C3

1. (a)

0	0
$\pi/9$	$-0{\cdot}062202456$
$2\pi/9$	$-0{\cdot}266515091$
$\pi/3$	$-0{\cdot}693147181$
$4\pi/9$	$-1{\cdot}750723994$

(5 values correct) B2

(**If B2 not awarded**, award B1 for either 3 or 4 values correct)

Correct formula with $h = \pi/9$ M1

$$I \approx \frac{\pi/9}{3} \times \{0 + (-1{\cdot}750723994) + 4[(-0{\cdot}062202456) + (-0{\cdot}693147181)] + 2(-0{\cdot}266515091)\}$$

$I \approx -5{\cdot}305152724 \times (\pi/9) \div 3$

$I \approx -0{\cdot}617282549$

$I \approx -0{\cdot}6173$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

(b)

$\int_0^{4\pi/9} \ln(\sec x)\,dx \approx 0{\cdot}6173$ (f.t. candidate's answer to (a)) B1

2. (*a*) $7\operatorname{cosec}^2\theta - 4(\operatorname{cosec}^2\theta - 1) = 16 + 5\operatorname{cosec}\theta$

(correct use of $\cot^2\theta = \operatorname{cosec}^2\theta - 1$) M1

An attempt to collect terms, form and solve quadratic equation in cosec θ, either by using the quadratic formula or by getting the expression into the form $(a\operatorname{cosec}\theta + b)(c\operatorname{cosec}\theta + d)$, with $a \times c$ = candidate's coefficient of $\operatorname{cosec}^2\theta$ and $b \times d$ = candidate's constant m1

$3\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta - 12 = 0 \Rightarrow (\operatorname{cosec}\theta - 3)(3\operatorname{cosec}\theta + 4) = 0$

$\Rightarrow \operatorname{cosec}\theta = 3,\ \operatorname{cosec}\theta = -\frac{4}{3}$

$\Rightarrow \sin\theta = \frac{1}{3},\ \sin\theta = -\frac{3}{4}$ (c.a.o.) A1

$\theta = 19{\cdot}47°,\ 160{\cdot}53°$ B1

$\theta = 311{\cdot}41°,\ 228{\cdot}59°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\sin\theta$ = +, –, f.t. for 3 marks, $\sin\theta$ = –, –, f.t. for 2 marks

$\sin\theta$ = +, +, f.t. for 1 mark

(*b*) $\sec\phi \geq 1, \operatorname{cosec}\phi \geq 1$ and thus $4\sec\phi + 3\operatorname{cosec}\phi \geq 7$ E1

3. (*a*) $\frac{d}{dx}(x^3) = 3x^2$ $\frac{d}{dx}(1) = 0$ $\frac{d}{dx}(\pi^2/4) = 0$ B1

$\frac{d}{dx}(2x\cos y) = 2x(-\sin y)\frac{dy}{dx} + 2\cos y$ B1

$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ B1

$\frac{dy}{dx} = \frac{3}{2 - \pi}$ (c.a.o.) B1

(*b*) $\frac{d^2y}{dx^2} = \frac{d}{dx}(x^2y) = x^2\frac{dy}{dx} + 2xy$ B1

Substituting x^2y for $\frac{dy}{dx}$ in candidate's derived expression for $\frac{d^2y}{dx^2}$ M1

$\frac{d^2y}{dx^2} = x^2(x^2y) + 2xy = x^4y + 2xy$ (o.e.) (c.a.o.) A1

4. (*a*) candidate's x-derivative = $\dfrac{1}{1+t^2}$ B1

candidate's y-derivative = $\dfrac{1}{t}$ B1

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{1+t^2}{t}$ A1

(*b*) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) = -t^{-2} + 1$ (o.e.) B1

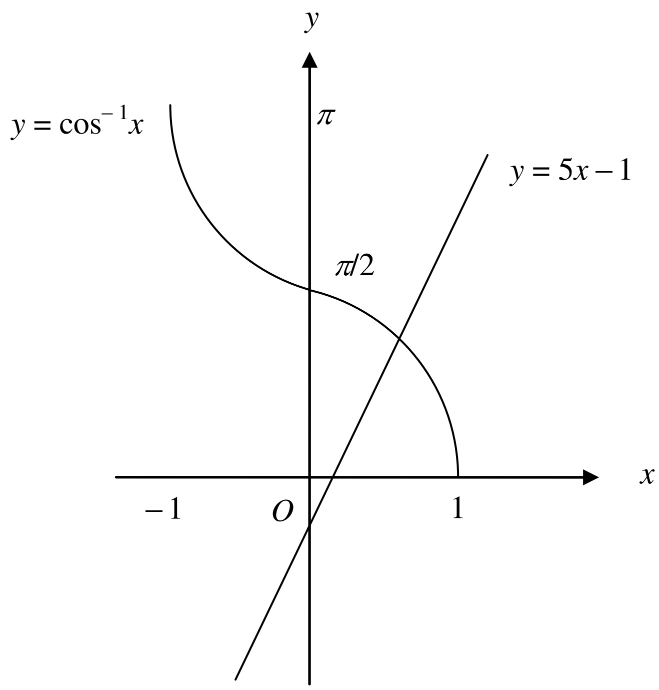
Use of $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{dy}{dx}\right) \div$ candidate's x-derivative M1

