



GCE AS/A level

0976/01

MATHEMATICS – C4
Pure Mathematics

A.M. MONDAY, 16 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The curve C is defined by

$$3x^3 - 5xy^2 + 2y^4 = 15.$$

The point P has coordinates $(1, 2)$ and lies on C .

Find the equation of the **normal** to C at P .

[5]

2. (a) Express $\frac{5x^2 + 7x + 17}{(x+1)^2(x-4)}$ in terms of partial fractions.

[4]

- (b) **Use your answer to part (a)** to express $\frac{5x^2 + 9x + 9}{(x+1)^2(x-4)}$ in terms of partial fractions.

[2]

3. (a) Find all values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan 2x = 3 \cot x.$$

[4]

- (b) (i) Express $21 \sin \theta - 20 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

- (ii) Use your results to part (i) to find the greatest value of

$$\frac{1}{21 \sin \theta - 20 \cos \theta + 31}.$$

Write down a value for θ for which this greatest value occurs.

[6]

4. The region R is bounded by the curve $y = 3 + 2 \sin x$, the x -axis and the lines $x = 0$, $x = \frac{\pi}{4}$.

Find the volume of the solid generated when R is rotated through four right angles about the x -axis. Give your answer correct to the nearest integer.

[6]

5. Expand

$$6\sqrt{1-2x} - \frac{1}{1+4x}$$

in ascending powers of x up to and including the term in x^2 .

State the range of values of x for which your expansion is valid.

[7]

6. The curve C has the parametric equations $x = 2t$, $y = 5t^3$. The point P lies on C and has parameter p .

(a) Show that the equation of the tangent to C at the point P is

$$2y = 15p^2x - 20p^3. \quad [4]$$

(b) The tangent to C at the point P intersects C again at the point $Q(2q, 5q^3)$. Given that $p = 1$, show that q satisfies the equation

$$q^3 - 3q + 2 = 0.$$

Hence find the value of q . [5]

7. (a) Find $\int x^4 \ln 2x \, dx$. [4]

(b) Use the substitution $u = 10 \cos x - 1$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sqrt{(10 \cos x - 1)} \sin x \, dx. \quad [4]$$

8. The value $\pounds V$ of a long term investment may be modelled as a continuous variable. At time t years, the rate of increase of V is directly proportional to the value of V .

(a) Write down a differential equation satisfied by V . [1]

(b) Show that $V = Ae^{kt}$, where A and k are constants. [3]

(c) The value of the investment after 2 years is $\pounds 292$ and its value after 28 years is $\pounds 637$.

(i) Show that $k = 0.03$, correct to two decimal places.

(ii) Find the value of A correct to the nearest integer.

(iii) Find the initial value of the investment. Give your answer correct to the nearest pound. [6]

TURN OVER

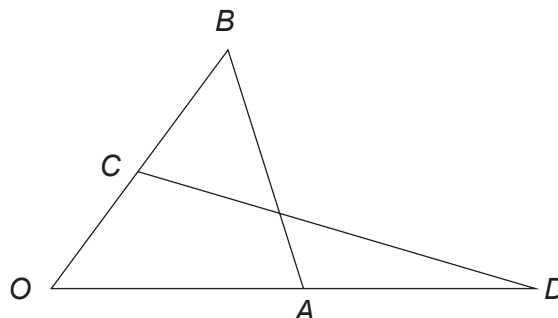
9. (a) The vectors $\mathbf{p}$ and $\mathbf{q}$ are given by

$$\mathbf{p} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} \\ \text{and} \quad \mathbf{q} = 5\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}.$$

Find the angle between $\mathbf{p}$ and $\mathbf{q}$.

[4]

- (b) In the diagram below, the points O , A , B , C and D are such that A is the mid-point of OD and C is the mid-point of OB .



Taking O as the origin, the position vectors of A and B are denoted by $\mathbf{a}$ and $\mathbf{b}$ respectively.

- (i) Show that $\mathbf{CD} = 2\mathbf{a} - \frac{1}{2}\mathbf{b}$.

Hence show that the vector equation of the line CD may be expressed in the form

$$\mathbf{r} = 2\lambda\mathbf{a} + \frac{1}{2}(1 - \lambda)\mathbf{b}.$$

The vector equation of the line L may be expressed in the form

$$\mathbf{r} = \frac{1}{3}\mu\mathbf{a} + \frac{1}{3}(\mu - 1)\mathbf{b}.$$

The lines CD and L intersect at the point E .

- (ii) By giving λ and μ appropriate values, or otherwise, show that E has position

vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

- (iii) Give a geometrical interpretation of the fact that E has position vector $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$.

[7]

10. Complete the following proof by contradiction to show that

$$\sin\theta + \cos\theta \leq \sqrt{2}$$

for all values of θ .

Assume that there is a value of θ for which $\sin\theta + \cos\theta > \sqrt{2}$.
Then squaring both sides, we have:

[3]

END OF PAPER