 m1 A1	sin/cos correct expression Accept rounding to 32.9.
3(b)	Apply N2L to A $T + mg \sin \alpha - F = 7a$ $T + 41.16 - 32.928 = 7a$ $T + 8.232 = 7a$ Apply N2L to B $3g - T = 3a$ $3g + 8.232 = 10a$ $a = \underline{3.7(632 \text{ ms}^{-2})}$ $T = \underline{18.1(104 \text{ N})}$	M1 A1 M1 A1 m1 A1 A1	dim correct equation Friction opposes motion 4 terms. Accept cos. ft (a) dim correct equation one variable eliminated Dep on both M's cao cao

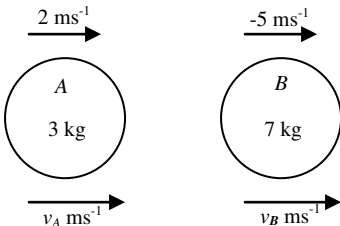
Q	Solution	Mark	Notes
---	----------	------	-------

4.



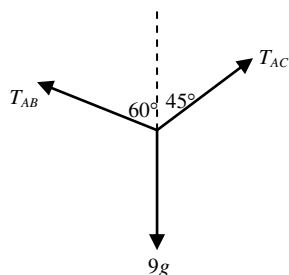
		B1	any 1 correct moment.
Take moments about C		M1	dim correct equation. oe
$0.4R_D = 3g \times 0.6 + 12g \times 1.5$		A1	correct equ any form
$0.4R_D = 19.8g = 194.04$		A1	cao
$R_D = 49.5g = \underline{485.1 \text{ (N)}}$			
Resolve vertically		M1	equation attempted. Or 2 nd moment equation.
$R_D = R_C + 15g$		A1	
$R_C = 34.5g = \underline{338.1 \text{ (N)}}$		A1	cao
<u>Alternative solution</u>			
Moment equation about A/centre/B		M1	
Correct equation		B1	
Second moment equation		M1	
Correct equation		A1	
Correct method for solving simultaneously	m1		Dep on both M's
$R_C = 34.5g = \underline{338.1 \text{ (N)}}$		A1	cao
$R_D = 49.5g = \underline{485.1 \text{ (N)}}$		A1	cao

Q	Solution	Mark	Notes
5(a)	Resolve perpendicular to motion $20\sin 60 + T\sin 30 = 28\sin 60$ $20\frac{\sqrt{3}}{2} + T \times \frac{1}{2} = 28\frac{\sqrt{3}}{2}$ $T = \underline{8\sqrt{3}}$	M1 A1 A1	equation, sin/cos convincing
5(b)	N2L in direction of motion $20\cos 60 + T\cos 30 + 28\cos 60 - 16 = 80a$ $20 \times \frac{1}{2} + 8\sqrt{3} \times \frac{\sqrt{3}}{2} + 28 \times \frac{1}{2} - 16 = 80a$ $a = \underline{0.25 \text{ (ms}^{-2}\text{)}}$	M1 A2 A1	dim correct all forces and No extra force -1 each error cao
5(c)	N2L $-16 = 80a$ $a = -0.2$ Use of $v = u + at$, $v=4$, $u=12$, $a=(+/-)0.2$ $4 = 12 - 0.2t$ $t = \underline{40 \text{ (s)}}$	M1 A1 m1 A1 A1	no extra force accept +/- ft if $a < 0$ ft if $a < 0$

Q	Solution	Mark	Notes
6(a)	 <p>Conservation of momentum</p> $2 \times 3 - 7 \times 5 = 3v_A + 7v_B$ $3v_A + 7v_B = -29$ <p>Restitution</p> $v_B - v_A = -0.6(-5 - 2)$ $v_B - v_A = 4.2$ $-7v_A + 7v_B = 29.4$ $3v_A + 7v_B = -29$ $10v_A = -58.4$ $v_A = \underline{(-)5.84}$ $v_B = \underline{(-)1.64}$	<p>M1 equation required Only one sign error. Ignore common factors</p> <p>A1</p> <p>M1 v_B, v_A opposing consistent with diagram, +/-7 with the 0.6.</p> <p>A1</p> <p>m1 one variable eliminated. Dep on both M's.</p> <p>A1 cao</p> <p>A1 cao</p>	
6(b)	<p>Impulse = change of momentum</p> $I = 7v_B - 7(-5)$ $I = -11.48 + 35$ $I = \underline{23.52 \text{ (Ns)}}$	<p>M1 used</p> <p>A1 ft their v_A or v_B</p>	
6(c)	$3.65 = e(5.84)$ $e = \underline{0.625}$	<p>B1 ft v_A if > 3.65.</p>	

Q**Solution****Mark****Notes**

7.