$\dfrac{d^2y}{dx^2} = (-t^{-2} + 1)(1 + t^2)$ (o.e.) (f.t. one slip) A1

$\dfrac{d^2y}{dx^2} = 0 \Rightarrow t = 1$ (c.a.o.) A1

$\dfrac{d^2y}{dx^2} = 0 \Rightarrow x = \dfrac{\pi}{4}$ (f.t. candidate's derived value for t) A1

5. (*a*)



Correct shape for $y = \cos^{-1}x$ B1

A straight line with negative y-intercept and positive gradient intersecting once with $y = \cos^{-1}x$ in the first quadrant. B1

(*b*) $x_0 = 0{\cdot}4$

$x_1 = 0{\cdot}431855896$ (x_1 correct, at least 4 places after the point) B1

$x_2 = 0{\cdot}424849379$

$x_3 = 0{\cdot}426400166$

$x_4 = 0{\cdot}426057413 = 0{\cdot}4261$ (x_4 correct to 4 decimal places) B1

Let $h(x) = \cos^{-1}x - 5x + 1$

An attempt to check values or signs of $h(x)$ at $x = 0{\cdot}42605$, $x = 0{\cdot}42615$ M1

$h(0{\cdot}42605) = 4{\cdot}24 \times 10^{-4} > 0$, $h(0{\cdot}42615) = -1{\cdot}86 \times 10^{-4} < 0$ A1

Change of sign $\Rightarrow \alpha = 0{\cdot}4261$ correct to four decimal places A1

6. (*a*) (i) $\frac{dy}{dx} = \frac{a + bx}{4x^2 - 3x - 5}$ (including $a = 1, b = 0$) M1

$\frac{dy}{dx} = \frac{8x - 3}{4x^2 - 3x - 5}$ A1

(ii) $\frac{dy}{dx} = e^{\sqrt{x}} \times f(x)$ $(f(x) \neq 1, 0)$ M1

$\frac{dy}{dx} = e^{\sqrt{x}} \times \frac{1}{2} x^{-1/2}$ A1

(iii) $\frac{dy}{dx} = \frac{(a - b\sin x) \times m\cos x - (a + b\sin x) \times k\cos x}{(a - b\sin x)^2}$

$(m = \pm b, k = \pm b)$ M1

$\frac{dy}{dx} = \frac{(a - b\sin x) \times b\cos x - (a + b\sin x) \times (-b)\cos x}{(a - b\sin x)^2}$ A1

$\frac{dy}{dx} = \frac{2ab\cos x}{(a - b\sin x)^2}$ A1

(*b*) $\frac{d}{dx}(\cot x) = \frac{d}{dx}(\tan x)^{-1} = (-1) \times (\tan x)^{-2} \times f(x)$ $(f(x) \neq 1, 0)$ M1

$\frac{d}{dx}(\tan x)^{-1} = (-1) \times (\tan x)^{-2} \times \sec^2 x$ A1

$\frac{d}{dx}(\tan x)^{-1} = -\text{cosec}^2 x$ (convincing) A1

7. (*a*) (i) $\int \frac{(7x^2 - 2)}{x}\,dx = \int 7x\,dx - \int \frac{2}{x}\,dx$

Correctly rewriting as two terms and an attempt to integrate M1

$\int \frac{(7x^2 - 2)}{x}\,dx = \frac{7}{2}x^2 - 2\ln x + c$ A1 A1

(ii) $\int \sin(2x/3 - \pi)\,dx = k \times \cos(2x/3 - \pi) + c$

$(k = -1, -3/2, 3/2, -2/3)$ M1

$\int \sin(2x/3 - \pi)\,dx = -3/2 \times \cos(2x/3 - \pi) + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (5x - 14)^{-1/4}\,dx = \frac{k \times (5x - 14)^{3/4}}{3/4}$ $(k = 1, 5, 1/5)$ M1

$\int (5x - 14)^{-1/4}\,dx = 1/5 \times \frac{(5x - 14)^{3/4}}{3/4}$ A1

A correct method for substitution of the correct limits limits in an expression of the form $m \times (5x - 14)^{3/4}$ M1

