



GCE MARKING SCHEME

**MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced**

SUMMER 2015

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2015 examination in GCE MATHEMATICS - C1-C4 & FP1-FP3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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C1

1. (a) (i) Gradient of $AB = \frac{\text{increase in } y}{\text{increase in } x}$ M1
 Gradient of $AB = -\frac{1}{3}$ (or equivalent) A1
- (ii) A correct method for finding the equation of AB using candidate's gradient for AB M1
 Equation of AB : $y - 3 = -\frac{1}{3}[x - (-7)]$ (or equivalent) A1
 (f.t. candidate's gradient of AB) A1
 Equation of AB : $x + 3y - 2 = 0$ (convincing) A1
- (iii) Use of $m_L \times m_{AB} = -1$ M1
 A correct method for finding the equation of L using candidate's gradient for L (M1)
(to be awarded only if corresponding M1 is not awarded in part (ii))
 Equation of L : $y - 5 = 3[x - (-3)]$ (or equivalent) A1
 (f.t. candidate's gradient of AB) A1

Note: Total mark for part (a) is 7 marks

- (b) An attempt to solve equations of AB and L simultaneously M1
 $x = -4, y = 2$ (convincing) (c.a.o.) A1
- (c) A correct method for finding at least one coordinate of the mid-point of AB M1
 y -coordinate of the mid-point of $AB = 1.5$ (or x -coordinate $= -2.5$)
 $\Rightarrow D$ is not the mid-point of AB **or**
 $\Rightarrow L$ is not the perpendicular bisector of AB **or**
 $\Rightarrow$ the mid-point does not lie on L A1

Alternative Mark Scheme

- A correct method for finding the lengths of two of AB, AD, BD M1
 Two of $AB = \sqrt{90}, AD = \sqrt{10}, BD = \sqrt{40}$
 $\Rightarrow D$ is not the mid-point of AB **or**
 $\Rightarrow L$ is not the perpendicular bisector of AB **or**
 $\Rightarrow$ the mid-point does not lie on L A1

- (d) A correct method for finding the length of $BD(CD)$ M1
 $BD = \sqrt{40}$ (or equivalent) A1
 $CD = \sqrt{10}$ A1
 Substitution of candidate's derived values in $\tan ABC = \frac{CD}{BD}$ m1
 $\tan ABC = \frac{1}{2}$ (c.a.o.) A1

Special Case

A candidate who has been awarded M0 A0 A0 m0 A0 may be awarded SC1 for one of $AB = \sqrt{90}$, $AC = \sqrt{20}$, $BC = \sqrt{50}$

2. (a) $\frac{4\sqrt{2} - \sqrt{11}}{3\sqrt{2} + \sqrt{11}} = \frac{(4\sqrt{2} - \sqrt{11})(3\sqrt{2} - \sqrt{11})}{(3\sqrt{2} + \sqrt{11})(3\sqrt{2} - \sqrt{11})}$ M1
 Numerator: $12 \times 2 - 4 \times \sqrt{2} \times \sqrt{11} - 3 \times \sqrt{11} \times \sqrt{2} + 11$ A1
 Denominator: $18 - 11$ A1
 $\frac{4\sqrt{2} - \sqrt{11}}{3\sqrt{2} + \sqrt{11}} = 5 - \sqrt{22}$ (c.a.o.) A1

Special case

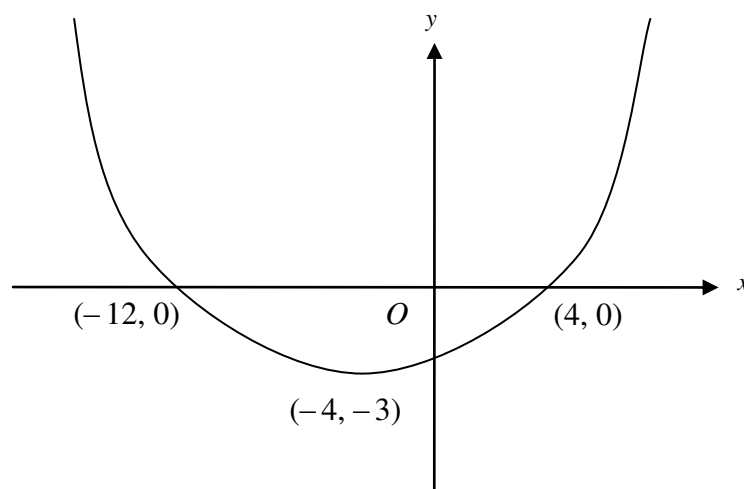
If M1 not gained, allow SC1 for correctly simplified numerator or denominator following multiplication of top and bottom by $3\sqrt{2} + \sqrt{11}$

- (b) $\frac{7}{2\sqrt{14}} = p\sqrt{14}$, where p is a fraction equivalent to $\frac{1}{4}$ B1
 $\left[\frac{\sqrt{14}}{2}\right]^3 = q\sqrt{14}$, where q is a fraction equivalent to $\frac{7}{4}$ B1
 $\frac{7}{2\sqrt{14}} + \left[\frac{\sqrt{14}}{2}\right]^3 = 2\sqrt{14}$ (c.a.o.) B1

3. (a) y-coordinate of $P = -4$ B1
 $\frac{dy}{dx} = 3x^2 - 2x - 13$
 (an attempt to differentiate, at least one non-zero term correct) M1
 An attempt to substitute $x = 2$ in candidate's expression for $\frac{dy}{dx}$ m1
 Value of $\frac{dy}{dx}$ at $P = -5$ (c.a.o.) A1
 Gradient of normal = $\frac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1
 Equation of normal to C at P : $y - (-4) = \frac{1}{5}(x - 2)$ (or equivalent) (f.t. candidate's value for $\frac{dy}{dx}$ and the candidate's derived y-value at $x = 2$ provided M1 and both m1's awarded) A1
- (b) Putting candidate's expression for $\frac{dy}{dx} = -8$ M1
 An attempt to collect terms, form and solve quadratic equation in a (or x) either by correct use of the quadratic formula or by getting the equation into the form $(ma + n)(pa + q) = 0$, with $m \times p =$ candidate's coefficient of a^2 and $n \times q =$ candidate's constant m1
 $3a^2 - 2a - 5 = 0 \Rightarrow a = -1$ or $\frac{5}{3}$ (both values) (c.a.o.) A1
4. (a) $4(x - 3)^2 - 225$ B1 B1 B1
- (b) $4(x - 3)^2 = 225$ (f.t. candidate's values for a, b, c) M1
 $(x - 3) = (\pm) \frac{15}{2}$ (f.t. candidate's values for a, b, c) m1
 $x = \frac{21}{2}, -\frac{9}{2}$ (both values) A1
5. (a) An expression for $b^2 - 4ac$, with at least two of a, b or c correct M1
 $b^2 - 4ac = (2k - 5)^2 - 4 \times k \times (k - 6)$ A1
 Putting $b^2 - 4ac < 0$ m1
 $k < -\frac{25}{4}$ (or equivalent) A1
- (b) $k = -\frac{25}{4}$ [f.t. the end point(s) of the candidate's range in (a)] B1

6. (a) $\left[1 - \frac{x}{2}\right]^8 = 1 - 4x + 7x^2 - 7x^3 + \dots$ B1 B1 B1 B1
(– 1 for further incorrect simplification)
- (b) First term = 2^n B1
 $2^n = 32 \Rightarrow n = 5$ B1
Second term = $n \times 2^{n-1} \times ax$ B1
 $a = -3$ (f.t. candidate's value for n) B1
7. (a) $y + \delta y = 9(x + \delta x)^2 - 8(x + \delta x) - 3$ B1
Subtracting y from above to find δy M1
 $\delta y = 18x\delta x + 9(\delta x)^2 - 8\delta x$ A1
Dividing by δx and letting $\delta x \rightarrow 0$ M1
 $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = 18x - 8$ (c.a.o.) A1
- (b) $\frac{dy}{dx} = 3 \times (-6) \times x^{-7} - 4 \times \frac{5}{3} \times x^{2/3}$ B1 B1
8. (a) Use of $f(3) = 0$ M1
 $27p - 117 - 57 + 12 = 0 \Rightarrow p = 6$ (convincing) A1
Special case
Candidates who assume $p = 6$ and show $f(3) = 0$ are awarded B1
- (b) $f(x) = (x - 3)(6x^2 + ax + b)$ with one of a, b correct M1
 $f(x) = (x - 3)(6x^2 + 5x - 4)$ A1
 $f(x) = (x - 3)(2x - 1)(3x + 4)$ (f.t. only $6x^2 - 5x - 4$ in above line) A1
Roots are $x = 3, \frac{1}{2}, -\frac{4}{3}$ (f.t. for factors $2x \pm 1, 3x \pm 4$) A1
- Special case**
Candidates who find one of the remaining factors, $(2x - 1)$ or $(3x + 4)$, using e.g. factor theorem, are awarded B1

9. (a)



Concave up curve and y-coordinate of minimum = -3	B1
x-coordinate of minimum = -4	B1
Both points of intersection with x-axis	B1

(b) **Either:**

Any graph of the form $y = af(x)$ (with $a \neq 0$) will intersect the x-axis at $(-6, 0)$ and $(2, 0)$ and thus not pass through the origin.

Or:

$f(0) \neq 0$ and since $a \neq 0$, $af(0) \neq 0$. Thus any graph of the form $y = af(x)$ will not pass through the origin. E1

10. (a)	$L = x + 2y$		
	$800 = xy$	(both equations)	M1
	$L = x + \frac{1600}{x}$	(convincing)	A1
(b)	$\frac{dL}{dx} = 1 + 1600 \times (-1) \times x^{-2}$		B1
	Putting derived $\frac{dL}{dx} = 0$		M1
	$x = 40, (-40)$	(f.t. candidate's $\frac{dL}{dx}$)	A1
	Stationary value of L at $x = 40$ is 80	(c.a.o)	A1
	A correct method for finding nature of the stationary point yielding a minimum value (for $x > 0$)		B1

C2

- 1.
- | | | |
|-----|--------------|-----------------------|
| 1 | 0.1111111111 | |
| 1.5 | 0.1709352011 | |
| 2 | 0.2329431339 | |
| 2.5 | 0.2969522777 | |
| 3 | 0.3628469322 | (5 values correct) B2 |
- (If B2 not awarded, award B1 for either 3 or 4 values correct)

Correct formula with $h = 0.5$ M1

$$I \approx \frac{0.5}{2} \times \{0.1111111111 + 0.3628469322 + 2(0.1709352011 + 0.2329431339 + 0.2969522777)\}$$

$$I \approx 1.875619269 \times 0.5 \div 2$$

$$I \approx 0.4689048172$$

$$I \approx 0.4689 \quad (\text{f.t. one slip}) \quad \text{A1}$$

Special case for candidates who put $h = 0.4$

1	0.1111111111	
1.4	0.1587880562	
1.8	0.2078915826	
2.2	0.2583141854	
2.6	0.3099833063	
3	0.3628469322	(all values correct) B1

Correct formula with $h = 0.4$ M1

$$I \approx \frac{0.4}{2} \times \{0.1111111111 + 0.3628469322 + 2(0.1587880562 + 0.2078915826 + 0.2583141854 + 0.3099833063)\}$$

$$I \approx 2.343912304 \times 0.4 \div 2$$

$$I \approx 0.4687824609$$

$$I \approx 0.4688 \quad (\text{f.t. one slip}) \quad \text{A1}$$

Note: Answer only with no working shown earns 0 marks

2. (a) $4(1 - \sin^2 \theta) - 2 \sin^2 \theta - \sin \theta + 8 = 0$,
 (correct use of $\cos^2 \theta = 1 - \sin^2 \theta$) M1
 An attempt to collect terms, form and solve quadratic equation
 in $\sin \theta$, either by using the quadratic formula or by getting the
 expression into the form $(a \sin \theta + b)(c \sin \theta + d)$,
 with $a \times c =$ candidate's coefficient of $\sin^2 \theta$ and $b \times d =$ candidate's
 constant m1
 $6 \sin^2 \theta + \sin \theta - 12 = 0 \Rightarrow (2 \sin \theta + 3)(3 \sin \theta - 4) = 0$
 $\Rightarrow \sin \theta = -\frac{3}{2}, \sin \theta = \frac{4}{3}$ (c.a.o.) A1
 $-1 \leq \sin \theta \leq 1 \Rightarrow$ no such θ can exist
 (f.t. only if candidate has 2 real values for $\sin \theta$, **neither** of which
 satisfies $-1 \leq \sin \theta \leq 1$) E1
- (b) $2x - 75^\circ = -31^\circ, 211^\circ, 329^\circ$, (one value) B1
 $x = 22^\circ, 143^\circ$ B1 B1
 Note: Subtract (from final two marks) 1 mark for each additional root
 in range, ignore roots outside range.
- (c) $4 \sin \phi + 7 \sin \phi \cos \phi = 0$ **or** $4 \tan \phi + 7 \tan \phi \cos \phi = 0$
or $\sin \phi \left[\frac{4}{\cos \phi} + 7 \right] = 0$ M1
 $\sin \phi = 0$ (or $\tan \phi = 0$), $\cos \phi = -\frac{4}{7}$ (both values) A1
 $\phi = 0^\circ, 180^\circ$ (both values) A1
 $\phi = 124.85^\circ$ (c.a.o.) A1
 Note: Subtract a maximum of 1 mark for each additional root in range
 for each branch, ignore roots outside range.
Special Case:
 No factorisation but division throughout by $\sin \phi$ (or $\tan \phi$) to yield
 $4 + 7 \cos \phi = 0$ (or equivalent) M1
 $\phi = 124.85^\circ$ A1
3. (a) $\frac{\sin ACB}{19} = \frac{\sin 25^\circ}{12}$
 (substituting the correct values in the correct places in the sin rule) M1
 $ACB = 42^\circ, 138^\circ$ (both values) A1
- (b) (i) $\hat{BAC} + 25^\circ + 138^\circ = 180^\circ$
 (f.t. either of candidate's values for ACB) M1
 $\hat{BAC} = 17^\circ$ (f.t. candidate's obtuse value for ACB) A1
 (ii) Area of triangle $ABC = \frac{1}{2} \times 19 \times 12 \times \sin 17^\circ$
 (substituting 19, 12 and candidate's derived value for $\hat{BAC}$ in
 the correct places in the area formula) M1
 Area of triangle $ABC = 33.33 \text{ cm}^2$. (c.a.o.) A1

