1. (*a*)

0	2·197224577		
0·75	2·314217179		
1·5	2·524262696		
2·25	2·861499826		
3	3·335254744	(5 values correct)	B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0{\cdot}75$ — M1

$I \approx \frac{0{\cdot}75}{3} \times \{2{\cdot}197224577 + 3{\cdot}335254744 + 4(2{\cdot}314217179 + 2{\cdot}861499826) + 2(2{\cdot}524262696)\}$

$I \approx 31{\cdot}28387273 \times 0{\cdot}75 \div 3$

$I \approx 7{\cdot}820968183$

$I \approx 7{\cdot}82$ (f.t. one slip) — A1

Note: Answer only with no working shown earns 0 marks

(*b*)

$\int_0^3 \ln(16 + 2e^x)\,dx = \int_0^3 \ln(8 + e^x)\,dx + \int_0^3 \ln 2\,dx$ — M1

$\int_0^3 \ln(16 + 2e^x)\,dx = 7{\cdot}82 + 2{\cdot}08 = 9{\cdot}90$

(f.t. candidate's answer to (*a*)) — A1

Note: Answer only with no working shown earns 0 marks

2. $8(\sec^2\theta - 1) - 5\sec^2\theta = 7 + 4\sec\theta$. (correct use of $\tan^2\theta = \sec^2\theta - 1$) — M1

An attempt to collect terms, form and solve quadratic equation in $\sec\theta$, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant — m1

$3\sec^2\theta - 4\sec\theta - 15 = 0 \Rightarrow (3\sec\theta + 5)(\sec\theta - 3) = 0$

$\Rightarrow \sec\theta = -\frac{5}{3}$, $\sec\theta = 3$

$\Rightarrow \cos\theta = -\frac{3}{5}$, $\cos\theta = \frac{1}{3}$ (c.a.o.) — A1

$\theta = 126{\cdot}87°, 233{\cdot}13°$ — B1 B1

$\theta = 70{\cdot}53°, 289{\cdot}47°$ — B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta = +, -$, f.t. for 3 marks, $\cos\theta = -, -$, f.t. for 2 marks
$\cos\theta = +, +$, f.t. for 1 mark

3. (*a*) $\frac{d}{dx}(y^4) = 4y^3\frac{dy}{dx}$ B1

$\frac{d}{dx}(8xy^2) = (8x)(2y)\frac{dy}{dx} + 8y^2$ B1

$\frac{d}{dx}(2x^2) = 4x,\ \frac{d}{dx}(9) = 0$ B1

$\frac{dy}{dx} = \frac{x - 2y^2}{y^3 + 4xy}$ (convincing) (c.a.o.) B1

(*b*) $\frac{dy}{dx} = 0 \Rightarrow x = 2y^2$ B1

Substitute $2y^2$ for x in equation of C M1

$9y^4 + 9 = 0$ (o.e.) (c.a.o.) A1

$9y^4 + 9 > 0$ for any real y (o.e.) and thus no such point exists A1

4. candidate's x-derivative $= 2e^t$ B1

candidate's y-derivative $= -8e^{-t} + 3e^t$ B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{-8e^{-t} + 3e^t}{2e^t}$ (o.e.) (c.a.o.) A1

Putting candidate's $\frac{dy}{dx} = -1$, rearranging and obtaining either an equation in **both** e^t and e^{-t}, or an equation in e^{2t}, or an equation in e^{-2t}. M1

Either $e^{2t} = \frac{8}{5}$ or $e^{-2t} = \frac{5}{8}$

(f.t. one numerical slip in candidate's derived expression for $\frac{dy}{dx}$) A1

$t = 0{\cdot}235$ (c.a.o.) A1

5. (*a*) $\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{ax + b}{3x^2 - 2x - 1}$ (including $a = 0, b = 1$) M1

$\frac{d}{dx}[\ln(3x^2 - 2x - 1)] = \frac{6x - 2}{3x^2 - 2x - 1}$ A1

$6x - 2 = 8x(3x^2 - 2x - 1)$ (o.e.) (f.t. candidate's a, b) A1

$12x^3 - 8x^2 - 7x + 1 = 0$ (convincing) A1

(*b*) $x_0 = -0{\cdot}6$

$x_1 = -0{\cdot}578232165$ (x_1 correct, at least 4 places after the point) B1

$x_2 = -0{\cdot}582586354$

$x_3 = -0{\cdot}581770386$

$x_4 = -0{\cdot}581925366 = -0{\cdot}5819$ (x_4 correct to 4 decimal places) B1

Let $g(x) = 12x^3 - 8x^2 - 7x + 1$

An attempt to check values or signs of $g(x)$ at $x = -0{\cdot}58185$, $x = -0{\cdot}58195$ M1

$g(-0{\cdot}58185) = 7{\cdot}35 \times 10^{-4}$, $g(-0{\cdot}58195) = -7{\cdot}15 \times 10^{-4}$ A1

Change of sign $\Rightarrow \alpha = -0{\cdot}5819$ correct to four decimal places A1

6. (*a*) (i) $\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = -\frac{1}{4} \times (9 - 4x^5)^{-5/4} \times (-20x^4)$