Resolve horizontally

$$T_{AB} \sin 60 = T_{AC} \sin 45$$

$$\frac{\sqrt{3}}{2} T_{AB} = \frac{1}{\sqrt{2}} T_{AC}$$

$$T_{AB} = \sqrt{\frac{2}{3}} T_{AC}$$

Resolve vertically

$$T_{AB} \cos 60 + T_{AC} \cos 45 = 9g$$

$$T_{AB} + \sqrt{2} T_{AC} = 18g$$

$$\sqrt{\frac{2}{3}} T_{AC} + \sqrt{2} T_{AC} = 18g$$

$$T_{AC} = \underline{79.(078) \text{ (N)}}$$

$$T_{AB} = \underline{64.(567) \text{ (N)}}$$

Alternative MethodThird angle $75^\circ/105^\circ$

$$\frac{T_{AB}}{\sin 45} = \frac{9g}{\sin 75}$$

$$T_{AB} = \frac{9g \times \sin 45}{\sin 75}$$

$$T_{AB} = \underline{64.(567) \text{ (N)}}$$

$$\frac{T_{AC}}{\sin 60} = \frac{9g}{\sin 75}$$

$$T_{AC} = \frac{9g \times \sin 60}{\sin 75}$$

$$T_{AC} = \underline{79.(078) \text{ (N)}}$$

M1 equation, no extra force

A1

M1 equation, no extra force

A1

m1

A1 cao allow 79

A1 cao allow 65

B1

M1 sine rule attempted

A1 si

A1 cao allow 65

M1 sine rule attempted

A1 si

A1 cao allow 79

Q	Solution	Mark	Notes
8(a)	mass	<i>AD</i> <i>AB</i>	
	<i>ABCD</i> 72	6 3	B1
	<i>XYZ</i> 12	6 2	B1
	<i>E</i> 24	3 4	
	<i>F</i> 36	9 4	B1 both <i>E</i> and <i>F</i> correct
	Jewel 120	<i>x</i> <i>y</i>	B1 masses in correct proportions.
8(a)(i)	Moments about <i>AD</i>	M1	masses and moments consistent.
	$120x + 12 \times 6 = 72 \times 6 + 24 \times 3 + 36 \times 9$	A1	ft table if triangle subt.
	$120x = 756$		
	$x = \frac{63}{10} = \underline{6.3(\text{cm})}$	A1	cao
8(a)(ii)	Moments about <i>AB</i>	M1	masses & moments consistent
	$120y + 12 \times 2 = 72 \times 3 + 24 \times 4 + 36 \times 4$	A1	ft table if triangle subt.
	$120y = 432$		
	$y = \frac{18}{5} = \underline{3.6(\text{cm})}$	A1	cao
8(b)	$PC = 12 - x$ $PC = \underline{5.7(\text{cm})}$	B1	ft their <i>x</i> if < 12.

M2

Q	Solution	Mark	Notes
1(a)	$EE = \frac{1}{2} \times \frac{\lambda x^2}{l}, \lambda=625, x=(+/-)0.1, l=0.2$ $EE = \frac{1}{2} \times \frac{625 \times 0.1^2}{0.2}$ $EE = \underline{15.625 \text{ (J)}}$	M1 A1	
1(b)	$KE = \frac{1}{2} \times 0.8 v^2 (= 0.4 v^2)$ $WD \text{ by resistance} = 46 \times 0.1 (= 4.6)$ <p>Work-energy Principle</p> $\frac{1}{2} 0.8 v^2 + 46 \times 0.1 = 15.625$ $0.4 v^2 = 15.625 - 4.6$ $0.4 v^2 = 11.025$ $v = \sqrt{\frac{11.025}{0.4}}$ $v = \underline{5.25 \text{ (ms}^{-1}\text{)}}$	B1 B1 M1 A1 A1	3 terms, no PE. FT their EE cao

Q	Solution	Mark	Notes
2(a)	$F - R = ma$ $30t^{-2} - 150 = 5a$ $6t^{-2} - 30 = a$ $\frac{dv}{dt} = 6t^{-2} - 30$	M1 A1	used, F and R opposing. Answer given
(b)	$24 = \frac{6}{t^2} - 30$ $\frac{6}{t^2} = 54$ $t = \frac{1}{3}$	M1 A1	Ft (a) if same form cao, accept 0.3.
2(c)	Integrate w.r.t. t $v = -6t^{-1} - 30t (+ C)$ $t = \frac{1}{3}, v = 18$ $18 = -18 - 10 + C$ $C = 46$ $v = -6t^{-1} - 30t + 46$	M1 A1 m1	Increase in powers
	When $v = 10$ $10 = -\frac{6}{t} - 30t + 46$ $5t^2 - 6t + 1 = 0$ $(5t - 1)(t - 1) = 0$ $t = \frac{1}{5}, 1$	m1 m1 A1	recognition of quadratic Some attempt to solve. cao

Q	Solution	Mark	Notes
3(a)	$T = \frac{P}{v}, P = 90 \times 1000, v = 4.8$ $T = \frac{90 \times 1000}{4.8}$ $T = 18750$	M1 A1	si si
	N2L $T - mgsin\alpha - R = ma$ $18750 - 4000 \times 9.8 \times \frac{2}{49} - R = 4000 \times 1.2$ $R = 18750 - 1600 - 4800$ $R = \underline{12350 \text{ (N)}}$	M1 A1 A1 A1	dim correct, all forces T, R opposing. cao
3(b)	N2L with $a = 0$ $T = \frac{90 \times 1000}{v}$ $T - 1600 - 12800 = 0$ $v = \underline{6.25 \text{ ms}^{-1}}$	M1 B1 A1 A1	all forces. si

