## C4

**1.** (*a*) $f(x) \equiv \frac{A}{(x+3)^2} + \frac{B}{(x+3)} + \frac{C}{(x-1)}$ (correct form) M1

$2x^2 + 5x + 25 \equiv A(x-1) + B(x+3)(x-1) + C(x+3)^2$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = -7$, $C = 2$, $B = 0$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $\int f(x)\,dx = \frac{7}{(x+3)} + 2\ln(x-1)$ B1 B1

(f.t. candidate's values for $A$, $B$, $C$)

$\int_3^{10} f(x)\,dx = \left[\frac{7}{13} + 2\ln 9\right] - \left[\frac{7}{6} + 2\ln 2\right] = 2{\cdot}38$ (c.a.o.) B1

**Note: Answer only with no working earns 0 marks**

**2.** (*a*) $4x^3 + 3x^2\frac{dy}{dx} + 6xy - 4y\frac{dy}{dx} = 0$

$\left[3x^2\frac{dy}{dx} + 6xy\right]$ B1

$\left[4x^3 - 4y\frac{dy}{dx}\right]$ B1

$\frac{dy}{dx} = \frac{4x^3 + 6xy}{4y - 3x^2}$ (convincing) B1

(*b*) $4y - 3x^2 = 0$ M1

**Either:** Substituting $\frac{3x^2}{4}$ for $y$ in the equation of $C$ and an attempt to collect terms m1

$x^4 = 16 \Rightarrow x = (\pm)\,2$ A1

$y = 3$ (for both values of $x$)

(f.t. $x^4 = a$, $a \neq 16$, provided both $x$ values are checked) A1

**Or:** Substituting $\frac{4y}{3}$ for $x^2$ in the equation of $C$ and an attempt to collect terms m1

$y^2 = 9 \Rightarrow y = (\pm)\,3$ A1

$y = 3 \Rightarrow x = \pm\,2$ (f.t. $y^2 = b$, $b \neq 9$) A1

**3.** (*a*) $\dfrac{\tan x + \tan 45°}{1 - \tan x \tan 45°} = 8 \tan x$ (correct use of formula for $\tan(x + 45°)$) M1

Use of $\tan 45° = 1$ and an attempt to form a quadratic in $\tan x$ by cross multiplying and collecting terms M1

$8\tan^2 x - 7 \tan x + 1 = 0$ (c.a.o.) A1

Use of a correct method to solve the candidate's derived quadratic in $\tan x$ m1

$x = 34{\cdot}8°,\ 10{\cdot}2°$ (both values)

(f.t. one slip in candidate's derived quadratic in $\tan x$ provided all three method marks have been awarded) A1

(*b*) (i) $R = 7$ B1

Correctly expanding $\sin(\theta - \alpha)$, correctly comparing coefficients and using either $7\cos\alpha = \sqrt{13}$ **or** $7 \sin \alpha = 6$ **or** $\tan\alpha = \dfrac{6}{\sqrt{13}}$ to find $\alpha$ (f.t. candidate's value for $R$) M1

$\alpha = 59°$ (c.a.o) A1

(ii) $\sin(\theta - \alpha) = -\dfrac{4}{7}$ (f.t. candidate's values for $R$, $\alpha$) B1

$\theta - 59° = -34{\cdot}85°,\ 214{\cdot}85°,\ 325{\cdot}15°,$

(at least one value, f.t. candidate's values for $R$, $\alpha$) B1

$\theta = 24{\cdot}15°,\ 273{\cdot}85°$ (c.a.o.) B1

**4.** (*a*)

$V = \pi \int_0^a (mx)^2 \, dx$ M1

$\int (mx)^2 \, dx = \dfrac{m^2x^3}{3}$ B1

$V = \dfrac{\pi m^2 a^3}{3}$ (c.a.o.) A1

(*b*) (i) Substituting $\dfrac{b}{a}$ for $m$ in candidate's derived expression for $V$ M1

$V = \dfrac{\pi b^2 a}{3}$ (c.a.o.) A1

(ii) This is the volume of a cone of (vertical) height $a$ and (base) radius $b$ E1

**5.** $\left(1 + \frac{x}{8}\right)^{-1/2} = 1 - \frac{x}{16} + \frac{3x^2}{512}$ — $\left(1 - \frac{x}{16}\right)$ B1

$\left(\frac{3x^2}{512}\right)$ B1

$|x| < 8$ or $-8 < x < 8$ B1

$\frac{2}{3}\sqrt{2} \approx 1 - \frac{1}{16} + \frac{3}{512}$ (f.t. candidate's coefficients) B1

**Either:** $\sqrt{2} \approx \frac{1449}{1024}$ (c.a.o.)

**Or:** $\sqrt{2} \approx \frac{2048}{1449}$ (c.a.o.) B1

**6.** (*a*) (i) candidate's *x*-derivative = $2at$
candidate's *y*-derivative = $2a$ (at least one term correct)
and use of
$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$

