1. $9x^2 - 5x \times 2y\frac{dy}{dx} - 5y^2 + 8y^3\frac{dy}{dx} = 0$ $\left[-5x \times 2y\frac{dy}{dx} - 5y^2\right]$ B1

$\left[9x^2 + 8y^3\frac{dy}{dx}\right]$ B1

Either $\frac{dy}{dx} = \frac{9x^2 - 5y^2}{10xy - 8y^3}$ **or** $\frac{dy}{dx} = \frac{1}{4}$ (o.e.) (c.a.o.) B1

Attempting to substitute $x = 1$ and $y = 2$ in candidate's expression **and** the use of $\text{grad}_{normal} \times \text{grad}_{tangent} = -1$ M1

Equation of normal: $y - 2 = -4(x - 1)$

$\left[\text{f.t. candidate's value for } \frac{dy}{dx}\right]$ A1

2. (*a*) $f(x) \equiv \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x-4)}$ (correct form) M1

$5x^2 + 7x + 17 \equiv A(x - 4) + B(x + 1)(x - 4) + C(x + 1)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -3$, $C = 5$, $B = 0$ (all three coefficients correct) A2

(**If A2 not awarded**, award A1 for either 1 or 2 correct coefficients)

(*b*) $\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{5x^2 + 7x + 17}{(x+1)^2(x-4)} + \frac{2}{(x+1)^2}$ M1

$\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)} = \frac{-1}{(x+1)^2} + \frac{5}{(x-4)}$

(f.t. candidates values for A, B, C) A1

3. (*a*) $\dfrac{2\tan x}{1-\tan^2 x} = 3\cot x$ (correct use of formula for $\tan 2x$) M1

$\dfrac{2\tan x}{1-\tan^2 x} = \dfrac{3}{\tan x}$ (correct use of $\cot x = \dfrac{1}{\tan x}$) M1

$\tan^2 x = {}^3/_5$ (o.e.) A1

$x = 37{\cdot}76°, 142{\cdot}24°$ (both values)

(f.t. $a\tan^2 x = b$ provided both M1's are awarded) A1

(*b*) (i) $R = 29$ B1

Correctly expanding $\sin(\theta - \alpha)$ and using either $29\cos\alpha = 21$ **or** $29\sin\alpha = 20$ **or** $\tan\alpha = \dfrac{20}{21}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 43{\cdot}6°$ (c.a.o) A1

(ii) Greatest value of $\dfrac{1}{21\sin\theta - 20\cos\theta + 31} = \dfrac{1}{29\times(\pm 1) + 31}$

(f.t. candidate's value for R) M1

Greatest value = $\dfrac{1}{2}$ (f.t. candidate's value for R) A1

Corresponding value for $\theta = 313{\cdot}6°$ (o.e.)

(f.t. candidate's value for α) A1

4.

Volume = $\pi\displaystyle\int_0^{\pi/4}(3 + 2\sin x)^2\,dx$ B1

Correct use of $\sin^2 x = \dfrac{(1-\cos 2x)}{2}$ M1

Integrand = $(9 + 2 + 12\sin x - 2\cos 2x)$ (c.a.o.) A1

$\displaystyle\int(a + b\sin x + c\cos 2x)\,dx = (ax - b\cos x + \frac{c}{2}\sin 2x)$ ($a \neq 0, b \neq 0, c \neq 0$) B1

Correct substitution of correct limits in candidate's integrated expression of form $(ax - b\cos x + \frac{c}{2}\sin 2x)$ ($a \neq 0, c \neq 0$) M1

Volume = 35 (c.a.o.) A1

Note: Answer only with no working earns 0 marks

5. $(1-2x)^{1/2} = 1 + (1/2) \times (-2x) + \dfrac{(1/2) \times (1/2-1) \times (-2x)^2}{1 \times 2} + \ldots$

(– 1 each incorrect term) B2

$\dfrac{1}{1+4x} = 1 + (-1) \times (4x) + \dfrac{(-1) \times (-2) \times (4x)^2}{1 \times 2} + \ldots$

(– 1 each incorrect term) B2

$6\sqrt{1-2x} - \dfrac{1}{1+4x} = 5 - 2x - 19x^2 + \ldots$

(– 1 each incorrect term) B2

Expansion valid for $|x| < 1/4$ (o.e.) B1

6. (*a*) candidate's *x*-derivative = 2

candidate's *y*-derivative = $15t^2$ (at least one term correct)

and use of

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{15}{2}t^2$ (o.e.) (c.a.o.) A1