Q	Solution	Mark	Notes
4(a)	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$ $\mathbf{r}_A = (3 - t)\mathbf{i} + (5 + 2t)\mathbf{j} + (20 + t)\mathbf{k}$ $\mathbf{r}_B = (-2 + 3t)\mathbf{i} + (x - 4t)\mathbf{j} + (15 + 2t)\mathbf{k}$	M1 A1 A1	used
4(b)	$\mathbf{r}_B - \mathbf{r}_A =$ $(-5 + 4t)\mathbf{i} + (x - 5 - 6t)\mathbf{j} + (-5 + t)\mathbf{k}$ $AB^2 = x^2 + y^2 + z^2$ $AB^2 = (-5 + 4t)^2 + (x - 5 - 6t)^2 + (-5 + t)^2$	M1 A1 M1 A1	 ft (a) similar expressions. cao
4(c)	Differentiate $\frac{dAB^2}{dt} = 2(-5 + 4t)(4) + 2(x - 5 - 6t)(-6)$ $\quad\quad\quad + 2(-5 + t)(1)$ $-40 + 32t - 12x + 60 + 72t - 10 + 2t = 0$ $106t + 10 = 12x$ When $t = 5$ $x = \underline{45}$	M1 m1 A1	powers reduced equating to 0. cao

Q	Solution	Mark	Notes
5(a)	$u_H = \frac{42}{2 \cdot 5} = \underline{16.8 \text{ (ms}^{-1}\text{)}}$ $s = u_V t + 0.5at^2, s = 3, t = 2.5, a = (\pm)9.8$ $3 = 2.5u_V - 4.9 \times 2.5^2$ $u_V = \underline{13.45 \text{ (ms}^{-1}\text{)}}$	B1 M1 A1 A1	 cao, accept 13.4, 13.5.
5(b)	$v_V = u_V + at, u_V = 13.45, a = (\pm)9.8, t = 2.5$ $v_V = 13.45 - 9.8 \times 2.5$ $v_V = -11.05$ magnitude of vel = $\sqrt{u_H^2 + v_V^2}$ $= \underline{20.11 \text{ (ms}^{-1}\text{)}}$ $\theta = \tan^{-1}\left(\frac{11.05}{16.8}\right)$ $\theta = \underline{33.33^\circ}$ (below horizontal)	M1 A1 m1 A1 m1 A1	 ft from (a) cao cao
5(c)	$s = ut + 0.5at^2, s = 0, u = 13.45, a = (\pm)9.8$ $0 = 13.45t - 4.9t^2$ $t = 2.7449$ Distance = 2.7449×16.8 Distance = 46.11 Required distance = $46.11 - 42 = \underline{4.11 \text{ (m)}}$	M1 m1 A1	 cao

Q	Solution	Mark	Notes
6(a)	$\mathbf{a} = \frac{dv}{dt}$ $\mathbf{a} = 8\cos 2t \mathbf{i} - 75\sin 5t \mathbf{j}$ $\text{At } t = \frac{3\pi}{2}, (\mathbf{a} = -8\mathbf{i} + 75\mathbf{j})$ $\text{Magnitude of force} = 3 \times \sqrt{8^2 + 75^2}$ $= \underline{226.28 \text{ (N)}}$	M1 A1 m1 M1 A1	differentiation attempted. Vectors required. substitution of t . or $\mathbf{F} = 3(-8\mathbf{i} + 75\mathbf{j})$ cao
6(b)	$\mathbf{r} = \int 4\sin 2t \mathbf{i} + 15\cos 5t \mathbf{j} \, dt$ $\mathbf{r} = -2\cos 2t \mathbf{i} + 3\sin 5t \mathbf{j} (+ \mathbf{c})$ $\text{At } t = 0,$ $-2\mathbf{i} + 3\mathbf{j} = -2\mathbf{i} + \mathbf{c}$ $\mathbf{c} = 3\mathbf{j}$ $\mathbf{r} = -2\cos 2t \mathbf{i} + 3\sin 5t \mathbf{j} + 3\mathbf{j}$	M1 A1 m1 A1	integration attempted
6(c)	<p>Particle crosses the y-axis when</p> $-2\cos 2t = 0$ $2t = \frac{\pi}{2}$ $t = \frac{\pi}{4}$ $\text{Distance from origin} = 3\sin(5 \times \frac{\pi}{4}) + 3$ $= \underline{0.88 \text{ (m)}}$	M1 A1 m1 A1	 cao substitute t into \mathbf{r} cao

Q	Solution	Mark	Notes
7(a)	Conservation of energy $0.5m(4u)^2 = mg(2l) + 0.5mu^2$ $16u^2 = 4gl + u^2$ $u^2 = \frac{4}{15}gl$	M1 A1 A1	 convincing
7(b)(i)	Conservation of energy $0.5m(4u)^2 = 0.5mv^2 + mgl(1 - \cos\theta)$ $v^2 = 16u^2 - 2gl + 2gl\cos\theta$ $v^2 = \frac{34}{15}gl + 2gl\cos\theta$	M1 A1 A1	
	N2L towards centre of circle $T - mg\cos\theta = \frac{mv^2}{l}$	M1 A1	
	$T = \frac{34}{15}mg + 3mg\cos\theta$	m1	If M1s gained, substitute for v^2 .
	$T = \frac{mg}{15}(34 + 45\cos\theta)$	A1	any correct form
7(b)(ii)	when $T = 0$, $\cos\theta = -\frac{34}{45}$ $\theta = 139.1^\circ$	M1 A1	putting $T = 0$ in $a\cos\theta \pm b$ $Ft\cos\theta = a$, $a < 0$.