Gradient of tangent at $P = \frac{1}{p}$ (c.a.o.) A1

(ii) Equation of tangent at *P*: $y - 2ap = \frac{1}{p}(x - ap^2)$

(f.t. candidate's expression for $\frac{dy}{dx}$) m1

Equation of tangent at *P*: $py = x + ap^2$ A1

(*b*) (i) Gradient $PQ = \frac{2ap - 2aq}{ap^2 - aq^2}$ B1

Use of $ap^2 - aq^2 = a(p + q)(p - q)$ B1

Gradient $PQ = \frac{2}{p + q}$ (c.a.o.) B1

(ii) As the point *Q* approaches *P*, *PQ* becomes a tangent
$\lim_{q \to p}$ (gradient $PQ$) $= \frac{2}{2p} = \frac{1}{p}$. E1

**7.** (*a*) $\int \frac{x^2}{(12-x^3)^2}\,dx = \int \frac{k}{u^2}\,du$ $(k = \frac{1}{3}, -\frac{1}{3}, 3 \text{ or } -3)$ M1

$\int \frac{a}{u^2}\,du = a \times \frac{u^{-1}}{-1}$ B1

**Either:** Correctly inserting limits of 12, 4 in candidate's $bu^{-1}$
**or:** Correctly inserting limits of 0, 2 in candidate's $b(12-x^3)^{-1}$
M1

$\int_0^2 \frac{x^2}{(12-x^3)^2}\,dx = \frac{1}{18}$ (c.a.o.) A1

**Note: Answer only with no working earns 0 marks**

(*b*) (i) $u = x \Rightarrow du = dx$ (o.e.) B1

$dv = \cos 2x\,dx \Rightarrow v = \frac{1}{2}\sin 2x$ (o.e.) B1

$\int x\cos 2x\,dx = x \times \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \times dx$ M1

$\int x\cos 2x\,dx = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$ (c.a.o.) A1

(ii) $\int x\sin^2 x\,dx = \int x\left[\frac{k}{2} - \frac{m}{2}\cos 2x\right]dx$ (o.e.)

$(k = \mathbf{1}, -1, m = \mathbf{1}, -1)$ M1

$\int x\sin^2 x\,dx = \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos 2x\,dx$ A1

$\int x\sin^2 x\,dx = \frac{x^2}{4} - \frac{1}{4}x\sin 2x - \frac{1}{8}\cos 2x + c$

(f.t. only candidate's answer to (*b*)(*i*)) A1

**8.** (*a*) (i) **AB** = $-$**i** $-$ 2**j** + 7**k** B1

(ii) Use of **a** + $\lambda$**AB**, **a** + $\lambda$(**b** $-$ **a**), **b** + $\lambda$**AB** or **b** + $\lambda$(**b** $-$ **a**) to find vector equation of *AB* M1

**r** = 5**i** $-$ **j** $-$ **k** + $\lambda$ ($-$**i** $-$ 2**j** + 7**k**) (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*) $5 - \lambda = 2 + \mu$
$-1 - 2\lambda = -3 + \mu$
$-1 + 7\lambda = -4 - \mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving two of the equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her equation of *AB*)

$\lambda = -1, \mu = 4$ (o.e.) (c.a.o.) A1

Correct verification that values of $\lambda$ and $\mu$ satisfy third equation A1

Position vector of point of intersection is 6**i** + **j** $-$ 8**k** (f.t. one slip) A1

**9.** (*a*) $\frac{\mathrm{d}P}{\mathrm{d}t} = kP^2$ B1

(*b*) $\int \frac{\mathrm{d}P}{P^2} = \int k \,\mathrm{d}t$ M1

$-\frac{1}{P} = kt + c$ (o.e.) A1

$c = -\frac{1}{A}$ (c.a.o.) A1

$-\frac{1}{P} = kt - \frac{1}{A} \Rightarrow kt = \frac{1}{A} - \frac{1}{P} \Rightarrow \frac{1}{k}\left(\frac{P-A}{PA}\right) = t$ (convincing) A1

(*c*) $\frac{1}{k}\left(\frac{800-A}{800A}\right) = 3,\quad \frac{1}{k}\left(\frac{900-A}{900A}\right) = 4$ (both equations) B1

An attempt to solve these equations simultaneously by eliminating $k$ M1

$A = 600$ (c.a.o.) A1

**10.** Assume that 4 is a factor of $a + b$.
Then there exists an integer $c$ such that $a + b = 4c$.
Similarly, there exists an integer $d$ such that $a - b = 4d$. B1
Adding, we have $2a = 4c + 4d$. B1
Therefore $a = 2c + 2d$, an even number, which contradicts the fact that $a$ is odd. B1