4. (a) (i) n th term $= 4 + 6(n - 1) = 4 + 6n - 6 = 6n - 2$ (convincing) B1
- (ii) $S_n = 4 + 10 + \dots + (6n - 8) + (6n - 2)$
 $S_n = (6n - 2) + (6n - 8) + \dots + 10 + 4$
 Reversing and adding M1
Either:
 $2S_n = (6n + 2) + (6n + 2) + \dots + (6n + 2) + (6n + 2)$
Or:
 $2S_n = (6n + 2) + \dots$ (n times) A1
 $2S_n = n(6n + 2)$
 $S_n = n(3n + 1)$ (convincing) A1
- (b) (i) $a + 9d = 4 \times (a + 4d)$ B1
 $3a + 7d = 0$
 $\frac{15}{2} \times (2a + 14d) = 210$ B1
 $a + 7d = 14$
 An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1
 $d = 3, a = -7$ (c.a.o.) A1
- (ii) $-7 + (k - 1) \times 3 = 200$
 (f.t. candidate's derived values for a and d) M1
 $k = 70$ (c.a.o.) A1
5. (a) $r = \frac{2304}{576} = 4$ (c.a.o.) B1
 $t_5 = \frac{576}{4^3}$ (f.t. candidate's value for r) M1
 $t_5 = 9$ (c.a.o.) A1
- (b) (i) $ar^2 = 24$ B1
 $ar + ar^2 + ar^3 = -56$ B1
 An attempt to solve the candidate's equations simultaneously by eliminating a M1
 $\frac{r^2}{r + r^2 + r^3} = -\frac{24}{56} \Rightarrow 3r^2 + 10r + 3 = 0$ (convincing) A1
- (ii) $r = -\frac{1}{3}$ ($r = -3$ discarded, c.a.o.) B1
 $a = 216$
 (f.t. candidate's derived value for r , provided $|r| < 1$) B1
 $S_\infty = \frac{216}{1 - (-1/3)}$ (use of formula for sum to infinity)
 (f.t. candidate's derived values for r and a) M1
 $S_\infty = 162$ (f.t. candidate's derived values for r and a) A1

6. (a) $3 \times \frac{x^{1/2}}{1/2} - 6 \times \frac{x^{7/3}}{7/3} + c$ B1, B1
(-1 if no constant term present)
- (b) (i) $6 + 5x - x^2 = 4x$ M1
An attempt to rewrite and solve quadratic equation in x , either by using the quadratic formula or by getting the expression into the form $(x + a)(x + b)$, with $a \times b =$ candidate's constant m1
 $(x + 2)(x - 3) = 0 \Rightarrow x = 3$ (c.a.o.) A1
- (ii) Use of integration to find the area under the curve M1
 $\int 6 \, dx = 6x, \quad \int 5x \, dx = \frac{5x^2}{2}, \quad \int x^2 \, dx = (1/3)x^3,$
(correct integration) B1
Correct method of substitution of candidate's limits m1
 $[6x + (5/2)x^2 - (1/3)x^3]_{-1}^3$
 $= (18 + 45/2 - 9) - (-6 + 5/2 - (-1/3)) = 104/3$
Use of a correct method to find the area of the triangle (f.t. candidate's coordinates for A) M1
Use of -1 and candidate's value for x_A as limits and trying to find total area by subtracting area of triangle from area under curve m1
Shaded area $= 104/3 - 18 = 50/3$ (c.a.o.) A1
7. (a) Let $p = \log_a x, q = \log_a y$
Then $x = a^p, y = a^q$ (the relationship between log and power) B1
 $\frac{x}{y} = \frac{a^p}{a^q} = a^{p-q}$ (the laws of indices) B1
 $\log_a x/y = p - q$ (the relationship between log and power)
 $\log_a x/y = p - q = \log_a x - \log_a y$ (convincing) B1
- (b) $\log_a(6x^2 + 9x + 2) - \log_a x = \log_a \left[\frac{6x^2 + 9x + 2}{x} \right]$
(subtraction law) B1
 $4 \log_a 2 = \log_a 2^4$ (power law) B1
 $\frac{6x^2 + 9x + 2}{x} = 2^4$ (removing logs) M1
An attempt to solve quadratic equation with three terms in x , either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c =$ candidate's coefficient of x^2 and $b \times d =$ candidate's constant m1
 $6x^2 - 7x + 2 = 0 \Rightarrow (2x - 1)(3x - 2) = 0 \Rightarrow x = 1/2, 2/3$
(both values, c.a.o.) A1

Note: Answer only with no working earns 0 marks

8. (a) (i) $A(3, -1)$ B1
(ii) A correct method for finding radius M1
Radius = $\sqrt{29}$ (convincing) A1
- (b) **Either:**
 $RQ = \sqrt{18}$ or $RP = \sqrt{98}$ (o.e.) B1
Correct substitution of candidate's values in an expression for $\sin Q$,
 $\cos Q$ or $\tan Q$ M1
 $PQR = 66.8^\circ$ (c.a.o) A1
Or:
 $RQ = \sqrt{18}$ or $RP = \sqrt{98}$ B1
Correct substitution of candidate's values in the cos rule to find $\cos Q$ M1
 $PQR = 66.8^\circ$ (c.a.o) A1
- (c) $AT^2 = 65$ (f.t. candidate's coordinates for A) B1
Use of $ST^2 = AT^2 - AS^2$ with candidate's derived value for AT M1
 $ST = 6$ (f.t. one slip) A1
9. Area of sector $AOB = \frac{1}{2} \times r^2 \times 2.6$ B1
Area of triangle $AOB = \frac{1}{2} \times r^2 \times \sin 2.6$ B1
Area of minor segment = $\frac{1}{2} \times r^2 \times 2.6 - \frac{1}{2} \times r^2 \times \sin 2.6 = 1.0422r^2$ B1
Use of a valid method for finding the area of the major segment M1
Area of major segment = $2.099r^2$
 $\Rightarrow$ area of major segment $\approx 2 \times$ area of minor segment (convincing) A1

C3

1. (a)
- | | | | | | |
|--|---|----------------|--------------------|--|--------------------|
| | 0 | 0 | | | |
| | $\pi/9$ | -0.062202456 | | | |
| | $2\pi/9$ | -0.266515091 | | | |
| | $\pi/3$ | -0.693147181 | | | |
| | $4\pi/9$ | -1.750723994 | (5 values correct) | | B2 |
| | (If B2 not awarded, award B1 for either 3 or 4 values correct) | | | | |
| | Correct formula with $h = \pi/9$ | | | | M1 |
| | $I \approx \frac{\pi/9}{3} \times \{0 + (-1.750723994)$ <div style="text-align: right; margin-right: 100px;"> $+ 4[(-0.062202456) + (-0.693147181)]$ $+ 2(-0.266515091)\}$ </div> | | | | |
| | $I \approx -5.305152724 \times (\pi/9) \div 3$ | | | | |
| | $I \approx -0.617282549$ | | | | |
| | $I \approx -0.6173$ | | | | (f.t. one slip) A1 |

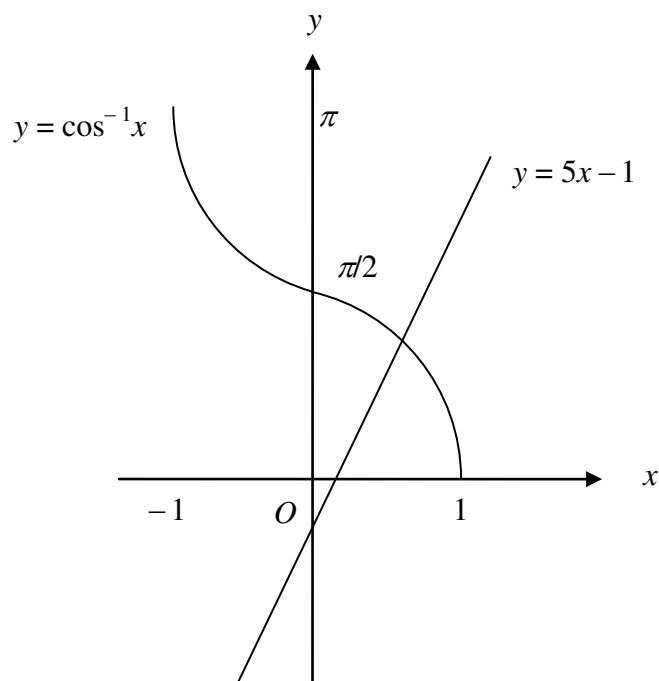
Note: Answer only with no working shown earns 0 marks

- (b)
- | | | | | | |
|--|----------------------------------|------------------|----------------------------------|--|----|
| | $\int_0^{4\pi/9} \ln(\sec x) dx$ | ≈ 0.6173 | (f.t. candidate's answer to (a)) | | B1 |
|--|----------------------------------|------------------|----------------------------------|--|----|

2. (a) $7 \operatorname{cosec}^2 \theta - 4 (\operatorname{cosec}^2 \theta - 1) = 16 + 5 \operatorname{cosec} \theta$
 (correct use of $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$) M1
 An attempt to collect terms, form and solve quadratic equation
 in $\operatorname{cosec} \theta$, either by using the quadratic formula or by getting the
 expression into the form $(a \operatorname{cosec} \theta + b)(c \operatorname{cosec} \theta + d)$,
 with $a \times c =$ candidate's coefficient of $\operatorname{cosec}^2 \theta$ and $b \times d =$ candidate's
 constant m1
 $3 \operatorname{cosec}^2 \theta - 5 \operatorname{cosec} \theta - 12 = 0 \Rightarrow (\operatorname{cosec} \theta - 3)(3 \operatorname{cosec} \theta + 4) = 0$
 $\Rightarrow \operatorname{cosec} \theta = 3, \operatorname{cosec} \theta = -\frac{4}{3}$
 $\Rightarrow \sin \theta = \frac{1}{3}, \sin \theta = -\frac{3}{4}$ (c.a.o.) A1
 $\theta = 19.47^\circ, 160.53^\circ$ B1
 $\theta = 311.41^\circ, 228.59^\circ$ B1 B1
 Note: Subtract 1 mark for each additional root in range for each
 branch, ignore roots outside range.
 $\sin \theta = +, -, \text{f.t. for 3 marks, } \sin \theta = -, -, \text{f.t. for 2 marks}$
 $\sin \theta = +, +, \text{f.t. for 1 mark}$
- (b) $\sec \phi \geq 1, \operatorname{cosec} \phi \geq 1$ and thus $4 \sec \phi + 3 \operatorname{cosec} \phi \geq 7$ E1
3. (a) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(1) = 0$ $\frac{d}{dx}(\pi^2/4) = 0$ B1
 $\frac{d}{dx}(2x \cos y) = 2x(-\sin y) \frac{dy}{dx} + 2 \cos y$ B1
 $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$ B1
 $\frac{dy}{dx} = \frac{3}{2 - \pi}$ (c.a.o.) B1
- (b) $\frac{d^2 y}{dx^2} = \frac{d}{dx}(x^2 y) = x^2 \frac{dy}{dx} + 2xy$ B1
 Substituting $x^2 y$ for $\frac{dy}{dx}$ in candidate's derived expression for $\frac{d^2 y}{dx^2}$ M1
 $\frac{d^2 y}{dx^2} = x^2(x^2 y) + 2xy = x^4 y + 2xy$ (o.e.) (c.a.o.) A1

4. (a) candidate's x -derivative = $\frac{1}{1+t^2}$ B1
- candidate's y -derivative = $\frac{1}{t}$ B1
- $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
- $\frac{dy}{dx} = \frac{1+t^2}{t}$ A1
- (b) $\frac{d}{dt}\left[\frac{dy}{dx}\right] = -t^{-2} + 1$ (o.e.) B1
- Use of $\frac{d^2y}{dx^2} = \frac{d}{dt}\left[\frac{dy}{dx}\right] \div \text{candidate's } x\text{-derivative}$ M1
- $\frac{d^2y}{dx^2} = (-t^{-2} + 1)(1+t^2)$ (o.e.) (f.t. one slip) A1
- $\frac{d^2y}{dx^2} = 0 \Rightarrow t = 1$ (c.a.o.) A1
- $\frac{d^2y}{dx^2} = 0 \Rightarrow x = \frac{\pi}{4}$ (f.t. candidate's derived value for t) A1

5. (a)



Correct shape for $y = \cos^{-1}x$ B1

A straight line with negative y-intercept and positive gradient intersecting once with $y = \cos^{-1}x$ in the first quadrant. B1