$\int_3^6 (5x - 14)^{-1/4}\,dx = \frac{28}{15}$ $(= 1{\cdot}867)$

(f.t. only for solutions of $\frac{28}{3}$ $(= 9{\cdot}333)$ and $\frac{140}{3}$ $(= 46{\cdot}667)$ from $k = 1$, $k = 5$ respectively) A1

Note: Answer only with no working shown earns 0 marks

8. (*a*) Trying to solve either $3x - 5 \leq 1$ or $3x - 5 \geq -1$ M1

$3x - 5 \leq 1 \Rightarrow x \leq 2$

$3x - 5 \geq -1 \Rightarrow x \geq 4/3$ (both inequalities) A1

Required range: $4/3 \leq x \leq 2$ (f.t. one slip) A1

Alternative mark scheme

$(3x - 5)^2 \leq 1$

(squaring both sides, forming and trying to solve quadratic) M1

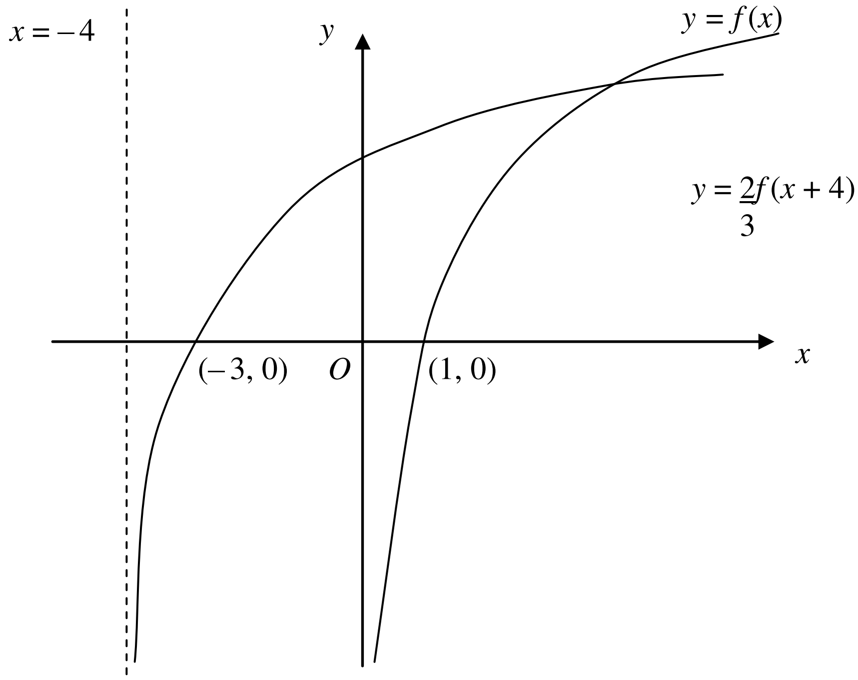
Critical values $x = 4/3$ and $x = 2$ A1

Required range: $4/3 \leq x \leq 2$ (f.t. one slip in critical values) A1

(*b*) $4/3 \leq 1/y \leq 2$ (f.t. candidate's $a \leq x \leq b$, $a > 0$, $b > 0$) M1

$1/2 \leq y \leq 3/4$ (f.t. candidate's $a \leq x \leq b$, $a > 0$, $b > 0$) A1

9.



Correct shape, including the fact that the y-axis is an asymptote for $y = f(x)$ at $-\infty$ — B1

$y = f(x)$ cuts x-axis at (1, 0) — B1

Correct shape, including the fact that $x = -4$ is an asymptote for $y = \frac{2}{3}f(x+4)$ at $-\infty$ — B1

$y = \frac{2}{3}f(x+4)$ cuts x-axis at (– 3, 0) (f.t. candidate's x-intercept for $f(x)$) — B1

The diagram shows that the graph of $y = f(x)$ is steeper than the graph of $y = \frac{2}{3}f(x+4)$ in the first quadrant — B1

10. (*a*) Choice of h, k such that $h(x) = k(x) + c$, $c \neq 0$ — M1

Convincing verification of the fact that $h'(x) = k'(x)$ — A1

(*b*) (i) $y - 3 = 2\ln(4x + 5)$ — B1

An attempt to express candidate's equation as an exponential equation — M1

$x = \dfrac{(e^{(y-3)/2} - 5)}{4}$ (c.a.o.) — A1

$f^{-1}(x) = \dfrac{(e^{(x-3)/2} - 5)}{4}$

(f.t. one slip in candidate's expression for x) — A1

(ii) $D(f^{-1}) = [10, 14]$ — B1 B1

(iii) $gf(x) = e^{2\ln(4x+5)+3}$ — B1

$e^{2\ln(4x+5)} = (4x+5)^2$ — B1

$gf(x) = e^3(4x+5)^2$ (c.a.o.) — B1