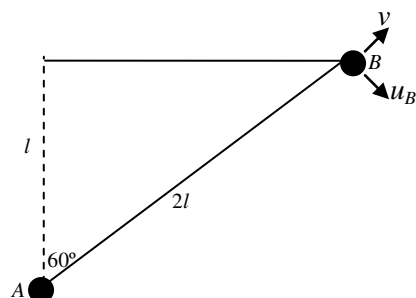
M3

Q	Solution	Mark	Notes
1(a)	N2L $500 - 100v = 1200 \frac{dv}{dt}$	M1	
	$\frac{dv}{dt} = \frac{500 - 100v}{1200} = \frac{5 - v}{12}$	A1	convincing
1(b)	$\int 12 \frac{dv}{5 - v} = \int dt$	M1	sep. var. (5-v) together.
	$-12 \ln(5 - v) = t + (C)$	A1	correct integration
	When $t = 0$, $v = 0$, $C = -12 \ln 5$	m1	allow +/-, oe
	$t = 12 \ln \left(\frac{5}{5 - v} \right)$		
	$\frac{5}{5 - v} = e^{\frac{t}{12}}$	m1	inversion ft similar exp.
	$v = 5(1 - e^{-t/12})$	A1	cao
	limiting speed = 5 (ms^{-1})	B1	Ft similar expression
1(c)	When $v = 4$, $t = 12 \ln \left(\frac{5}{5 - 4} \right)$	M1	
	$t = 12 \ln 5 (= 19.31\text{s})$	A1	cao

Q	Solution	Mark	Notes
2(a)	$\text{Period} = \frac{2\pi}{\omega} = 2$ $k = \omega = \pi$	M1 A1	
2(b)	$x = 0.52\cos\pi t$ When $t = \frac{1}{3}$, $x = 0.52\cos\frac{\pi}{3}$ $x = 0.26$	B1 M1 A1	for amp=0.52 allow asin/acos, c's a cao
2(c)	$0.4 = 0.52\cos\pi t$ $\cos\pi t = \frac{0.4}{0.52}$ $t = 0.22$ $t = 1.78$	M1 A1 A1	allow sin/cos cao FT t , ie 2-first t .
2(d)	$v^2 = \omega^2(0.52^2 - x^2)$ $v^2 = \pi^2(0.52^2 - 0.2^2)$ $v = \pi(0.48) (= 1.508 \text{ ms}^{-1})$	M1 m1 A1	used. oe sub $x = 0.2$ cao
2(e)	$\max v = a\omega$ $= 0.52\pi (= 1.634 \text{ ms}^{-1})$	M1 A1	used cao

Q**Solution****Mark****Notes**

3



Impulse = change in momentum

$$J = 2u \cos 30 - 2v$$

$$J = 3v$$

M1 used

A1

B1

Eliminating J

$$3v = 2u \cos 30 - 2v$$

m1 one variable eliminated

$$5v = 2u \cos 30$$

$$v = 0.4u \cos 30$$

$$v = 2.77 \text{ (ms}^{-1}\text{)} \text{ (speed of A)}$$

A1 cao

$$J = 1.2 u \cos 30 = 8.31 \text{ (Ns)}$$

A1 ft 3 x c's v.

$$u_B = u \sin 30 = 4 \text{ (ms}^{-1}\text{)}$$

B1

$$\text{Speed of } B = \sqrt{(2.77^2 + 4^2)}$$

m1

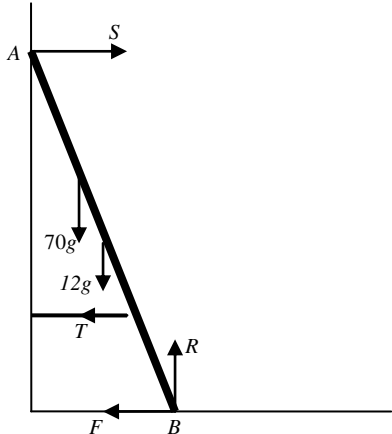
$$\text{Speed of } B = 4.87 \text{ (ms}^{-1}\text{)}$$

A1 cao

Q	Solution	Mark	Notes
4(a)	<p>Auxiliary equation $2m^2 + 6m + 5 = 0$ $m = -1.5 \pm 0.5i$ C.F. is $x = e^{-1.5t}(A\sin 0.5t + B\cos 0.5t)$</p> <p>For PI, try $x = a$ $5a = 1$ $a = 0.2$</p> <p>GS is $x = e^{-1.5t}(A\sin 0.5t + B\cos 0.5t) + 0.2$</p>	<p>B1 B1 B1</p> <p>B1</p> <p>B1</p>	<p>ft complex roots</p> <p>ft CF + a</p>
4(b)	<p>$e^{-1.5t} \rightarrow 0$ as $t \rightarrow \infty$ x tends to 0.2 as t tends to infinity Limiting value = 0.2</p>	<p>M1 A1</p>	<p>si ft similar expression</p>
4(c)(i)	<p>$x = 0.5$ and $\frac{dx}{dt} = 0$ when $t = 0$ $B + 0.2 = 0.5$ $B = 0.3$</p> <p>$\frac{dx}{dt} = -1.5e^{-1.5t}(A\sin 0.5t + B\cos 0.5t)$ $+ e^{-1.5t}(0.5A\cos 0.5t - 0.5B\sin 0.5t)$ $0 = -1.5B + 0.5A$ $A = 3B = 0.9$</p> <p>$x = e^{-1.5t}(0.9\sin 0.5t + 0.3\cos 0.5t) + 0.2$</p>	<p>M1 A1 B1 A1</p>	<p>used cao ft similar expressions cao</p>
4(c)(ii)	<p>When $t = \frac{\pi}{3}$ $x = e^{-\pi/2}(0.9\sin \frac{\pi}{6} + 0.3\cos \frac{\pi}{6}) + 0.2$ $x = 0.348$</p>	<p>A1</p>	<p>cao</p>