(b) $x_0 = 0.4$

$x_1 = 0.431855896$ (x_1 correct, at least 4 places after the point) B1

$x_2 = 0.424849379$

$x_3 = 0.426400166$

$x_4 = 0.426057413 = 0.4261$ (x_4 correct to 4 decimal places) B1

Let $h(x) = \cos^{-1}x - 5x + 1$

An attempt to check values or signs of $h(x)$ at $x = 0.42605$,
 $x = 0.42615$ M1

$h(0.42605) = 4.24 \times 10^{-4} > 0$, $h(0.42615) = -1.86 \times 10^{-4} < 0$ A1

Change of sign $\Rightarrow \alpha = 0.4261$ correct to four decimal places A1

6. (a) (i) $\frac{dy}{dx} = \frac{a+bx}{4x^2-3x-5}$ (including $a=1, b=0$) M1
 $\frac{dy}{dx} = \frac{8x-3}{4x^2-3x-5}$ A1
- (ii) $\frac{dy}{dx} = e^{\sqrt{x}} \times f(x)$ ($f(x) \neq 1, 0$) M1
 $\frac{dy}{dx} = e^{\sqrt{x}} \times \frac{1}{2} x^{-1/2}$ A1
- (iii) $\frac{dy}{dx} = \frac{(a-b \sin x) \times m \cos x - (a+b \sin x) \times k \cos x}{(a-b \sin x)^2}$
 $(m = \pm b, k = \pm b)$ M1
 $\frac{dy}{dx} = \frac{(a-b \sin x) \times b \cos x - (a+b \sin x) \times (-b) \cos x}{(a-b \sin x)^2}$ A1
 $\frac{dy}{dx} = \frac{2ab \cos x}{(a-b \sin x)^2}$ A1
- (b) $\frac{d}{dx}(\cot x) = \frac{d}{dx}(\tan x)^{-1} = (-1) \times (\tan x)^{-2} \times f(x)$ ($f(x) \neq 1, 0$) M1
 $\frac{d}{dx}(\tan x)^{-1} = (-1) \times (\tan x)^{-2} \times \sec^2 x$ A1
 $\frac{d}{dx}(\tan x)^{-1} = -\operatorname{cosec}^2 x$ (convincing) A1

7. (a) (i) $\int \frac{(7x^2 - 2)}{x} dx = \int 7x dx - \int \frac{2}{x} dx$
 Correctly rewriting as two terms and an attempt to integrate M1
 $\int \frac{(7x^2 - 2)}{x} dx = \frac{7x^2}{2} - 2 \ln x + c$ A1 A1
- (ii) $\int \sin(2x/3 - \pi) dx = k \times \cos(2x/3 - \pi) + c$
 $(k = -1, -3/2, 3/2, -2/3)$ M1
 $\int \sin(2x/3 - \pi) dx = -3/2 \times \cos(2x/3 - \pi) + c$ A1

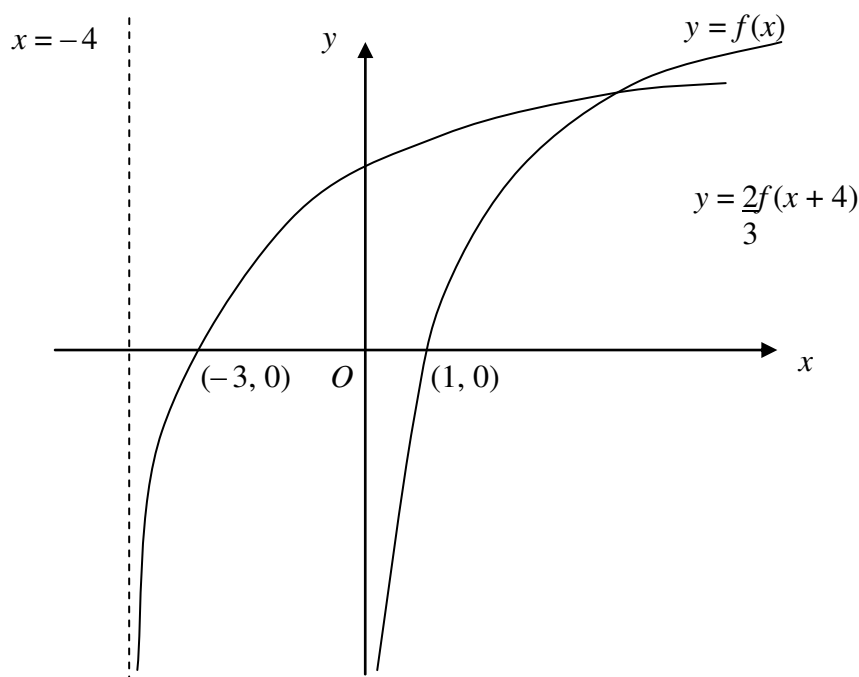
Note: The omission of the constant of integration is only penalised once.

- (b) $\int (5x - 14)^{-1/4} dx = \frac{k \times (5x - 14)^{3/4}}{3/4}$ ($k = 1, 5, 1/5$) M1
 $\int (5x - 14)^{-1/4} dx = 1/5 \times \frac{(5x - 14)^{3/4}}{3/4}$ A1
 A correct method for substitution of the correct limits in an expression of the form $m \times (5x - 14)^{3/4}$ M1
 $\int_3^6 (5x - 14)^{-1/4} dx = \frac{28}{15}$ ($= 1.867$)
 (f.t. only for solutions of $\frac{28}{3}$ ($= 9.333$) and $\frac{140}{3}$ ($= 46.667$)
 from $k = 1, k = 5$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (a) Trying to solve either $3x - 5 \leq 1$ or $3x - 5 \geq -1$ M1
 $3x - 5 \leq 1 \Rightarrow x \leq 2$
 $3x - 5 \geq -1 \Rightarrow x \geq 4/3$ (both inequalities) A1
 Required range: $4/3 \leq x \leq 2$ (f.t. one slip) A1
- Alternative mark scheme**
 $(3x - 5)^2 \leq 1$
 (squaring both sides, forming and trying to solve quadratic) M1
 Critical values $x = 4/3$ and $x = 2$ A1
 Required range: $4/3 \leq x \leq 2$ (f.t. one slip in critical values) A1
- (b) $4/3 \leq 1/y \leq 2$ (f.t. candidate's $a \leq x \leq b, a > 0, b > 0$) M1
 $1/2 \leq y \leq 3/4$ (f.t. candidate's $a \leq x \leq b, a > 0, b > 0$) A1

9.



Correct shape, including the fact that the y -axis is an asymptote for

$y = f(x)$ at $-\infty$

B1

$y = f(x)$ cuts x -axis at $(1, 0)$

B1

Correct shape, including the fact that $x = -4$ is an asymptote for

$y = \frac{2}{3}f(x + 4)$ at $-\infty$

B1

3

$y = \frac{2}{3}f(x + 4)$ cuts x -axis at $(-3, 0)$ (f.t. candidate's x -intercept for $f(x)$)

B1

3

The diagram shows that the graph of $y = f(x)$ is steeper than the graph of $y = \frac{2}{3}f(x + 4)$ in the first quadrant

B1

3

10. (a) Choice of h, k such that $h(x) = k(x) + c, c \neq 0$

M1

Convincing verification of the fact that $h'(x) = k'(x)$

A1

(b) (i) $y - 3 = 2 \ln(4x + 5)$

B1

An attempt to express candidate's equation as an exponential equation

M1

$$x = \frac{(e^{(y-3)/2} - 5)}{4}$$

(c.a.o.)

A1

$$f^{-1}(x) = \frac{(e^{(x-3)/2} - 5)}{4}$$

(f.t. one slip in candidate's expression for x)

A1

(ii) $D(f^{-1}) = [10, 14]$

B1 B1

(iii) $gf(x) = e^{2 \ln(4x+5)+3}$

B1

$$e^{2 \ln(4x+5)} = (4x+5)^2$$

B1

$$gf(x) = e^3(4x+5)^2$$

(c.a.o.)

B1

C4

1. (a) $f(x) \equiv \frac{A}{(x+3)^2} + \frac{B}{(x+3)} + \frac{C}{(x-1)}$ (correct form) M1

$2x^2 + 5x + 25 \equiv A(x-1) + B(x+3)(x-1) + C(x+3)^2$
(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -7, C = 2, B = 0$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(b) $\int \frac{f(x)}{(x+3)} dx = \frac{7}{(x+3)} + 2 \ln(x-1)$ B1 B1
(f.t. candidate's values for A, B, C)

$\int_3^{10} f(x) dx = \left[\frac{7}{13} + 2 \ln 9 \right] - \left[\frac{7}{6} + 2 \ln 2 \right] = 2.38$ (c.a.o.) B1

Note: Answer only with no working earns 0 marks

2. (a) $4x^3 + 3x^2 \frac{dy}{dx} + 6xy - 4y \frac{dy}{dx} = 0$ $\left[\frac{3x^2 \frac{dy}{dx} + 6xy}{dx} \right]$ B1

$\left[\frac{4x^3 - 4y \frac{dy}{dx}}{dx} \right]$ B1

$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$ (convincing) B1

(b) $4y - 3x^2 = 0$ M1

Either: Substituting $\frac{3x^2}{4}$ for y in the equation of C and an

attempt to collect terms m1

$x^4 = 16 \Rightarrow x = (\pm) 2$ A1

$y = 3$ (for both values of x)

(f.t. $x^4 = a, a \neq 16$, provided both x values are checked)

A1

Or: Substituting $\frac{4y}{3}$ for x^2 in the equation of C and an

attempt to collect terms m1

$y^2 = 9 \Rightarrow y = (\pm) 3$ A1

$y = 3 \Rightarrow x = \pm 2$ (f.t. $y^2 = b, b \neq 9$) A1

3. (a) $\frac{\tan x + \tan 45^\circ}{1 - \tan x \tan 45^\circ} = 8 \tan x$ (correct use of formula for $\tan(x + 45^\circ)$) M1
 Use of $\tan 45^\circ = 1$ and an attempt to form a quadratic in $\tan x$ by cross multiplying and collecting terms M1
 $8 \tan^2 x - 7 \tan x + 1 = 0$ (c.a.o.) A1
 Use of a correct method to solve the candidate's derived quadratic in $\tan x$ m1
 $x = 34.8^\circ, 10.2^\circ$ (both values)
 (f.t. one slip in candidate's derived quadratic in $\tan x$ provided all three method marks have been awarded) A1
- (b) (i) $R = 7$ B1
 Correctly expanding $\sin(\theta - \alpha)$, correctly comparing coefficients and using either $7 \cos \alpha = \sqrt{13}$ **or** $7 \sin \alpha = 6$ **or** $\tan \alpha = \frac{6}{\sqrt{13}}$ to find α (f.t. candidate's value for R) M1
 $\alpha = 59^\circ$ (c.a.o.) A1
- (ii) $\sin(\theta - \alpha) = -\frac{4}{7}$
 (f.t. candidate's values for R, α) B1
 $\theta - 59^\circ = -34.85^\circ, 214.85^\circ, 325.15^\circ,$
 (at least one value, f.t. candidate's values for R, α) B1
 $\theta = 24.15^\circ, 273.85^\circ$ (c.a.o.) B1
4. (a) $V = \pi \int_0^a (mx)^2 dx$ M1
 $\int_0^a (mx)^2 dx = \frac{m^2 x^3}{3}$ B1
 $V = \pi \frac{m^2 a^3}{3}$ (c.a.o.) A1
- (b) (i) Substituting $\frac{b}{a}$ for m in candidate's derived expression for V M1
 $V = \pi \frac{b^2 a}{3}$ (c.a.o.) A1
- (ii) This is the volume of a cone of (vertical) height a and (base) radius b E1

5. $\left(1 + \frac{x}{8}\right)^{-1/2} = 1 - \frac{x}{16} + \frac{3x^2}{512}$ $\left(1 - \frac{x}{16}\right)$ B1
 $\left[\frac{3x^2}{512}\right]$ B1
- $|x| < 8$ or $-8 < x < 8$ B1
 $\frac{2\sqrt{2}}{3} \approx 1 - \frac{1}{16} + \frac{3}{512}$ (f.t. candidate's coefficients) B1
- Either:** $\sqrt{2} \approx \frac{1449}{1024}$ (c.a.o.)
- Or:** $\sqrt{2} \approx \frac{2048}{1449}$ (c.a.o.) B1
6. (a) (i) candidate's x -derivative = $2at$
candidate's y -derivative = $2a$ (at least one term correct)
and use of
 $\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1
 $\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$
Gradient of tangent at $P = \frac{1}{p}$ (c.a.o.) A1
- (ii) Equation of tangent at P : $y - 2ap = \frac{1}{p}(x - ap^2)$
(f.t. candidate's expression for $\frac{dy}{dx}$) m1
Equation of tangent at P : $py = x + ap^2$ A1
- (b) (i) Gradient $PQ = \frac{2ap - 2aq}{ap^2 - aq^2}$ B1
Use of $ap^2 - aq^2 = a(p + q)(p - q)$ B1
Gradient $PQ = \frac{2}{p + q}$ (c.a.o.) B1
- (ii) As the point Q approaches P , PQ becomes a tangent
Limit (gradient PQ) = $\frac{2}{2p} = \frac{1}{p}$. E1
 $q \rightarrow p$

$$7. \quad (a) \quad \int \frac{x^2}{(12-x^3)^2} dx = \int \frac{k}{u^2} du \quad (k = 1/3, -1/3, 3 \text{ or } -3) \quad \text{M1}$$

$$\int \frac{a}{u^2} du = a \times \frac{u^{-1}}{-1} \quad \text{B1}$$

Either: Correctly inserting limits of 12, 4 in candidate's bu^{-1}
or: Correctly inserting limits of 0, 2 in candidate's $b(12-x^3)^{-1}$ M1

$$\int_0^2 \frac{x^2}{(12-x^3)^2} dx = \frac{1}{18} \quad (\text{c.a.o.}) \quad \text{A1}$$