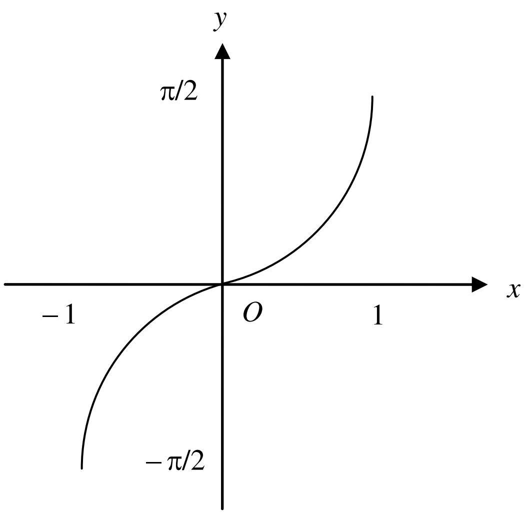
$\frac{dy}{dx} = 5x^4 \times (9 - 4x^5)^{-5/4}$ A1

(ii) $\frac{dy}{dx} = \frac{(7 - x^3) \times f(x) - (3 + 2x^3) \times g(x)}{(7 - x^3)^2}$ $(f(x), g(x) \neq 1)$ M1

$\frac{dy}{dx} = \frac{(7 - x^3) \times 6x^2 - (3 + 2x^3) \times (-3x^2)}{(7 - x^3)^2}$ A1

$\frac{dy}{dx} = \frac{51x^2}{(7 - x^3)^2}$ (c.a.o.) A1

(*b*) (i)



G1

(ii) $x = \sin y \Rightarrow \frac{dx}{dy} = \cos y$ B1

$\frac{dx}{dy} = \pm\sqrt{(1 - \sin^2 y)}$ B1

The +ive sign is chosen because the graph shows the gradient to be positive E1

$\frac{dx}{dy} = \sqrt{(1 - x^2)}$ B1

$\frac{dy}{dx} = \frac{1}{\sqrt{(1 - x^2)}}$ B1

7. (*a*) (i) $\int \cos(2-5x)\,dx = k \times \sin(2-5x) + c$ $(k = 1, {}^1/_5, -5, -{}^1/_5)$ M1

$\int \cos(2-5x)\,dx = -{}^1/_5 \times \sin(2-5x) + c$ A1

(ii) $\int \frac{4}{e^{3x-2}}\,dx = k \times 4 \times e^{2-3x} + c$ $(k = 1, -3, {}^1/_3\ -{}^1/_3)$ M1

$\int \frac{4}{e^{3x-2}}\,dx = -{}^4/_3 \times e^{2-3x} + c$ A1

(iii) $\int \frac{5}{{}^1/_6x - 3}\,dx = k \times 5 \times \ln\left|{}^1/_6x - 3\right| + c$ $(k = 1, {}^1/_6, \mathbf{6})$ M1

$\int \frac{5}{{}^1/_6x - 3}\,dx = 30 \times \ln\left|{}^1/_6x - 3\right| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (4x+1)^{1/2}\,dx = k \times \frac{(4x+1)^{3/2}}{3/2}$ $(k = 1, 4, {}^1/_4)$ M1

$\int_2^6 (4x+1)^{1/2}\,dx = \left[{}^1/_4 \times \frac{(4x+1)^{3/2}}{3/2}\right]_2^6$ A1

A correct method for substitution of limits in an expression of the form $m \times (4x+1)^{3/2}$ M1

$\int_2^6 (4x+1)^{1/2}\,dx = \frac{125}{6} - \frac{27}{6} = \frac{98}{6} = 16{\cdot}33$

(f.t. only for solutions of $\frac{392}{6}$ and $\frac{1568}{6}$ from $k = 1, 4$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) Choice of a, b, with one positive and one negative and one side correctly evaluated M1

Both sides of identity evaluated correctly A1

(*b*) Trying to solve $3x - 2 = 7x$ M1

Trying to solve $3x - 2 = -7x$ M1

$x = -0{\cdot}5,\ x = 0{\cdot}2$ (both values) (c.a.o.) A1

Alternative mark scheme

$(3x-2)^2 = 7^2 \times x^2$ (squaring both sides) M1

$40x^2 + 12x - 4 = 0$ (o.e.) (c.a.o.) A1

$x = -0{\cdot}5,\ x = 0{\cdot}2$ (both values, f.t. one slip in quadratic) A1

9. (*a*) $f(x) = (x - 4)^2 - 9$ B1

(*b*) $y = (x - 4)^2 - 9$ and an attempt to isolate x
(f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) M1
$x = (\pm)\sqrt{(y + 9)} + 4$
(f.t. candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1
$x = -\sqrt{(y + 9)} + 4$ (o.e.) (c.a.o.) A1
$f^{-1}(x) = -\sqrt{(x + 9)} + 4$ (o.e.)
(f.t. only incorrect choice of sign in front of the $\sqrt{}$ sign and candidate's expression for $f(x)$ of form $(x + a)^2 + b$, with a, b derived) A1

10. (*a*) $R(g) = [2k - 4, \infty)$ B1

(*b*) (i) $2k - 4 \geq -2$ M1
$k \geq 1$ ($\Rightarrow$ least value of k is 1)
(f.t. candidate's $R(g)$ provided it is of form $[a, \infty)$) A1

(ii) $fg(x) = (kx - 4)^2 + k(kx - 4) - 8$ B1

(iii) $(3k - 4)^2 + k(3k - 4) - 8 = 0$
(substituting 3 for x in candidate's expression for $fg(x)$ and putting equal to 0) M1
Either $12k^2 - 28k + 8 = 0$ or $6k^2 - 14k + 4 = 0$
or $3k^2 - 7k + 2 = 0$ (c.a.o.) A1
$k = {}^1/_3, 2$ (f.t. candidate's quadratic in k) A1
$k = 2$ (c.a.o.) A1