Q	Solution	Mark	Notes
5(a)	Using $F = ma$ $1200(v+3)^{-1} = 800 a$ $2v \frac{dv}{dx} = \frac{3}{v+3}$	M1 A1	convincing
5(b)	$\int 3dx = \int 2v(v+3)dv$ $3x = \frac{2v^3}{3} + 3v^2 + (C)$ $x = 0, v = 0$, hence $C = 0$ When $v = 3$, $3x = 18 + 27$ $x = 15$	M1 A1 B1 m1 A1	separate variables correct integration convincing
5(c)	$\frac{dv}{dt} = \frac{3}{2(v+3)}$ $\int 2(v+3)dv = \int 3dt$ $v^2 + 6v = 3t + (C)$ $t = 0, v = 0$, hence $C = 0$ When $v = 3$ $3t = 9 + 18 = 27$ $t = 9$	M1 A1 B1 A1	cao
5(d)(i)	$v^2 + 6v - 3t = 0$ $v = 0.5(-6 \pm \sqrt{6^2 - 4 \times -3t})$ $v = -3 + \sqrt{9 + 3t}$	M1 A1 A1	recognition of quadratic And attempt to solve si
(ii)	$\frac{dx}{dt} = -3 + (9 + 3t)^{\frac{1}{2}}$ $x = -3t + \frac{2}{9}(9 + 3t)^{\frac{3}{2}} + (C)$ $x = 0, t = 0$, (hence $C = -6$) $x = -3t + \frac{2}{9}(9 + 3t)^{\frac{3}{2}} + (-6)$ When $t = 7$ $x = -21 - 6 + 2 \times 30^{1.5}/9 = 9.5148$ x is approximately 9.5	M1 A1 m1 A1	correct integration cao

Q	Solution	Mark	Notes
5(d)(ii)	$v = -3 + \sqrt{9 + 3t}$ When $t=7$, $v = -3 + \sqrt{9+21}$ $v = -3 + \sqrt{30}$ $v = 2.4723$	M1 A1 si	
	$x = \frac{2}{9}(-2.4723)^3 + (2.4723)^2$ $x = \underline{9.51 \text{ (m)}}$	m1 A1 cao	

Q	Solution	Mark	Notes
6(a)		<p>B2 B1 if one error. B0 more than one error.</p>	
6(b)	<p>Resolve vertically $R = 12g + 70g = 82g$</p>	<p>M1 all forces A1</p>	
6(c)	<p>Moments about B</p> $3T\sin 75 + 12g \times 4\cos 75 + 70gx \cos 75 = 8S\sin 75$ <p>Resolve horizontally $T + F = S$ $F = 0.1R = 8.2g$ $S = T + 8.2g$</p> $8(8.2g + T)\sin 75 - 3T\sin 75 - 48g\cos 75 = 70gx\cos 75$ $5T\sin 75 = 48g\cos 75 - 65.6g\sin 75 + 70gx\cos 75$ $T = 100$ $x = 5.53 \text{ m}$	<p>M1 dim correct equation All terms A4 -1 each incorrect term Accept $T=100$.</p> <p>B1 ft R B1 ft F</p> <p>A1 cao</p>	

Q	Solution	Mark	Notes
	<u>OR</u>		
	Moments about A	M1	dim correct equation All terms
	$5T\sin 75 + 12g \times 4\cos 75 + 70g(8-x)\cos 75$ $+ 8F\sin 75 = 8R\cos 75$	A5	-1 each incorrect term Accept $T=100$.
	$F = 0.1R = 80.36 \text{ N}$	B1	Ft R
	$T = 100$ $x = 5.53 \text{ m}$	A1	cao
6(d)	Ladder modelled as a rigid rod.	B1	

S1

Ques	Solution	Mark	Notes
1(a)	EITHER $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= 0.2$	M1 A1	Award M1 for using formula
(b)	This is not equal to $P(A) \times P(B)$ therefore not independent. OR Assume A,B are independent so that $P(A \cap B) = P(A) + P(B) - P(A)P(B)$ $= 0.58$ Since $P(A \cup B) \neq 0.58$, A,B are not independent.	A1 M1 A1 A1	Award M1 for using formula
	$P(A B') = \frac{P(A \cap B')}{P(B')}$ $= \frac{0.3 - 0.2}{0.6}$ $= \frac{1}{6}$	M1 A1 A1	Award M1 for using formula FT their $P(A \cap B)$ if independence not assumed Accept Venn diagram
2	$np = 0.9, npq = 0.81$ Dividing, $q = 0.9, p = 0.1$ $n = 9$	B1B1 M1A1 A1	
3(a)	P(1 of each) = $\frac{3}{9} \times \frac{3}{8} \times \frac{3}{7} \times 6 \text{ or } \binom{3}{1} \times \binom{3}{1} \times \binom{3}{1} \div \binom{9}{3}$ $= \frac{9}{28}$	M1A1 A1	M1A0 if 6 omitted
(b)	P(2 particular colour and 1 different) = $\frac{3}{9} \times \frac{2}{8} \times \frac{6}{7} \times 3 \text{ or } \binom{3}{2} \times \binom{6}{1} \div \binom{9}{3}$ $= \frac{3}{14}$ P(2 of any colour and 1 different) = $\frac{9}{14}$	M1A1 A1 B1	M1A0 if 3 omitted Allow 3/28 FT previous line
4(a)	Let X denote the number of goals scored in the first 15 minutes so that X is Po(1.5) si $P(X = 2) = \frac{e^{-1.5} \times 1.5^2}{2!}$ $= 0.251$	B1 M1 A1	Award M0 if no working seen
(b)	$P(X > 2) = 1 - e^{-1.5} \left(1 + 1.5 + \frac{1.5^2}{2!} \right)$ $= 0.191$	M1A1 A1	