Note: Answer only with no working earns 0 marks

$$(b) \quad (i) \quad u = x \Rightarrow du = dx \quad (\text{o.e.}) \quad \text{B1}$$

$$dv = \cos 2x dx \Rightarrow v = \frac{1}{2} \sin 2x \quad (\text{o.e.}) \quad \text{B1}$$

$$\int x \cos 2x dx = x \times \frac{1}{2} \sin 2x - \int \frac{1}{2} \sin 2x \times dx \quad \text{M1}$$

$$\int x \cos 2x dx = \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c \quad (\text{c.a.o.}) \quad \text{A1}$$

$$(ii) \quad \int x \sin^2 x dx = \int x \left[\frac{k}{2} - \frac{m}{2} \cos 2x \right] dx \quad (\text{o.e.})$$

$$(k = 1, -1, m = 1, -1) \quad \text{M1}$$

$$\int x \sin^2 x dx = \frac{1}{2} \int x dx - \frac{1}{2} \int x \cos 2x dx \quad \text{A1}$$

$$\int x \sin^2 x dx = \frac{x^2}{4} - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + c$$

(f.t. only candidate's answer to (b)(i)) A1

$$8. \quad (a) \quad (i) \quad \mathbf{AB} = -\mathbf{i} - 2\mathbf{j} + 7\mathbf{k} \quad \text{B1}$$

$$(ii) \quad \text{Use of } \mathbf{a} + \lambda\mathbf{AB}, \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}), \mathbf{b} + \lambda\mathbf{AB} \text{ or } \mathbf{b} + \lambda(\mathbf{b} - \mathbf{a}) \text{ to find vector equation of } AB \quad \text{M1}$$

$$\mathbf{r} = 5\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}) \quad (\text{o.e.})$$

(f.t. if candidate uses his/her expression for $\mathbf{AB}$) A1

$$(b) \quad 5 - \lambda = 2 + \mu$$

$$-1 - 2\lambda = -3 + \mu$$

$$-1 + 7\lambda = -4 - \mu \quad (\text{o.e.})$$

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her equation of AB)

$$\lambda = -1, \mu = 4 \quad (\text{o.e.}) \quad (\text{c.a.o.}) \quad \text{A1}$$

Correct verification that values of λ and μ satisfy third equation A1

Position vector of point of intersection is $6\mathbf{i} + \mathbf{j} - 8\mathbf{k}$

9. (a) $\frac{dP}{dt} = kP^2$ (f.t. one slip) A1
B1
- (b) $\int \frac{dP}{P^2} = \int k \, dt$ M1
 $-\frac{1}{P} = kt + c$ (o.e.) A1
 $c = -\frac{1}{A}$ (c.a.o.) A1
 $-\frac{1}{P} = kt - \frac{1}{A} \Rightarrow kt = \frac{1}{A} - \frac{1}{P} \Rightarrow \frac{1}{k} \left[\frac{P-A}{PA} \right] = t$ (convincing) A1
- (c) $\frac{1}{k} \left[\frac{800-A}{800A} \right] = 3, \quad \frac{1}{k} \left[\frac{900-A}{900A} \right] = 4$ (both equations) B1
 An attempt to solve these equations simultaneously by eliminating k M1
 $A = 600$ (c.a.o.) A1
10. Assume that 4 is a factor of $a + b$.
 Then there exists an integer c such that $a + b = 4c$.
 Similarly, there exists an integer d such that $a - b = 4d$. B1
 Adding, we have $2a = 4c + 4d$. B1
 Therefore $a = 2c + 2d$, an even number, which contradicts the fact that a is odd. B1

FP1

Ques	Solution	Mark	Notes
1	$f(x+h) - f(x) = \frac{1}{(x+h)^2 - (x+h)} - \frac{1}{x^2 - x}$ $= \frac{x^2 - x - [(x+h)^2 - (x+h)]}{[(x+h)^2 - (x+h)](x^2 - x)}$ $= \frac{x^2 - x - [x^2 + 2hx + h^2 - x - h]}{[(x+h)^2 - (x+h)](x^2 - x)}$ $= \frac{-2hx - h^2 + h}{[(x+h)^2 - (x+h)](x^2 - x)}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-2x - h + 1}{[(x+h)^2 - (x+h)](x^2 - x)} = \frac{-2x+1}{(x^2 - x)^2}$	M1A1 A1 A1 A1 M1 A1	oe
2(a)	The reflection matrix for $y = x$ is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ The reflection matrix for $y = -x$ is $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ It follows that $\mathbf{T} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \text{ cao}$	B1 B1 M1 A1 B1	Allow the use of 3×3 matrices Special case B1 for matrices the wrong way round Do not award this A1 for a 3×3 matrix
(b)	T therefore corresponds to a rotation through 180° about the origin. cao		
3(a)	$\frac{2+i}{1-i} = \frac{(2+i)(1+i)}{(1-i)(1+i)}$ $= \frac{2+3i+i^2}{1-i+i-i^2}$ $= \frac{1}{2} + \frac{3}{2}i$ Let $z = x + iy$ so that $\bar{z} = x - iy$ $2(x + iy) - i(x - iy) = \frac{1}{2} + \frac{3}{2}i$ $2x - y = \frac{1}{2}; 2y - x = \frac{3}{2}$ $x = \frac{5}{6}; y = \frac{7}{6} \left(\text{so } z = \frac{5}{6} + \frac{7}{6}i \right)$	M1 A1 A1 M1 A1 A1	FT their above result

	METHOD 2 Let the result be true for $n = k$, ie $\mathbf{A}^k = k\mathbf{A} - (k - 1)\mathbf{I}$ $= \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix}$ Consider, for $n = k + 1$, $\mathbf{A}^{k+1} = \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 2(k+1) & 1 \end{bmatrix}$ Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since trivially true for $n = 1$ ($\mathbf{A} = \mathbf{A}$), the result is proved by induction. METHOD 3 Let the result be true for $n = k$, ie $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^k = k \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - (k - 1) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Consider, for $n = k + 1$, $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{k+1} = \left\{ k \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} - (k - 1) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2(k+1) & 1 \end{bmatrix}$ But the assumed result for $n = k$ can be written as $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & 0 \\ 2k & 1 \end{bmatrix}$ Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since trivially true for $n = 1$ ($\mathbf{A} = \mathbf{A}$), the result is proved by induction.	M1 A1 M1 A1 A1 A1 A1 M1 M1 A1 A1 A1 A1	Award this A1 for a correct concluding statement and correct presentation of proof by induction Award this A1 for a correct concluding statement and correct presentation of proof by induction
9(a) Taking logs correctly, $\ln f(x) = x \ln 2 + \ln \sin x$ Differentiating, $\frac{f'(x)}{f(x)} = \ln 2 + \cot x$ $f'(x) = 2^x \sin x (\ln 2 + \cot x)$ (b) Stationary value where $f'(x) = 0$ $x = \cot^{-1}(-\ln 2) \text{ rad}$ $= 2.18$	M1 A1A1 A1 M1 A1 A2	Condone ignoring $\sin x = 0$ Award A1 for – 0.96	

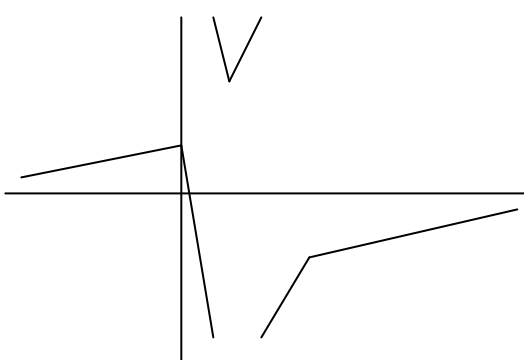
10(a)(i) (ii) (b)(i) (ii)	$ z + 3 = k z - i $ Putting $z = x + iy$, $(x + 3)^2 + y^2 = k^2 x^2 + k^2 (y - 1)^2$ $x^2 + 6x + 9 + y^2 = k^2 x^2 + k^2 y^2 - 2k^2 y + k^2$ $(k^2 - 1)x^2 + (k^2 - 1)y^2 - 6x - 2k^2 y + k^2 - 9 = 0$ (which is the equation of the circle.) Rewriting the equation in the form $x^2 + y^2 - \frac{6}{(k^2 - 1)}x - \frac{2k^2}{(k^2 - 1)}y = \frac{9 - k^2}{(k^2 - 1)}$ Completing the square, $\left(x - \frac{3}{k^2 - 1}\right)^2 + \left(y - \frac{k^2}{k^2 - 1}\right)^2 = \text{terms involving } k$ $\text{Centre} = \left(\frac{3}{k^2 - 1}, \frac{k^2}{k^2 - 1}\right)$ $6x + 2y + 8 = 0$ It is the perpendicular bisector of the line joining the points $(-3, 0)$ and $(0, 1)$	M1 A1 A1 M1 m1 A1 A1 B1 B1	Award full credit for the use of the standard result for the coordinates of the centre
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FP2

Ques	Solution	Mark	Notes
1(a)	Let $\frac{5}{(x^2+1)(2-x)} = \frac{Ax+B}{x^2+1} + \frac{C}{2-x}$ $= \frac{(Ax+B)(2-x) + C(x^2+1)}{(x^2+1)(2-x)}$ $A=1; B=2; C=1$ $\left(\frac{5}{(x^2+1)(2-x)} = \frac{x+2}{x^2+1} + \frac{1}{2-x} \right)$	M1 A1A1A1	
(b)	$u = \tan x \Rightarrow du = \sec^2 x dx$ $[0, \pi/4] \rightarrow [0, 1]$ $I = \int_0^1 \frac{5}{(2-u)} \times \frac{du}{(1+u^2)}$ $= \int_0^1 \left(\frac{u+2}{u^2+1} + \frac{1}{2-u} \right) du$ $= \left[\frac{1}{2} \ln(u^2+1) + 2 \tan^{-1} u - \ln(2-u) \right]_0^1$ $= 2.61 \text{ cao}$	B1 B1 M1A1 A1 B1B1B1 A1	Award M0 if no working
2(a)	Denoting the two functional expressions by f_1, f_2 $f_1(-1) = 4, f_2(-1) = -a - b$ Therefore $a + b = -4$ $f_1'(x) = 2x - 1, f_2'(x) = 3ax^2 + b$ $f_1'(-1) = -3, f_2'(1) = 3a + b$ Therefore $3a + b = -3$ Solving, $a = \frac{1}{2}, b = -\frac{9}{2}$	M1 A1 M1 A1 A1A1	FT one slip in equations
(b)	Solving $\frac{1}{2}x^3 - \frac{9}{2}x = 0; x = -3$	M1A1	FT if possible Award M1 for attempting to solve this equation
3(a)	Modulus of cube roots $= \sqrt[3]{2}$ $R1 = \sqrt[3]{2}(\cos \pi/4 + i \sin \pi/4)$ $= 0.891 + 0.891i$ $R2 = \sqrt[3]{2}(\cos 11\pi/12 + i \sin 11\pi/12)$ $= -1.217 + 0.326i$ $R3 = \sqrt[3]{2}(\cos 19\pi/12 + i \sin 19\pi/12)$ $= 0.326 - 1.217i$	B1 M1 A1 M1 A1 A1	Use of de Moivre's Theorem FT their modulus Addition of $2\pi/3$ to argument Penalise accuracy only once

(b)(i) (ii)	z^n is real when $n = 4$ and imaginary when $n = 2$.	B2 B1	Award B1 for $n = 8$
4	METHOD 1 Combining the first and third terms, $2\cos\left(2\theta + \frac{\pi}{6}\right)\cos\theta + \cos\left(2\theta + \frac{\pi}{6}\right) = 0$ $\cos\left(2\theta + \frac{\pi}{6}\right)(2\cos\theta + 1) = 0$ Either $\cos\theta = -\frac{1}{2}$, $\theta = 2n\pi \pm \frac{2\pi}{3} \text{ or } (2n+1)\pi \pm \frac{\pi}{3}$ Or $\cos\left(2\theta + \frac{\pi}{6}\right) = 0$ $2\theta + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{2} \text{ or } \left(n + \frac{1}{2}\right)\pi$ $\theta = n\pi \pm \frac{\pi}{4} - \frac{\pi}{12} \text{ or } \frac{n\pi}{2} + \frac{\pi}{6}$ METHOD 2 $\cos\theta\cos\frac{\pi}{6} - \sin\theta\sin\frac{\pi}{6} + \cos 2\theta\cos\frac{\pi}{6}$ $- \sin 2\theta\sin\frac{\pi}{6} + \cos 3\theta\cos\frac{\pi}{6} - \sin 3\theta\sin\frac{\pi}{6} = 0$ Combining appropriate terms, $\cos\frac{\pi}{6}(2\cos\theta\cos 2\theta + \cos 2\theta)$ $= \sin\frac{\pi}{6}[2\sin 2\theta\cos\theta + \sin 2\theta]$ $\frac{\sqrt{3}}{2}\cos 2\theta(2\cos\theta + 1) = \frac{1}{2}\sin 2\theta(2\cos\theta + 1)$ Either $\cos\theta = -\frac{1}{2}$, $\theta = 2n\pi \pm \frac{2\pi}{3} \text{ or } (2n+1)\pi \pm \frac{\pi}{3}$ Or $\tan 2\theta = \sqrt{3}$ $2\theta = n\pi + \frac{\pi}{3}$ $\theta = \frac{n\pi}{2} + \frac{\pi}{6}$	M1A1 A1 M1 A1 M1 A1 A1 M1 A1 M1 A1 A1 M1 A1 A1	M1 for combining two terms Accept equivalent answers Accept equivalent answers Accept equivalent answers Accept equivalent answers