Equation of tangent at *P*: $y - 5p^3 = \dfrac{15}{2}p^2(x-2p)$

(f.t. candidate's expression for $\dfrac{dy}{dx}$) m1

$2y = 15p^2x - 20p^3$ (convincing) A1

(*b*) Substituting $p = 1$, $x = 2q$, $y = 5q^3$ in equation of tangent M1

$q^3 - 3q + 2 = 0$ (convincing) A1

Putting $f(q) = q^3 - 3q + 2$

Either $f(q) = (q-1)(q^2 + q - 2)$ **or** $f(q) = (q+2)(q^2 - 2q + 1)$ M1

Either $f(q) = (q-1)(q-1)(q+2)$ **or** $q = 1$, $q = -2$ A1

$q = -2$ A1

7. (*a*) $u = \ln 2x \Rightarrow du = 2 \times \frac{1}{2x} dx$ (o.e.) B1

$dv = x^4 dx \Rightarrow v = \frac{1}{5}x^5$ (o.e.) B1

$\int x^4 \ln 2x \, dx = \ln 2x \times \frac{1}{5}x^5 - \int \frac{1}{5}x^5 \times \frac{1}{x} dx$ (o.e.) M1

$\int x^4 \ln 2x \, dx = \ln 2x \times \frac{1}{5}x^5 - \frac{1}{25}x^5 + c$ (c.a.o.) A1

(*b*) $\int \sqrt{(10\cos x - 1)} \sin x \, dx = \int k \times u^{1/2} du$ ($k = -\frac{1}{10}, \frac{1}{10}$ or ± 10) M1

$\int a \times u^{1/2} du = a \times \frac{u^{3/2}}{3/2}$ B1

$\int_0^{\pi/3} \sqrt{(10\cos x - 1)} \sin x \, dx = k\left[\frac{u^{3/2}}{3/2}\right]_9^4$ or $k\left[\frac{(10\cos x - 1)^{3/2}}{3/2}\right]_0^{\pi/3}$ B1

$\int_0^{\pi/3} \sqrt{(10\cos x - 1)} \sin x \, dx = \frac{19}{15} = 1\cdot27$ (c.a.o.) A1

8. (*a*) $\frac{dV}{dt} = kV$ B1

(*b*) $\int \frac{dV}{V} = \int k \, dt$ M1

$\ln V = kt + c$ A1

$V = e^{kt + c} = Ae^{kt}$ (convincing) A1

(*c*) (i) $292 = Ae^{2k}$

$637 = Ae^{28k}$ (both values) B1

Dividing to eliminate A M1

$\frac{637}{292} = e^{26k}$ A1

$k = \frac{1}{26} \ln\left[\frac{637}{292}\right] = 0\cdot03$ A1

(ii) $A = 275$ B1

(iii) When $t = 0$, initial value of investment = £275
(f.t. candidate's derived value for A) B1

9. (*a*) $\mathbf{p.q} = -18$ B1

$|\mathbf{p}| = \sqrt{14}$, $|\mathbf{q}| = \sqrt{105}$ (at least one correct) B1

Correctly substituting candidate's derived values in the formula

$\mathbf{p.q} = |\mathbf{p}| \times |\mathbf{q}| \times \cos\theta$ M1

$\theta = 118°$ (c.a.o.) A1

(*b*) (i) Use of $\mathbf{CD} = \mathbf{CO} + \mathbf{OD}$ and the fact that $\mathbf{OC} = \frac{1}{2}\mathbf{b}$ and $\mathbf{OD} = 2\mathbf{a}$, leading to printed answer $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$

(convincing) B1

Use of $\frac{1}{2}\mathbf{b} + \lambda\mathbf{CD}$ (o.e.) to find vector equation of *CD* M1

Vector equation of *CD*: $\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1-\lambda)\mathbf{b}$ (convincing) A1

(ii) **Either:**

Either substituting $\frac{1}{3}$ for λ in the vector equation of *CD*

or substituting 2 for μ in the vector equation of *L* M1

At least one of these position vectors $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ A1

Both position vectors $= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b} \Rightarrow$ this must be the position vector of the point of intersection *E* A1

Or:

$2\lambda = \frac{\mu}{3}$

$\frac{1}{2}(1-\lambda) = \frac{1}{3}(\mu - 1)$

(comparing candidate's coefficients of **a** and **b** and an attempt to solve) M1

$\lambda = \frac{1}{3}$ or $\mu = 2$ A1

$\mathbf{OE} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ (convincing) A1

(iii) **Either**: *E* lies on *AB* and is such that *AE* : *EB* = 1 : 2 (o.e.)

Or: *E* is the point of intersection of *AB* and *CD* B1

10. Squaring both sides we have

$1 + 2\sin\theta\cos\theta > 2$ B1

$\sin 2\theta > 1$ B1

Contradiction, since the sine of any angle ≤ 1 B1