Ques	Solution	Mark	Notes
5(a)	Let X = number of female dogs so X is $B(20,0.55)$	B1	si
(i)	$P(X = 12) = \binom{20}{12} \times 0.55^{12} \times 0.45^8$ $= 0.162$	M1 A1	Accept 0.4143 – 0.2520 or 0.7480 – 0.5857
(ii)	Let Y = number of male dogs so Y is $B(20,0.45)$ $P(8 \leq X \leq 16) = P(4 \leq Y \leq 12)$ $= 0.9420 - 0.0049$ or $0.9951 - 0.0580$ $= 0.9371$	M1 A1 A1A1 A1	Award M0 if no working seen
(b)	Let U = number of yellow dogs so U is $B(60,0.05) \approx \text{Po}(3)$ $P(U < 5) = 0.8153$	M1 m1A1	
6(a)	$P(\text{head}) = \frac{3}{4} \times \frac{1}{2} + \frac{1}{4} \times 1$ $= \frac{5}{8}$	M1A1 A1	M1 Use of Law of Total Prob (Accept tree diagram)
(b)(i)	$P(DH \text{head}) = \frac{1/4}{5/8}$ $= \frac{2}{5} \text{ cao}$	B1B1 B1	B1 num, B1 denom FT denominator from (a)
(ii)	EITHER $P(\text{head}) = \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times 1$ $= \frac{7}{10}$ OR $P(\text{Head}) = \frac{\frac{3}{4} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \times 1}{\frac{5}{8}}$ $= \frac{7}{10}$	M1A1 A1 B1B1 B1	M1 Use of Law of Total Prob (Accept tree diagram) B1 num, B1 denom FT denominator from (a)

Ques	Solution	Mark	Notes
7(a)	[0,0.4]	B1	Allow(0,0.4)
(b)	$E(X) = 0.1 + 0.6 + 3\theta + 0.8 + 5(0.4 - \theta)$ $= 3.5 - 2\theta$ The range is [2.7,3.5]	M1 A1 A1	FT the range from (a)
(c)	$E(X^2) = 0.1 + 1.2 + 9\theta + 3.2 + 25(0.4 - \theta)$ $\text{Var}(X) = 0.1 + 1.2 + 9\theta + 3.2 + 25(0.4 - \theta)$ $\quad - (3.5 - 2\theta)^2$ $= 2.25 - 2\theta - 4\theta^2$ $\text{Var}(X) = 1.5$ gives $4\theta^2 + 2\theta - 0.75 = 0$ $16\theta^2 + 8\theta - 3 = 0$ $(4\theta + 3)(4\theta - 1) = 0$ $\theta = 0.25$	M1A1 M1 A1 M1 A1 M1 A1	Must be in terms of θ Allow use of formula
8(a)	<p>EITHER the sample space contains 64 pairs of which 8 are equal OR whatever number one of them obtains, 1 number out of 8 obtained by the other one gives equality.</p> $P(\text{equal numbers}) = \frac{1}{8}$	M1 A1	
(b)	<p>The possible pairs are (4,8);(5,7);(6,6);(7,5);(8,4)</p> <p>EITHER the sample space contains 64 pairs of which 5 give a sum of 12 OR each pair has probability 1/64.</p> $P(\text{sum} = 12) = \frac{5}{64}$	B1 M1 A1	
(c)	<p>EITHER reduce the sample space to (4,8);(5,7);(6,6);(7,5);(8,4)</p> <p>OR $P(\text{equal numbers}) = \frac{P(6,6)}{P(\text{sum}=12)} = \frac{1/64}{5/64}$</p> <p>Therefore $P(\text{equal numbers}) = \frac{1}{5}$</p>	 M1 A1	

Ques	Solution	Mark	Notes
9(a)(i)	$P(0.4 \leq X \leq 0.6) = F(0.6) - F(0.4)$ $= 0.261$	M1 A1	
(ii)	The median m satisfies $2m^3 - m^6 = 0.5$ $2m^6 - 4m^3 + 1 = 0$ $m^3 = \frac{4 \pm \sqrt{8}}{4} \quad (0.293)$ $m = 0.664$	B1 M1A1 A1	Award M1 for a valid attempt to solve the equation Do not award A1 if both roots given
(b)(i)	Attempting to differentiate $F(x)$ $f(x) = 6x^2 - 6x^5$	M1 A1	
(ii)	$E(X^3) = \int_0^1 x^3 (6x^2 - 6x^5) dx$ $= \left[\frac{6x^6}{6} - \frac{6x^9}{9} \right]_0^1$ $= 1/3$	M1A1 A1 A1	M1 for the integral of $x^3 f(x)$ A1 for completely correct although limits may be left until 2 nd line. FT their $f(x)$ if M1 awarded in (i)