5(a) (b) (c)	$\frac{d}{dx} \left(\int_0^x e^{\sqrt{u}} du \right) = e^{\sqrt{x}}$ Put $y = x^2; \frac{dy}{dx} = 2x$ $\frac{d}{dx} \left(\int_0^{x^2} e^{\sqrt{u}} du \right) = \frac{d}{dy} \left(\int_0^y e^{\sqrt{u}} du \right) \times \frac{dy}{dx}$ $= 2xe^x$ $\int_x^{x^2} e^{\sqrt{u}} du = \int_0^{x^2} e^{\sqrt{u}} du - \int_0^x e^{\sqrt{u}} du$ $\frac{d}{dx} \left(\int_x^{x^2} e^{\sqrt{u}} du \right) = 2xe^x - e^{\sqrt{x}} \quad \text{cao}$	B1 M1 A1 A1 M1 A1	Do not accept integration followed by differentiation Do not accept integration followed by differentiation Award this M1 for the difference of integrals
6(a) (b)(i) (ii) (iii) (iv)	We are given that $x^2 + (y - 3)^2 = (y + 3)^2$ $x^2 + y^2 - 6y + 9 = y^2 + 6y + 9$ $x^2 = 12y$ $x^2 = 36t^2; 12y = 36t^2$ showing that the point $(6t, 3t^2)$ lies on C. $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ $= \frac{6t}{6} = t$ The equation of the tangent is $y - 3t^2 = t(x - 6t)$ $y = tx - 3t^2$ Substituting $(0, -12)$ into the equation, $-12 = -3t^2$ $t = \pm 2$ Since the positive gradient of the tangent is equal to 2, the angle between the tangent and the y-axis is equal to $\tan^{-1}(1/2)$. The angle between the tangents is therefore equal to $2 \tan^{-1}(1/2) = 53.1^\circ$ or 0.927 rad	M1 A1 B1 M1 A1 M1A1 M1 A1 M1 A1	Do not accept solutions which assume the equation given the focus and directrix Award M1 for any valid method Accept 126.9° or 2.21 rad

7(a)	$x = 1, x = 2$	B1	
(b)	$f(0) = 1$ giving the point (0,1) $f(x) = 0 \Rightarrow x = 2/3$ giving the point (2/3,0)	B1 M1A1	
(c)	$f'(x) = -\frac{1}{(x-1)^2} + \frac{4}{(x-2)^2}$ At a stationary point, $\frac{1}{(x-1)^2} = \frac{4}{(x-2)^2}$ $\frac{1}{(x-1)} = \pm \frac{2}{(x-2)}$ giving (0,1) and (4/3,9) $f''(x) = \frac{2}{(x-1)^3} - \frac{8}{(x-2)^3}$ $f''(0) < 0$ so that (0,1) is a maximum $f''(4/3) > 0$ so that (4/3,9) is a minimum	B1 M1 A1 A1A1 M1 A1 A1	Award A1A0 if only x values given Accept any valid method including looking at appropriate values of $f(x)$ or $f'(x)$
(d)		G1 G1	Award G1 for 2 correct branches
(e)(i)	$f(-1) = 5/6 ; f(0) = 1$ $f(S) = [5/6, 1]$	M1 A1	
(ii)	Solve $\frac{1}{x-1} - \frac{4}{x-2} = -1$ $x^2 - 6x + 4 = 0$ $x = 3 \pm \sqrt{5}$ $f^{-1}(S) = [2/3, 3 - \sqrt{5}] \cup [3 + \sqrt{5}, \infty)$	M1 A1 A1 A1	

FP3

Ques	Solution	Mark	Notes	
1(a)	Expanding the right hand side, $5\cosh\theta + 3\sinh\theta = r\cosh\theta\cosh\alpha + r\sinh\theta\sinh\alpha$ Therefore $r\cosh\alpha = 5$ and $r\sinh\alpha = 3$ Squaring and subtracting, $r^2(\cosh^2\alpha - \sinh^2\alpha) = 5^2 - 3^2$ so that $r = 4$ Dividing, $\frac{\sinh\alpha}{\cosh\alpha} = \tanh\alpha = \frac{3}{5}$ $\alpha = \tanh^{-1}\left(\frac{3}{5}\right) = 0.693$	M1 A1 A1	Condone the absence of $\pm$ here	
(b)	Substituting, $4\cosh(\theta + 0.693) = 10$ $(\theta + 0.693) = \pm \cosh^{-1}\left(\frac{10}{4}\right)$ $\theta = -0.693 \pm \cosh^{-1}\left(\frac{10}{4}\right)$ $= -2.26, 0.874$	M1 A1 A1A1		
2	EITHER $I = \int_0^{\pi/2} e^{2x} d(\sin x)$ $= \left[e^{2x} \sin x \right]_0^{\pi/2} - 2 \int_0^{\pi/2} e^{2x} \sin x dx$ $= e^{\pi} - 2 \int_0^{\pi/2} e^{2x} d(-\cos x)$ $= e^{\pi} + 2 \left[e^{2x} \cos x \right]_0^{\pi/2} - 4I$ $= e^{\pi} - 2 - 4I$ $I = \frac{e^{\pi} - 2}{5}$	M1 A1 A1A1 A1 A1 A1		

	OR $I = \int_0^{\pi/2} \cos x \, d\left(\frac{e^{2x}}{2}\right)$ $= \left[\frac{e^{2x}}{2} \cos x\right]_0^{\pi/2} + \frac{1}{2} \int_0^{\pi/2} e^{2x} \sin x \, dx$ $= -\frac{1}{2} + \frac{1}{2} \int_0^{\pi/2} \sin x \, d\left(\frac{e^{2x}}{2}\right)$ $= -\frac{1}{2} + \frac{1}{4} [e^{2x} \sin x]_0^{\pi/2} - \frac{1}{4} I$ $= -\frac{1}{2} + \frac{1}{4} e^{\pi} - \frac{1}{4} I$ $I = \frac{e^{\pi}/4 - 1/2}{5/4} = \frac{e^{\pi} - 2}{5}$	M1 A1 A1A1 A1 A1 A1	
3(a)(i) (ii) (b)(i) (ii)	$f'(x) = 12x^3 - 12x^2 - 6x - 6$ $f'(1.4) = -4.99... f'(1.6) = 2.83...$ The change in sign shows that α lies between 1.4 and 1.6. Since α satisfies $f'(\alpha) = 0$, it follows that $12\alpha^3 - 12\alpha^2 - 6\alpha - 6 = 0$ so that $2\alpha^3 = 2\alpha^2 + \alpha + 1$ $\alpha = \left(\frac{2\alpha^2 + \alpha + 1}{2}\right)^{\frac{1}{3}}$ Let $F(x) = \left(\frac{2x^2 + x + 1}{2}\right)^{\frac{1}{3}}$ $F'(x) = \frac{1}{3} \left(\frac{2x^2 + x + 1}{2}\right)^{-\frac{2}{3}} \times \left(\frac{4x + 1}{2}\right)$ $F'(1.5) = 0.506...$ The sequence converges because $F'(1.5) < 1$ Using the iterative formula, successive values are 1.5 1.518294486 1.527545210 etc $\alpha = 1.537$ (to 3 dps)	B1 B1 B1 M1 A1 M1A1 A1 A1 M1 A1 A1 A1	

(ii)	$\text{CSA} = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$ $= 2\pi \int_0^\pi (1 - \cos t) \times 2 \cos \frac{1}{2}t dt$ $= 4\pi \int_0^\pi \left(\cos \frac{1}{2}t dt - \frac{1}{2} \left(\cos \frac{3}{2}t + \cos \frac{1}{2}t \right) \right) dt$ $= 4\pi \left[\sin \frac{1}{2}t - \frac{1}{3} \sin \frac{3}{2}t \right]_0^\pi$ $= \frac{16\pi}{3}$	M1 A1 A1 A1 A1	Or $8\pi \int_0^\pi \sin^2 \frac{1}{2}t \cos \frac{1}{2}t dt$ $= \frac{16\pi}{3} \left[\sin^3 \frac{1}{2}t \right]_0^\pi$
6(a) (b) (c)(i) (ii)	$\frac{d}{dx} \left((4 - x^2)^{\frac{3}{2}} \right) = \frac{3}{2} (4 - x^2)^{\frac{1}{2}} \times (-2x)$ $= -3x(4 - x^2)^{\frac{1}{2}}$ $I_n = -\frac{1}{3} \int_0^2 x^{n-1} \frac{d}{dx} ((4 - x^2)^{3/2}) dx$ $= -\frac{1}{3} \left[x^{n-1} (4 - x^2)^{3/2} \right]_0^2 + \frac{n-1}{3} \int_0^2 x^{n-2} (4 - x^2)^{3/2} dx$ $= \left(\frac{n-1}{3} \right) \int_0^2 x^{n-2} (4 - x^2)^{3/2} dx$ $= \frac{n-1}{3} (4I_{n-2} - I_n)$ $I_n = \left(\frac{4(n-1)}{n+2} \right) I_{n-2}$ Evaluate $I_0 = \int_0^2 \sqrt{4 - x^2} dx$ Put $x = 2 \sin \theta$, $dx = 2 \cos \theta d\theta$, $[0, 2] \rightarrow [0, \pi/2]$ $I_0 = 4 \int_0^{\pi/2} \cos^2 \theta d\theta$ $= 2 \int_0^{\pi/2} (1 + \cos 2\theta) d\theta$ $= 2 \left[\theta + \frac{1}{2} \sin 2\theta \right]_0^{\pi/2}$ $= \pi$ $I_4 = 2I_2$ $= 2 \times 1 \times I_0$ $= 2\pi$	B1 M1 A1A1 A1 A1 M1 M1A1 A1 A1 M1 A1 A1	Convincing



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GCE MARKING SCHEME

**MATHEMATICS - M1-M3 & S1-S3
AS/Advanced**

SUMMER 2015

INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2015 examination in GCE MATHEMATICS M1-M3 & S1-S3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

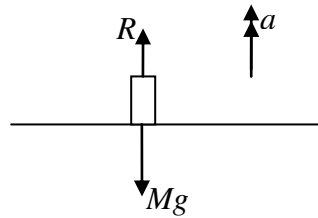
WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

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M2	10
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M1

Q	Solution	Mark	Notes
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1.



N2L applied to man

M1 R and Mg opposing.
dim correct

$$R - Mg = Ma$$

A1

$$680 = M(9.8 + 0.2)$$

$$M = \underline{68}$$

A1 cao

N2L applied to Lift and Man

M1 T and weight opposing.
dim correct.

$$T - 1868g = 1868a$$

A1 ft M

$$T = \underline{18680 \text{ (N)}}$$

A1 ft M

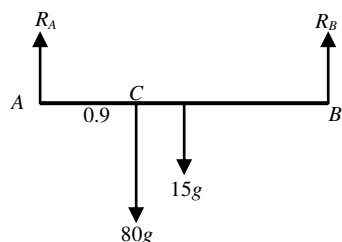
Q	Solution	Mark	Notes
2.	Apply N2L to B	M1	dim correct, all forces.
	$5g - T = 0$	A1	allow $5a$ RHS $5g$ and T opposing.
	Resolve perpendicular to plane for A	M1	allow sin
	$R = 4g \cos \alpha$	A1	
	Apply N2L to A	M1	Friction opposes motion.
	$T - 4g \sin \alpha - F = 0$	A1	Allow $4a$ RHS and/or cos
	At limiting equilibrium $F = \mu R$	M1	used
	$\mu = \frac{F}{R} = \frac{45g}{48g} = \frac{15}{16}$	A1	convincing
	$T = 5g = 49$		
	$F = T - 4g \sin \alpha = \frac{45g}{13} = \frac{441}{13} = 33.9231$		
	$R = 4g \times \frac{12}{13} = \frac{48g}{13} = \frac{2352}{65} = 36.1846$		

Q	Solution	Mark	Notes
3(a)	Conservation of momentum	M1	attempted, equation, dim correct.
	$3 \times 8 + 5 \times 2 = 3v_A + 5v_B$	A1	
	$3v_A + 5v_B = 34$		
	Restitution	M1	
	$v_B - v_A = -\frac{1}{3}(2 - 8)$	A1	
	$v_B - v_A = 2$		
	$3v_A + 5v_B = 34$		
	$-3v_A + 3v_B = 6$		
	Adding	m1	dep on both M's
	$8v_B = 40$		
	$v_B = \underline{5 \text{ (ms}^{-1}\text{)}}$	A1	cao
	$v_A = \underline{3 \text{ (ms}^{-1}\text{)}}$	A1	cao
3(b)	Impulse = change of momentum	M1	used
	$I = 5 \times 5 - 5 \times 2 = \underline{15 \text{ (Ns)}}$	A1	ft v_A or v_B

Q	Solution	Mark	Notes
4	Moments about x-axis $= 5 \times (-1) + 2 \times (3) + 3 \times 5 + 6 \times 0$ $16y = 16$ $y = 1$	B1 M1 si A1 cao	
	Moments about y-axis $= 5 \times 4 + 2 \times 2 + 3 \times (-2) + 6 \times (-3)$ $16x = 0$ $x = 0$	B1 M1 si A1 cao	

Q	Solution	Mark	Notes
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5(a)



Moments about A

$$2.8R_B = 80g \times 0.9 + 15g \times 1.4$$

$$R_B = \underline{325.5 \text{ (N)}}$$

Vertical forces in equilibrium

$$R_A + R_B = 80g + 15g$$

$$R_A = \underline{605.5 \text{ (N)}}$$

M1 3 terms, dim correct
Equation required
A1 correct equation
B1 any correct moment

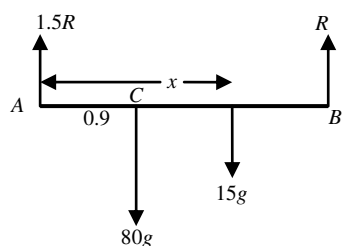
A1 cao

M1 all forces, no extra

A1

A1 cao

5(b)



Resolve vertically

$$1.5R + R = 95g$$

$$R = 38g$$

Moments about A

$$2.8 \times R = 80g \times 0.9 + 15g \times x$$

$$x = \frac{172}{75} = \underline{2.3 \text{ (m)}}$$

M1

A1

M1 3 terms, dim correct

A1 oe

A1 cao

Q	Solution	Mark	Notes
6(a)		B1 labels, units and shape B1 (0, 0) to (10, 20) B1 (10, 20) to (10+T, 20)	
6(b)	$v = u + at, v=20, u=0, t=10$ $20 = 0 + 10a$ $a = \underline{2 \text{ (ms}^{-2}\text{)}}$	M1 A1	
6(c)	Total distance = area under graph $D = 0.5 \times 10 \times 20 + 20T$ $D = 100 + 20T \text{ (m)}$	M1 attempted B1 one correct area A1 cao	
6(d)	$s = ut + 0.5at^2, u=0, t=5+T, a=2$ $s = 0.5 \times 2 \times (5+T)^2$ $D = 25 + 10T + T^2$	M1 A1	
	$25 + 10T + T^2 = 100 + 20T$ $T^2 - 10T - 75 = 0$ $(T + 5)(T - 15) = 0$ $T = 15$ $D = \underline{400 \text{ (m)}}$	M1 Ft exp for D in (d) and (c) A1 cao A1 cao	

Q	Solution	Mark	Notes
7	Resolve in 80 N direction $80 = P\cos 60^\circ + Q\cos 45^\circ$	M1 A1	Equation required
	Resolve in 25 N direction $25 = P\sin 60^\circ - Q\sin 45^\circ$	M1 A1	Equation required
	$160 = P + Q\sqrt{2}$ $50 = P\sqrt{3} - Q\sqrt{2}$		
Adding		m1	dep on both M's
	$(1 + \sqrt{3})P = 210$		
	$P = \underline{76.9}$	A1	cao
	$Q = \underline{58.8}$	A1	cao
			penalise once if not 1 d.p.

Q	Solution	Mark	Notes
8(a)	Use of $v^2=u^2+2as$ with $u=(\pm)2.1, a=(\pm)9.8,$ $s=(\pm)4.$ $v^2 = 2.1^2 + 2 \times 9.8 \times 4$ $v = 9.1$ speed of rebound $= 9.1 \times \frac{4}{7}$ $= \underline{5.2 \text{ (ms}^{-1}\text{)}}$	M1 A1 A1 m1 A1	 allow - convincing
8(b)	We require smallest n st $\left(\frac{4}{7}\right)^n \times 9.1 < 1$ 4 bounces	M1 A1	 oe, si trial & error

Q	Solution				Mark	Notes
9	<i>BCD</i>	45	19	(5)	B1	for 19
	<i>ABDE</i>	160	8	(5)		
	Circle	9π	7	(5)	B1	both 8 and 7 required
	Lamina	$205-9\pi$	x	(y)	B1	expressions for areas, oe
	Moments about <i>AE</i>				M1	
	$(205-9\pi)x + 9\pi \times 7 = 160 \times 8 + 45 \times 19$				A1	signs correct. Ft table if at least one B1 for c of m gained.
	$x = \underline{10.96}$				A1	cao
	$y = \underline{5}$				B1	

M2

Q	Solution	Mark	Notes
1.	$\mathbf{x} \cdot \mathbf{y} = 0$ $(\sin\theta \mathbf{i} + 2\cos 2\theta \mathbf{j}) \cdot (2\mathbf{i} - \mathbf{j}) = 0$ $2\sin\theta - 2\cos 2\theta = 0$ $\sin\theta - (1 - 2\sin^2\theta) = 0$ $2\sin^2\theta + \sin\theta - 1 = 0$ $(2\sin\theta - 1)(\sin\theta + 1) = 0$ $\sin\theta = 0.5$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\sin\theta = -1$ $\theta = \frac{3\pi}{2}$	M1 M1 A1 m1 A1 A1	used correct method for dot product, no $\mathbf{i}, \mathbf{j}$'s $\cos 2\theta = 1 - 2\sin^2\theta$ depends on both M's both values

Q	Solution	Mark	Notes
2(a)	Apply N2L to object $1600 - R = 50a$ $1600 - kt = 50a$ When $t = 2$, $a = -4$ $1600 - 2k = 50 \times (-4)$ $k = 900$ $1600 - 900t = 50 \frac{dv}{dt}$ $\frac{dv}{dt} = 32 - 18t$	M1	
		B1 m1	$R = kt$ used
		A1	convincing
2(b)	$\int dv = \int 32 - 18t \, dt$ $v = 32t - 9t^2 (+ C)$ When $t = 2$, $v = 41$ $C = 9 \times 2^2 - 32 \times 2 + 41$ $C = 13$ $v = -9t^2 + 32t + 13$ When $v = 28$, $28 = -9t^2 + 32t + 13$ $9t^2 - 32t + 15 = 0$ $(9t - 5)(t - 3) = 0$ $t = \frac{5}{9}, 3$	M1	increase in power at least once
		A1 m1	used
		A1	cao
		m1	substitution of $v=28$ in c's expression for $v(t)$.
		A1	cao

Q	Solution	Mark	Notes
3.			
	N2L $T - mgsin\alpha - R = ma$ $T = \frac{P}{v}$ $\frac{5P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 6000 \times 2$ $\frac{5P}{16} - R = 19200$	M1 A1 B1 A1	dim correct, all forces correct equation used si correct equation in P & R
	N2L with $a = 0$ $T - mgsin\alpha - R = 0$ $\frac{3P}{16} - 6000 \times 9.8 \times \frac{6}{49} - R = 0$ $\frac{3P}{16} - R = 7200$	M1 A1 A1	dim correct, all forces correct equation correct equation in P & R
	Solving simultaneously $\frac{2P}{16} = 12000$ $P = 96000; R = 10800$	m1 A1	eliminating one variable, depends on both M's both answers cao

Q	Solution	Mark	Notes
4(a)	N2L $(4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j} = 0.5\mathbf{a}$ $\mathbf{a} = (8t - 6)\mathbf{i} + (6t^2 - 10t)\mathbf{j}$ $\mathbf{v} = \int \mathbf{a} \, dt$ $\mathbf{v} = (4t^2 - 6t)\mathbf{i} + (2t^3 - 5t^2)\mathbf{j} + (\mathbf{c})$ When $t = 0$, $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$ $\mathbf{c} = 8\mathbf{i} - 7\mathbf{j}$ $\mathbf{v} = (4t^2 - 6t)\mathbf{i} + (2t^3 - 5t^2)\mathbf{j} + 8\mathbf{i} - 7\mathbf{j}$ $\mathbf{v} = (4t^2 - 6t + 8)\mathbf{i} + (2t^3 - 5t^2 - 7)\mathbf{j}$	M1 A1 M1 A1 A1	use of $\mathbf{F} = m\mathbf{a}$ cao attempted, $\mathbf{i}$, $\mathbf{j}$ retained, power of t increased once ft $\mathbf{a}$ of same diff, not $\mathbf{F}$
4(b)	Impulse = change in momentum When $t = 3$, $\mathbf{v} = 26\mathbf{i} + 2\mathbf{j}$ $0.5(x\mathbf{i} + y\mathbf{j}) - 0.5(26\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} - 9\mathbf{j}$ $(x\mathbf{i} + y\mathbf{j}) = 30\mathbf{i} - 16\mathbf{j}$ Speed = $\sqrt{30^2 + (-16)^2}$ Speed = <u>34 ms⁻¹</u>	M1 B1 A1 M1 A1	attempted, vector form required si ft c's $\mathbf{v}$ cao ft c's x, y cao

Q	Solution	Mark	Notes
5(a)	$T = 15g$ Hooke's Law $T = \frac{\lambda x}{l} = \frac{1470 \times x}{0.4}$ $x = \frac{15 \times 9.8 \times 0.4}{1470}$ $x = \underline{0.04 \text{ (m)}}$	B1 M1 A1	si cao
5(b)	Let PE be zero at the natural length level. $PE = mgh$ Initial PE = $15 \times 9.8 \times (-0.16)$ Initial PE = -23.52 J Initial EE = $\frac{1}{2} \times \frac{\lambda (x)^2}{l}$ Initial EE = $\frac{1}{2} \times \frac{1470(0.16)^2}{0.4}$ Initial EE = 47.04 J Final KE = $0.5mv^2$ Final KE = $7.5v^2$ Final PE = $15 \times 9.8 \times -0.05$ Final PE = -7.35 J Final EE = $\frac{1}{2} \times \frac{1470(0.05)^2}{0.4}$ Final EE = 4.59375 J Conservation of energy $7.5v^2 - 7.35 + 4.59375 = 47.04 - 23.52$ $v^2 = 3.5035$ $v = \underline{1.8718} = \underline{1.87 \text{ (ms}^{-1}\text{)(to 2 d.p.)}}$	M1 A1 M1 A1 B1 M1 A1 A1	used used equation, all 3 types all correct, any form

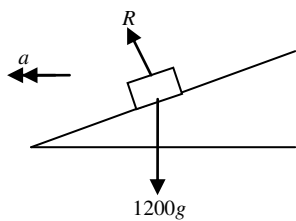
Q	Solution	Mark	Notes
6(a)	Initial $u_H = 35\cos\alpha = (35 \times 0.6 = 21) \text{ (ms}^{-1}\text{)}$ Initial $u_V = 35\sin\alpha = (35 \times 0.8 = 28) \text{ (ms}^{-1}\text{)}$	B1 B1	si si
	use of $s = ut + 0.5at^2$ with $s=0, u=28\text{(c)}, a=(\pm)9.8$ $0 = 28t + 0.5(-9.8)t^2$ $t(28 - 4.9t) = 0$ $t = (0), \frac{40}{7}$	M1 A1 A1	complete method ft u_V
	Total distance travelled by ball = $\frac{40}{7} \times 21$ = 120 (m)		
	Ball will not fall into lake.	A1	
6(b)	time to tree = $\frac{17.5}{21} = \frac{5}{6}$	B1	
	Use $v = u + at$ with $u=28\text{(c)}, a=(\pm)9.8, t=5/6\text{(c)}$	M1	oe complete method
	$v = 28 - 9.8 \times \frac{5}{6}$	A1	
	$v = \frac{119}{6} (= 19.8333)$		
	speed = $\sqrt{\left(\frac{119}{6}\right)^2 + (21)^2}$	m1	
	speed = <u>28.89 (ms⁻¹)</u>	A1	cao
	$\theta = \tan^{-1}\left(\frac{119}{6 \times 21}\right)$	m1	
	$\theta = \underline{43.36^\circ}$	A1	cao

Q

Solution

Mark Notes

7



Resolve vertically

M1 equation, dim correct
No extra force

$$R \cos 12^\circ = 1200g$$

$$R = \underline{12022.73 \text{ (N)}}$$

A1

N2L towards the centre of motion

M1 dim correct,
no extra force

$$R \sin 12^\circ = \frac{1200 \times v^2}{80}$$

A1

$$v = \underline{12.91}$$

A1 cao

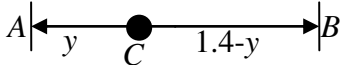
Q	Solution	Mark	Notes
8(a)(i)	Conservation of energy $0.5 \times 3 \times 5^2 =$ $3 \times 9.8 \times 0.8(1 - \cos \theta) + 0.5 \times 3 \times v^2$ $25 = v^2 + 1.6 \times 9.8 - 1.6 \times 9.8 \cos \theta$ $v^2 = \underline{9.32 + 15.68 \cos \theta}$	M1 A1A1 A1	KE and PE cao
8(a)(ii)	N2L towards centre of motion $T - 3g \cos \theta = \frac{3v^2}{0.8}$ $T = 3g \cos \theta + 3.75(9.32 + 15.68 \cos \theta)$ $T = \underline{34.95 + 88.2 \cos \theta}$	M1 A1 m1 A1	dim correct, 3 terms T , $3g \cos \theta$ opposing ft v^2 of form $a \pm b \sin / \cos \theta$ cao
8(b)	Greatest value of θ occurs when $T=0$ $34.95 + 88.2 \cos \theta = 0$ $\cos \theta = - \frac{34.95}{88.2}$ $\theta = \underline{113.34^\circ}$ Motion stops being circular when $\theta = 113.34^\circ$ as string cannot support negative tension. P moves under the action of gravity only.	M1 A1 E1	ft T of form $a \pm b \sin / \cos \theta$ ft $a + b \cos \theta$ ft $\theta > 90^\circ$