Ques	Solution	Mark	Notes
1	$\bar{x} = \frac{405.6}{8} (= 50.7)$ $\text{SE of } \bar{X} = \frac{4}{\sqrt{8}} (= 1.4142\dots)$ <p>90% conf limits are $50.7 \pm 1.645 \times 1.4142\dots$ giving [48.4, 53.0] cao</p>	B1 M1A1 M1A1 A1	M1 correct form, A1 correct z. Award M0 if no working seen
2(a)	Upper quartile = mean + 0.6745 \times SD = 86.0	M1 A1	
(b)	Let X =weight of an orange, Y =weight of a lemon $E(\Sigma X) = 1984$ $\text{Var}(\Sigma X) = 512$ $z = \frac{2000 - 1984}{\sqrt{512}} = 0.71$	B1 B1 M1A1	Award M0 if no working seen
(c)	Prob = 0.7611 cao Let $U = X - 3Y$ $E(U) = -7$ $\text{Var}(U) = 64 + 9 \times 2.25 = 84.25$ We require $P(U > 0)$ $z = \frac{0 + 7}{\sqrt{84.25}} = 0.76$ Prob = 0.2236	A1 M1 A1 M1A1 m1A1 A1	Award m0 if no working seen
3(a)	$H_0 : \mu_M = \mu_F; H_1 : \mu_M \neq \mu_F$ Let X = male weight, Y =female weight ($\sum x = 39.2; \sum y = 46.6$) $\bar{x} = 4.9; \bar{y} = 4.66$	B1	
(b)	SE of diff of means = $\sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{10}}$ (0.237...)	B1B1 M1A1	
	Test statistic = $\frac{4.9 - 4.66}{0.237\dots}$ = 1.01 Prob from tables = 0.1562 p -value = 0.3124 Insufficient evidence to conclude that there is a difference in mean weight between males and females.	m1 A1 A1 B1 B1	Award m0 if no working seen FT line above FT their p -value

Ques	Solution	Mark	Notes
4(a)(i)	$H_0 : p = 0.6; H_1 : p < 0.6$	B1	
(ii)	Let X = Number of games won Under H_0 , X is $B(20, 0.6)$ si Let Y = Number of games lost Under H_0 , Y is $B(20, 0.4)$ $p\text{-value} = P(X \leq 7 (X \text{ is } B(20, 0.6)))$ $= P(Y \geq 13 Y \text{ is } B(20, 0.4))$ $= 0.021$	B1 B1 M1 A1 A1	Award M0 if no working seen
(b)	Strong evidence to reject Gwilym's claim (or to accept Huw's claim). X is now $B(80, 0.6)$ (under H_0) $\approx N(48, 19.2)$ $p\text{-value} = P(X \leq 37 X \text{ is } N(48, 19.2))$ $z = \frac{37.5 - 48}{\sqrt{19.2}}$ $= -2.40$ $p\text{-value} = 0.0082$ Very strong evidence to reject Gwilym's claim (or to accept Huw's claim).	B1 B1B1 M1 A1 A1 A1 B1	FT on p-value Award M0 if no working seen Award M1A0A1 for incorrect or no continuity correction No cc ; $z = -2.51, p = 0.00604$ $36.5 ; z = -2.62, p = 0.0044$ FT on p-value only if less than 0.01
5(a)	$E(X) = E(Y) = 1.2$ $E(U) = E(X)E(Y) = 1.44$ cao	B1 B1	
(b)	$\text{Var}(X) = \text{Var}(Y) = 0.96$ $E(X^2) (= E(Y^2)) = \text{Var}(X) + [E(X)]^2 = 2.4$ $\text{Var}(U) = E(X^2Y^2) - [E(XY)]^2$ $= E(X^2)E(Y^2) - [E(X)E(Y)]^2$ $= 3.69$ cao	B1 M1A1 M1 A1 A1	FT their values from (a)
6(a)(i)	Under H_0 , X is $\text{Po}(15)$ si $P(X \leq 10) = 0.1185 ; P(X \geq 20) = 0.1248$ Significance level = 0.2433	B1 B1 B1	Award B1 for either correct
(ii)	X is now $\text{Poi}(10)$ $P(\text{accept } H_0) = P(11 \leq X \leq 19)$ $= 0.9965 - 0.5830$ or $0.4170 - 0.0035$ $= 0.4135$ cao	B1 M1 A1 A1	Award M0 if no working seen
(b)	Under H_0 , X is now $\text{Po}(75) \approx N(75, 75)$ $z = \frac{91.5 - 75}{\sqrt{75}} = 1.91$ Prob from tables = 0.0281 $p\text{-value} = 0.056$ Insufficient evidence to reject H_0	B1 M1A1 A1 A1 B1	Award M1A0 for incorrect or no continuity correction but FT further work. FT from line above FT from line above No cc gives $z = 1.96, p = .05$ 92.5 gives $z = 2.02, p = 0.0434$

Ques	Solution	Mark	Notes
7(a)	$P(L \leq 4) = P(A \leq 4^2)$ $= \frac{16 - 15}{20 - 15}$ $= 0.2$	M1 A1 A1	
(b)	$E(L) = E(A^{1/2})$ $= \int_{15}^{20} a^{1/2} \times \frac{1}{5} da$ $= \frac{2}{15} [a^{3/2}]_{15}^{20}$ $= 4.18$	M1A1 A1 A1	Limits can be left until next line Do not accept $\sqrt{17.5} = 4.18$
(c)	$\text{Var}(L) = E(L^2) - [E(L)]^2$ $= 17.5 - 4.18^2$ $= 0.03$	M1 A1 A1	FT their $E(L)$