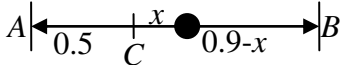
M3

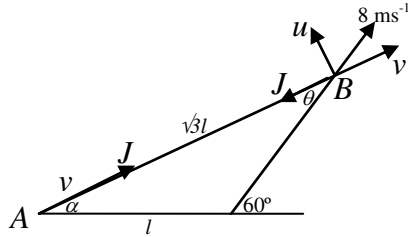
Q	Solution	Mark	Notes
1(a).	Use of N2L $F = 400a$ $500\left(\frac{x}{v+2}\right) = 400v \frac{dv}{dx}$ $5x = 4v(v+2) \frac{dv}{dx}$	M1 A1	use of $a = v \frac{dv}{dx}$
1(b)(i)	$\int 5x dx = \int 4(v^2 + 2v) dv$ $\frac{5}{2}x^2 = 4\left(\frac{v^3}{3} + v^2\right) + (C)$ When $x = 0, v = 0$, hence $C = 0$ $x = \sqrt{\frac{8}{5}\left(\frac{v^3}{3} + v^2\right)}$	M1 A1A1 m1 A1	sep variables any correct form
1(b)(ii)	When $v = 3$ $2.5x^2 = 4(9 + 9)$ $x = \frac{12}{\sqrt{5}} \text{ m} = \underline{5.37 \text{ m}}$ $a = \frac{5}{4}\left(\frac{12}{5\sqrt{5}}\right)$ $a = \frac{3}{\sqrt{5}} = \underline{1.34 \text{ (ms}^{-2}\text{)}}$	m1 A1 m1 A1	cao substitution of x and $v=3$. cao

Q	Solution	Mark	Notes
2(a)(i).	$N2L \ 0.5a = -6.5x - 2v$ $\frac{1}{2} \frac{d^2x}{dt^2} = -\frac{13}{2}x - 2\frac{dx}{dt}$ $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0$	M1 A1	dimensionally correct $a = \frac{d^2x}{dt^2}$, $v = \frac{dx}{dt}$.
2(a)(ii)	Axilliary equation $m^2 + 4m + 13 = 0$ $m = -2 \pm 3i$ C. F. is $x = e^{-2t}(A\sin 3t + B\cos 3t)$ When $t=0$, $x=6$, $\frac{dx}{dt}=3$ $B = 6$ $\frac{dx}{dt} = -2e^{-2t}(A\sin 3t + B\cos 3t)$ $+ e^{-2t}(3A\cos 3t - 3B\sin 3t)$ $-2B + 3A = 3$ $A = 5$ Solution is $x = e^{-2t}(5\sin 3t + 6\cos 3t)$ When t is large, $x \approx 0$	M1 A1 A1 m1 B1 A1 A1	ft m if complex used ft $e^{kt}(A\sin pt + B\cos pt)$ cao
2(b)	Try PI $x = at + b$ $4a + 13(at + b) = 91t + 15$ $13a = 91$ $a = 7$ $4a + 13b = 15$ $b = -1$ G.S. is $x = e^{-2t}(A\sin 3t + B\cos 3t) + 7t - 1$	M1 A1 m1 A1	equating coefficients cao both

Q	Solution	Mark	Notes
3(a)	$\text{N2L } 250a = 250g - 50v$ $5 \frac{dv}{dt} = 5g - v$	M1 A1	dimensionally correct convincing
3(b)	$\int \frac{5dv}{5g - v} = \int dt$ $-5 \ln 5g - v = t (+C)$ When $t = 0, v = 0$ $-5 \ln 5g = C$ $-\frac{t}{5} = \ln \left \frac{5g - v}{5g} \right $ $5ge^{-\frac{t}{5}} = 5g - v$ $v = 5g \left(1 - e^{-\frac{t}{5}} \right)$ When $t = 5, v = 5g(1 - e^{-1})$ $= 30.974 \text{ (ms}^{-1}\text{)}$	M1 A1 m1 A1 m1 A1 A1	separation of variables correct integration used correct inversion cao cao numerical answer.
3(c)	$\frac{dx}{dt} = 5g - 5ge^{-\frac{t}{5}}$ $x = 5gt + 25ge^{-\frac{t}{5}} (+C)$ When $t = 0, x = 0$ $C = -25g$ $x = 5gt + 25ge^{-\frac{t}{5}} - 25g$ When $t = 5,$ $x = 25ge^{-1} = 90.13 \text{ (m)}$	M1 A1 m1 A1 A1	$v = \frac{dx}{dt}$ correct integration ft similar expression used cao

Q	Solution	Mark	Notes
4(a)	 Tension of spring at A = $\frac{15(y - 0.3)}{0.3}$ Tension of spring at B = $\frac{20(1.4 - y - 0.6)}{0.6}$ When in equilibrium $T_A = T_B$ $\frac{15(y - 0.3)}{0.3} = \frac{20(1.4 - y - 0.6)}{0.6}$ $30y - 9 = 16 - 20y$ $50y = 25$ $y = \underline{0.5 \text{ (m)}}$	B1 B1 M1 A1 A1	all correct convincing

Q	Solution	Mark	Notes
4(b)(i)	 $T_A = \frac{15(0.2 + x)}{0.3}$ $T_B = \frac{20(0.3 - x)}{0.6}$ $\text{Force to right} = \frac{20(0.3 - x)}{0.6} - \frac{15(0.2 + x)}{0.3}$ $= -\frac{250x}{3}$ $\text{Apply N2L to } P, 7.5 \frac{d^2x}{dt^2} = -\frac{250x}{3}$ $\frac{d^2x}{dt^2} = -\frac{100}{9}x$ $\text{Therefore motion is SHM with } \omega = \frac{10}{3}.$ $\text{Period} = \frac{2\pi}{\omega} = \frac{3\pi}{5}$	B1 M1 M1 A1 B1	either allow +/- si or $\omega^2 = 100/9$ convincing
4(b)(ii)	Amplitude = <u>0.25 (m)</u>	B1	
4(b)(iii)	Use $v^2 = \omega^2(a^2 - x^2)$, $\omega = \frac{10}{3}$, $a = 0.25$, $x = 0.2$ $v^2 = \left(\frac{10}{3}\right)^2(0.25^2 - 0.2^2)$ $v = \underline{0.5 \text{ (ms}^{-1}\text{)}}$	M1 A1 A1	ft a and ω. oe cao
4(b)(iv)	$x = a \cos(\omega t)$ $0.2 = 0.25 \cos\left(\frac{10}{3} t\right)$ $t = \frac{3}{10} \cos^{-1}\left(\frac{0.2}{0.25}\right)$ $t = \underline{0.193 \text{ (s)}}$	M1 A1 A1	oe allow sin/cos, c's a, ω. cao

Q	Solution	Mark	Notes
5.			
5(a)	Sine rule $\frac{\sin \theta}{l} = \frac{\sin 120^\circ}{l\sqrt{3}}$ $\sin \theta = 0.5 = 30^\circ$ $\alpha = 60^\circ - 30^\circ = 30^\circ$	M1 A1	
5(b)	Impulse = change in momentum Apply to B $J = 5 \times 8 \cos 30^\circ - 5v$ Apply to A $J = 3v$ Solving simultaneously $40 \frac{\sqrt{3}}{2} - 5v = 3v$ Speed of A = $v = \frac{5\sqrt{3}}{2} = 4.33 \text{ (ms}^{-1}\text{)}$ $u = 8 \sin 30^\circ = 4 \text{ (ms}^{-1}\text{)}$ Speed of B = $\sqrt{4^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$ $= 5.9 \text{ (ms}^{-1}\text{)}$ $J = 3v = \underline{12.99 \text{ (Ns)}}$	M1 A1 B1 m1 A1 B1 m1 A1 A1	used. Allow sin/cos. cao cao ft c's 3v

Q	Solution	Mark	Notes
6	Resolve vertically $R = 80g + 20g \quad (= 100g)$ Resolve horizontally $S = 0.6R$ $= 60g = 588 \text{ (N)}$ Moments about B $80g \times 5 \cos \theta + 20g \times 3 \cos \theta = S \times 6 \sin \theta$ $360 \sin \theta = 460 \cos \theta$ $\theta = \tan^{-1} \left(\frac{460}{360} \right) = 51.95^\circ$ The ladder is modelled as a rigid rod.	M1 A1 M1 A1 M1 A2 A1 B1	equation, no missing and no extra force. equation, no missing and no extra force. equation, no missing and no extra force. Dimensionally correct. -1 each error cao

Ques	Solution	Mark	Notes
1(a)	$E(X) = 3, \text{Var}(X) = 2.1$ si $E(Y) = 2E(X) + 1$ $= 7$ $\text{Var}(Y) = 4\text{Var}(X)$ $= 8.4$	B1 M1 A1 M1 A1	
(b)	$P(Y = 7) = P(X = 3)$ $= \binom{10}{3} \times 0.3^3 \times 0.7^7$ $= 0.267$	M1 A1 A1 A1	Award M1 just for this line Award M0A0 for no working Accept 0.6496 – 0.3828 or 0.6172 – 0.3504
2(a)	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ oe $P(A \cap B) = 0.4 + 0.5 - 2P(A \cap B)$ $P(A \cap B) = 0.3$	M1 A1	Award B1 for a valid verification
(b)	$P(A B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{0.3}{0.5} = 0.6$	M1 A1	Accept the use of a Venn diagram in (b) and (c)
(c)	$P(B A') = \frac{P(B \cap A')}{P(A')} (= \frac{P(B) - P(B \cap A)}{1 - P(A)})$ $= \frac{0.5 - 0.3}{1 - 0.4}$ $= \frac{1}{3} \text{ (0.33)}$	M1 A1 A1	
3(a)	$P(A \text{ chooses } G) = 0.3$	B1	
(b)	$P(B \text{ chooses } Y) = \frac{8}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{1}{9}$ $= 0.2$	M1A1 A1	Accept 0.2 without working
(c)	$P(\text{Diff colours}) = \frac{3}{10} \times \frac{7}{9} + \frac{5}{10} \times \frac{5}{9} + \frac{2}{10} \times \frac{8}{9}$ $= \frac{31}{45}$	M1A1 A1	Accept $\frac{{}^5C_1 \times {}^3C_1 + {}^5C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1}{{}^{10}C_2}$
4(a)(i)	$P(X = 9) = \frac{e^{-10} \times 10^9}{9!}$ $= 0.1251$	M1 A1	Accept 0.4579 – 0.3328 or 0.6672 – 0.5421 Award M0 if no working seen Award M1A0 if in adjacent row or column
(ii)	$P(X < 12) = 0.6968$	M1A1	
(b)	Looking at the appropriate section of the table, $n = 19$	M1 A1	Award M1A0 for 18 or 20

5(a)(i)	$P(\text{male and bike}) = 0.6 \times 0.75$ $= 0.45$	M1A1	
(ii)	$P(\text{owns a bike}) = 0.6 \times 0.75 + 0.4 \times 0.3$ $= 0.57$	M1A1 A1	
(b)	$P(\text{female} \text{bike}) = \frac{0.12}{0.57}$ $= 0.211 \text{ (4/19) cao}$	B1B1 B1	B1 num, B1 denom FT denominator from (a)
6(a)	Let X = no. of defective cups so X is $B(50,0.05)$	B1	si
(i)	$P(X = 2) = \binom{50}{2} \times 0.05^2 \times 0.95^{48}$ $= 0.261$	M1 A1	Accept 0.5405 – 0.2794 or 0.7206 – 0.4595 M0A0 if no working
(ii)	$P(3 \leq X \leq 8) = 0.9992 - 0.5405$ or $0.4595 - 0.0008$ $= 0.4587$	B1B1 B1	Award no marks if no working seen
(b)	Let Y = no. of defective plates so Y is $B(250,0.015) \approx \text{Po}(3.75)$ si $P(Y = 4) = \frac{e^{-3.75} \times 3.75^4}{4!}$ $= 0.194$	B1 M1 A1	M0A0 if no working
7(a)	$k \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} \right) = 1$ $k \times \frac{15}{12} = 1$ $k = \frac{4}{5}$	M1 A1	Or equivalent Accept verification
(b)	$E(X) = \frac{4}{5} \left(\frac{2}{2} + \frac{3}{3} + \frac{4}{4} + \frac{6}{6} \right)$ $= 3.2$	M1 A1	
(c)	The possible pairs are (3,4), (4,3), (2,6), (6,2) $\text{Prob} = \frac{4}{5} \times \frac{1}{3} \times \frac{4}{5} \times \frac{1}{4} \times 2 + \frac{4}{5} \times \frac{1}{2} \times \frac{4}{5} \times \frac{1}{6} \times 2$ $= 0.213 \text{ (16/75)}$	B1 M1A1 A1	B1 for (3,4),(2,6) M1A0A0 if factor 2 missing

8(a)	$P(1^{\text{st}} \text{ hit with } 3^{\text{rd}} \text{ throw}) = 0.7 \times 0.7 \times 0.3$ $= 0.147$	M1 A1	
(b)(i)	$P(\text{F wins } 1^{\text{st}} \text{ throw}) = P(\text{G misses}) \times P(\text{F hits})$ $= 0.8 \times 0.3 = 0.24$	M1 A1	
(ii)	$P(\text{F wins with } 2^{\text{nd}} \text{ throw})$ $= P(\text{G miss}) \times P(\text{F miss}) \times P(\text{G miss}) \times P(\text{F hits})$ $= 0.8 \times 0.7 \times 0.8 \times 0.3 = 0.1344$	M1 A1	
(iii)	$P(\text{F wins}) = 0.24 + 0.24 \times 0.56 + 0.24 \times 0.56^2 + \dots$ $= \frac{0.24}{1 - 0.56}$ $= 0.545 \left(\frac{6}{11} \right)$	M1 B1 A1	Award this M1 for realising that the probability is the sum of an infinite geometric series
9(a)	$E\left(\frac{1}{X}\right) = \frac{4}{9} \int_1^2 \frac{1}{x} (4x - x^3) dx$ $= \frac{4}{9} \left[4x - \frac{x^3}{3} \right]_1^2$ $= 0.741 \quad (20/27)$	M1A1 A1 A1	M1 for the integral of $\frac{1}{x} f(x)$ A1 for completely correct although limits may be left until 2nd line Award M0 if no working
(b)(i)	$F(x) = \frac{4}{9} \int_1^x (4u - u^3) du$ $= \frac{4}{9} \left[2u^2 - \frac{u^4}{4} \right]_1^x$ $= \frac{8x^2}{9} - \frac{x^4}{9} - \frac{7}{9}$	M1 A1 A1	Allow x as dummy variable Limits may be left until next line but must then be applied
(ii)	$P(1.25 \leq X \leq 1.75) = F(1.75) - F(1.25)$ $= 0.5625 \quad (9/16)$	M1 A1	FT from (b)(i) if possible
(iii)	The median m satisfies $\frac{8m^2 - m^4 - 7}{9} = 0.5$ $m^4 - 8m^2 + 11.5 = 0$ $m^2 = \left(\frac{8 \pm \sqrt{64 - 46}}{2} \right) = 1.88$ $m = 1.37$	M1 A1 A1 A1	FT from (b)(i) if possible Condone the absence of $\pm$

[illegible]

4(a)	Under H_0 , X is $B(20, 0.4)$ si	B1	
	$\begin{pmatrix} P(X \geq 13) = 0.0210 \\ P(X \geq 14) = 0.0065 \end{pmatrix}$ $X \geq 14$ has significance level closest to 1%	M1 A1	Award M1 for valid attempt at using tables Award M1A0 for 13 or 15
(b)	Let Y = number of hits Under H_0 , Y is $B(120, 0.4)$ $\approx N(48, 28.8)$ si	B1 B1	
	Test statistic = $\frac{54.5 - 48}{\sqrt{28.8}}$ = 1.21 -value = 0.1131 Insufficient evidence to conclude that his shooting has improved	M1A1 A1 A1 B1	Award M1A0 for incorrect or no continuity correction but FT for following marks No cc gives $z = 1.30$, $p = 0.0968$ Wrong cc $z = 1.40$, $p = 0.0808$ FT the p-value
5	Let X = score on a single die. Then $E(X) = 3.5$ and $\text{Var}(X) = \frac{91}{6} - 3.5^2 = \frac{35}{12}$ Let Y = mean of scores on 100 dice. Then by the Central Limit Theorem, $Y \approx N(3.5, 35/1200)$. $z = \frac{3.75 - 3.5}{\sqrt{35/1200}}$ = ($\pm$)1.46 Prob = 0.0721	B1 M1A1 M1A1 m1A1 A1 A1	 FT their mean and variance Use of continuity correction gives $z = 1.43$, $p = 0.0764$
6(a)(i)	$H_0 : \mu = 1.2; H_1 : \mu < 1.2$	B1	
	(ii) Under H_0 , X is $Po(12)$ si $P(X \leq 9)$ = 0.2424 Insufficient evidence to conclude that the (mean) number of breakdowns has decreased.	B1 B1 M1 A1 B1	Accept 12 in place of 1.2 FT the p-value
(b)	Under H_0 , Y is $Po(120) \approx N(120, 120)$ $z = \frac{101.5 - 120}{\sqrt{120}}$ = -1.69 p-value = 0.0455 Strong evidence to conclude that the (mean) number of breakdowns has decreased.	B1 M1A1 A1 A1 B1	Award M1A0 for incorrect or no continuity correction but FT for following marks No cc gives $z = -1.73$, $p = 0.0418$ Wrong cc, $z = -1.78$, $p = 0.0375$ FT the p-value if less than 0.05

7(a)(i)	$P(Y \leq y) = P(\sqrt{X} \leq y)$	M1	
	$= P(X \leq y^2)$	A1	
	$= \frac{y^2 - a}{b - a}$	A1	
(ii)	Attempting to differentiate,	M1	
	giving $\frac{2y}{b-a}$	A1	
(b)	$f(y) = \frac{2y}{b-a} \text{ for } \sqrt{a} \leq y \leq \sqrt{b}$	A1	
	$= 0 \text{ otherwise}$		
	We are given that		
	$\frac{a+b}{2} = 5.5 \text{ and } \frac{(b-a)^2}{12} = 3$	B1B1	
	Solving,	M1	
	$a = 2.5, b = 8.5$	A1A1	

Ques	Solution	Mark	Notes																																																																																																											
1	The sample space is as follows. EITHER <table><tr><th>Samples</th><th>R</th><th>M</th></tr><tr><td>1,2,2</td><td>1</td><td>2</td></tr><tr><td>1,2,4</td><td>3</td><td>2</td></tr><tr><td>1,2,6</td><td>5</td><td>2</td></tr><tr><td>1,2,6</td><td>5</td><td>2</td></tr><tr><td>1,2,4</td><td>3</td><td>2</td></tr><tr><td>1,2,6</td><td>5</td><td>2</td></tr><tr><td>1,2,6</td><td>5</td><td>2</td></tr><tr><td>1,4,6</td><td>5</td><td>4</td></tr><tr><td>1,4,6</td><td>5</td><td>4</td></tr><tr><td>1,6,6</td><td>5</td><td>6</td></tr><tr><td>2,2,4</td><td>2</td><td>2</td></tr><tr><td>2,2,6</td><td>4</td><td>2</td></tr><tr><td>2,2,6</td><td>4</td><td>2</td></tr><tr><td>2,4,6</td><td>4</td><td>4</td></tr><tr><td>2,4,6</td><td>4</td><td>4</td></tr><tr><td>2,6,6</td><td>4</td><td>6</td></tr><tr><td>2,4,6</td><td>4</td><td>4</td></tr><tr><td>2,4,6</td><td>4</td><td>4</td></tr><tr><td>2,6,6</td><td>4</td><td>6</td></tr><tr><td>4,6,6</td><td>2</td><td>6</td></tr></table> OR <table><tr><th>Samples</th><th>R</th><th>M</th><th>No. of ways</th></tr><tr><td>1,2,2</td><td>1</td><td>2</td><td>1</td></tr><tr><td>1,2,4</td><td>3</td><td>2</td><td>2</td></tr><tr><td>1,2,6</td><td>5</td><td>2</td><td>4</td></tr><tr><td>1,4,6</td><td>5</td><td>4</td><td>2</td></tr><tr><td>1,6,6</td><td>5</td><td>6</td><td>1</td></tr><tr><td>2,2,4</td><td>2</td><td>2</td><td>1</td></tr><tr><td>2,2,6</td><td>4</td><td>2</td><td>2</td></tr><tr><td>2,4,6</td><td>4</td><td>4</td><td>4</td></tr><tr><td>2,6,6</td><td>4</td><td>6</td><td>2</td></tr><tr><td>4,6,6</td><td>2</td><td>6</td><td>1</td></tr></table>	Samples	R	M	1,2,2	1	2	1,2,4	3	2	1,2,6	5	2	1,2,6	5	2	1,2,4	3	2	1,2,6	5	2	1,2,6	5	2	1,4,6	5	4	1,4,6	5	4	1,6,6	5	6	2,2,4	2	2	2,2,6	4	2	2,2,6	4	2	2,4,6	4	4	2,4,6	4	4	2,6,6	4	6	2,4,6	4	4	2,4,6	4	4	2,6,6	4	6	4,6,6	2	6	Samples	R	M	No. of ways	1,2,2	1	2	1	1,2,4	3	2	2	1,2,6	5	2	4	1,4,6	5	4	2	1,6,6	5	6	1	2,2,4	2	2	1	2,2,6	4	2	2	2,4,6	4	4	4	2,6,6	4	6	2	4,6,6	2	6	1	M1 A1 A1 A1 M1 A1 A1 A1	A1 for the samples column A1 for the R column A1 for the M column Minus A1 if 1 or 2 rows omitted Minus A2 if 3 or 4 rows omitted A1 for columns 1 and 4 A1 for the R column A1 for the M column Minus A1 if 1 or 2 rows omitted Minus A2 if 3 or 4 rows omitted
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	The probability distribution of R is therefore <table><tr><td>r</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$P(R=r)$</td><td>1/20</td><td>2/20</td><td>2/20</td><td>8/20</td><td>7/20</td></tr></table> The probability distribution of M is therefore <table><tr><td>m</td><td>2</td><td>4</td><td>6</td></tr><tr><td>$P(M=m)$</td><td>10/20</td><td>6/20</td><td>4/20</td></tr></table>	r	1	2	3	4	5	$P(R=r)$	1/20	2/20	2/20	8/20	7/20	m	2	4	6	$P(M=m)$	10/20	6/20	4/20	M1 A1 M1 A1	FT for both tables from (a) if sum of probabilities is 1
r	1	2	3	4	5																		
$P(R=r)$	1/20	2/20	2/20	8/20	7/20																		
m	2	4	6																				
$P(M=m)$	10/20	6/20	4/20																				
2(a) (b)	$\sum x = 192.9; \sum x^2 = 3118.91$ UE of $\mu = 16.075$ UE of $\sigma^2 = \frac{3118.91}{11} - \frac{192.9^2}{132}$ $= 1.640$ DF = 11 si Crit value = 3.106 99% confidence limits are $16.075 \pm 3.106 \sqrt{\frac{1.640}{12}}$ giving [14.9,17.2]	B1B1 B1 M1 A1 B1 B1 M1 A1 A1	Must be seen No working need be seen M0 division by 12 Answer only no marks FT their s^2 and mean M0 use of z-values M0 if 12 omitted Answer only no marks																				
3(a) (b)	$H_0 : \mu_a = \mu_b; H_1 : \mu_a \neq \mu_b$ $SE = \sqrt{\frac{7.62}{100} + \frac{6.91}{100}} \quad (= 0.381\dots)$ Test stat = $\frac{161.17 - 160.53}{0.381}$ $= 1.68$ Tabular value = 0.04648 p-value = 0.09296 Insufficient evidence to conclude that there is a difference in mean weight.	B1 M1A1 M1A1 A1 A1 A1 B1	Treat taking the variances as SDs as a misread, giving SE = 1.029, Test stat = 0.62, p-value = 0.535 M0 if 100 omitted FT the p-value																				
4(a)	$\hat{p} = \frac{54}{90} = 0.6 \quad \text{si}$ $ESE = \sqrt{\frac{0.6 \times 0.4}{90}} = 0.0516.. \quad \text{si}$ 90% confidence limits are $0.6 \pm 1.645 \times 0.0516..$ giving [0.515,0.685]	B1 M1A1 M1A1 A1																					

6(a)(i)	$E(X) = \theta + 2 \times 2\theta + 3 \times 3\theta + 4(1 - 6\theta)$ $= 4 - 10\theta$ Therefore $E(\bar{X}) = 4 - 10\theta \text{ si}$ $E(U) = a(4 - 10\theta) + b = \theta \text{ for all } \theta$ $a = -\frac{1}{10}; b = \frac{4}{10}$ $\left(U = \frac{4}{10} - \frac{1}{10} \bar{X} \right)$	M1	
		A1	
		A1	
		M1	
		A1	
(ii)	$\text{Var}(X) = \theta + 4 \times 2\theta + 9 \times 3\theta + 16(1 - 6\theta) - (4 - 10\theta)^2$ $= 20\theta(1 - 5\theta)$ $\text{Var}(U) = a^2 \frac{\text{Var}(X)}{n}$ $= \frac{\theta(1 - 5\theta)}{5n}$	M1	
		A1	
		M1	
		A1	
(b)(i)	Y is $B(n, 1 - 6\theta)$ so $E(Y) = n(1 - 6\theta)$ Therefore $E(V) = cn(1 - 6\theta) + d = \theta \text{ (for all } \theta)$ $c = -\frac{1}{6n}; d = \frac{1}{6}$ $\left(V = \frac{1}{6} - \frac{1}{6n} Y \right)$	M1	
		A1	
		A1	
(ii)	$\text{Var}(V) = c^2 \text{Var}(Y) = c^2 npq$ $= \frac{\theta(1 - 6\theta)}{6n}$	M1	
		A1	
(c)	$\frac{\text{Var}(U)}{\text{Var}(V)} = \frac{\theta(1 - 5\theta)}{5n} \times \frac{6n}{\theta(1 - 6\theta)} = \frac{6 - 30\theta}{5 - 30\theta}$ This ratio is greater than 1 so that V is the better estimator.	B1	
		B1	Convincing



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