Ques	Solution	Mark	Notes
1	$\bar{x} = 52.0 \text{ si}$ $\text{Variance estimate} = \frac{162480}{59} - \frac{3120^2}{60 \times 59} = 4.068$ (Accept division by 60 which gives 4.0) 90% confidence limits are $52 \pm 1.645\sqrt{4.068/60}$ giving [51.6, 52.4]	B1 M1A1 M1A1 A1	
2(a)	$H_0 : \mu = 4.5; H_1 : \mu \neq 4.5$	B1	
(b)	$\sum x = 43.6; \sum x^2 = 190.3428$ UE of $\mu = 4.36$ $\text{UE of } \sigma^2 = \frac{190.3428}{9} - \frac{43.6^2}{90}$ $= 0.0274(22\dots)$	B1B1 B1 M1 A1	No working need be seen Answer only no marks
(c)	$\text{test-stat} = \frac{4.36 - 4.5}{\sqrt{0.0274222\dots/10}}$ $= -2.67 \text{ (Accept } +2.67)$ DF = 9 si Crit value = 3.25 This result suggests that we should accept H_0 , ie that the mean weight is 4.5 kg because $2.67 < 3.25$	M1A1 A1 B1 B1 B1	FT their values from (b) Answer only no marks FT their t -statistic
3(a)	$\hat{p} = \frac{654}{1500} = 0.436 \text{ si}$ $\text{ESE} = \sqrt{\frac{0.436 \times 0.564}{1500}} = 0.0128\dots \text{ si}$ 95% confidence limits are $0.436 \pm 1.96 \times 0.0128\dots$ giving [0.41, 0.46]	B1 M1A1 M1 A1 A1	M1 correct form A1 correct z
(b)	$\hat{p} = \frac{0.4348 + 0.4852}{2} = 0.46$ Number of people = $0.46 \times 1200 = 552$ $0.4852 - 0.4348 = 2z\sqrt{\frac{0.46 \times 0.54}{1200}}$ $z = 1.75$ Prob from tables = 0.0401 or 0.9599 Confidence level = 92%	B1 B1 M1A1 A1 A1 B1	FT line above

Ques	Solution	Mark	Notes
4(a) (b)	$H_0: \mu_a = \mu_b; H_1: \mu_a \neq \mu_b$ $SE = \sqrt{\frac{0.115}{80} + \frac{0.096}{70}} \quad (= 0.053)$ $\text{Test stat} = \frac{3.65 - 3.52}{0.053}$ $= 2.45 \quad (\text{Accept } 2.46)$ $\text{Tabular value} = 0.00714 \quad (0.00695)$ $p\text{-value} = 0.01428 \quad (0.0139)$ <p>Strong evidence to conclude that there is a difference in mean weight.</p>	B1 M1A1 M1A1 A1 A1 B1	 FT their p -value Accept the conclusion that the Variety B mean is greater than the Variety A mean
(c)	<p>Estimates of the variances of the sample means are used and not exact values.</p> <p>The sample means are assumed to be normally distributed (using the Central Limit Theorem).</p>	B1 B1	
5(a)	$\sum x = 42, \sum x^2 = 364, \sum y = 340.6, \sum xy = 2906.4$ $S_{xy} = 2906.4 - 42 \times 340.6 / 6 = 522.2$ $S_{xx} = 364 - 42^2 / 6 = 70$ $b = \frac{522.2}{70} = 7.46$ $a = \frac{340.6 - 7.46 \times 42}{6} = 4.55$	B2 B1 B1 M1 A1 M1 A1	Minus 1 each error Answers only no marks
(b)(i)	Unbiased estimate $= a + 5b = 41.85$	B1	FT their values of a, b if answer between 33.9 and 49.9
(ii)	$SE \text{ of } a + 5b = 0.5 \sqrt{\frac{1}{6} + \frac{(5-7)^2}{70}} \quad (0.2365\dots)$ <p>95% confidence limits for $a + 5b$ are</p> $41.85 \pm 1.96 \times 0.2365\dots$ <p>giving [41.4, 42.3]</p>	M1A1 m1A1 A1	And FT their value of S_{xx}
(iii)	$\text{Test stat} = \frac{7.6 - 7.46}{\sqrt{0.5^2 / 70}} = 2.34$ <p>Critical value = 1.96 or p-value = 0.01928</p> <p>We conclude that $\beta = 7.6$ is not consistent with the tabular values.</p>	M1A1 A1 B1	FT their values of b and S_{xx} if possible. FT their test statistic FT the line above

Ques	Solution	Mark	Notes
6(a)(i)	$E(Y) = kE(\bar{X}) = kE(X) = \frac{k\theta}{2}$ <p>For an unbiased estimator, $k = 2$.</p>	M1A1 A1	
(ii)	$\text{Var}(Y) = 4\text{Var}(\bar{X})$ $= \frac{4}{n} \text{Var}(X)$ $= \frac{4}{n} \times \frac{\theta^2}{12}$ $= \frac{\theta^2}{3n}$ $\text{SE} = \frac{\theta}{\sqrt{3n}}$	M1 A1 A1 A1 A1	FT their k
(b)(i)	<p>Using $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$</p> $E(Y^2) = \frac{\theta^2}{3n} + \theta^2$ <p>$\neq \theta^2$ therefore not unbiased</p>	M1 A1 B1	
(ii)	$E(Y^2) = \theta^2 \left(\frac{3n+1}{3n} \right)$ $E\left(\frac{3nY^2}{3n+1} \right) = \theta^2$ <p>Therefore $\frac{3nY^2}{3n+1}$ is an unbiased estimator for θ^2</p>	M1 A1 A1	FT the line above



WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk