GCE MARKING SCHEME

MATHEMATICS - C1-C4 & FP1-FP3
AS/Advanced

SUMMER 2015

M2

Q	Solution	Mark	Notes
1.	$\mathbf{x}.\mathbf{y} = 0$	M1	used
	$(\sin\theta\,\mathbf{i} + 2\cos2\theta\,\mathbf{j}).(2\mathbf{i} - \mathbf{j})\ (= 0)$	M1	correct method for dot product, no **i**, **j**'s
	$2\sin\theta - 2\cos2\theta\ (= 0)$	A1	
	$\sin\theta - (1 - 2\sin^2\theta) = 0$	m1	$\cos2\theta = 1 - 2\sin^2\theta$ depends on both M's
	$2\sin^2\theta + \sin\theta - 1 = 0$		
	$(2\sin\theta - 1)(\sin\theta + 1) = 0$		
	$\sin\theta = 0.5$		
	$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	A1	both values
	$\sin\theta = -1$		
	$\theta = \frac{3\pi}{2}$	A1	

Q	Solution	Mark	Notes
2(a)	Apply N2L to object $1600 - R = 50a$	M1	
	$1600 - kt = 50a$	B1	$R = kt$
	When $t = 2$, $a = -4$	m1	used
	$1600 - 2k = 50 \times (-4)$ $k = 900$		
	$1600 - 900t = 50\frac{dv}{dt}$		
	$\frac{dv}{dt} = 32 - 18t$	A1	convincing
2(b)	$\int dv = \int 32 - 18t \, dt$	M1	increase in power at least once
	$v = 32t - 9t^2 (+ C)$	A1	
	When $t = 2$, $v = 41$	m1	used
	$C = 9 \times 2^2 - 32 \times 2 + 41$		
	$C = 13$	A1	cao
	$v = -9t^2 + 32t + 13$		
	When $v = 28$, $28 = -9t^2 + 32t + 13$	m1	substitution of v=28 in c's expression for $v(t)$.
	$9t^2 - 32t + 15 = 0$ $(9t - 5)(t - 3) = 0$		
	$t = \frac{5}{9}, 3$	A1	cao

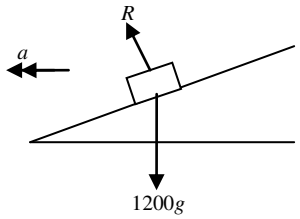
Q	Solution	Mark	Notes
3.			
	N2L	M1	dim correct, all forces
	$T - mg\sin\alpha - R = ma$	A1	correct equation
	$T = \frac{P}{v}$	B1	used si
	$\frac{5P}{16} - 6000\times9.8\times\frac{6}{49} - R = 6000\times2$	A1	correct equation in P & R
	$\frac{5P}{16} - R = 19200$		
	N2L with $a = 0$	M1	dim correct, all forces
	$T - mg\sin\alpha - R = 0$	A1	correct equation
	$\frac{3P}{16} - 6000\times9.8\times\frac{6}{49} - R = 0$	A1	correct equation in P & R
	$\frac{3P}{16} - R = 7200$		
	Solving simultaneously	m1	eliminating one variable, depends on both M's
	$\frac{2P}{16} = 12000$		
	$P = 96000$; $R = 10800$	A1	both answers cao

Q	Solution	Mark	Notes
4(a)	N2L		
	$(4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j} = 0.5\mathbf{a}$	M1	use of $\mathbf{F} = m\mathbf{a}$
	$\mathbf{a} = (8t - 6)\mathbf{i} + (6t^2 - 10t)\mathbf{j}$	A1	cao
	$\mathbf{v} = \int \mathbf{a}\, dt$	M1	attempted, **i**, **j** retained, power of t increased once
	$\mathbf{v} = (4t^2 - 6t)\mathbf{i} + (2t^3 - 5t^2)\mathbf{j} + (\mathbf{c})$	A1	ft **a** of same diff, not **F**
	When $t = 0$, $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$		
	$\mathbf{c} = 8\mathbf{i} - 7\mathbf{j}$	A1	
	$\mathbf{v} = (4t^2 - 6t)\mathbf{i} + (2t^3 - 5t^2)\mathbf{j} + 8\mathbf{i} - 7\mathbf{j}$		
	$\mathbf{v} = (4t^2 - 6t + 8)\mathbf{i} + (2t^3 - 5t^2 - 7)\mathbf{j}$		
4(b)	Impulse = change in momentum	M1	attempted, vector form required
	When $t = 3$, $\mathbf{v} = 26\mathbf{i} + 2\mathbf{j}$	B1	si ft c's **v**
	$0.5(x\mathbf{i} + y\mathbf{j}) - 0.5(26\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} - 9\mathbf{j}$		
	$(x\mathbf{i} + y\mathbf{j}) = 30\mathbf{i} - 16\mathbf{j}$	A1	cao
	Speed = $\sqrt{30^2 + (-16)^2}$	M1	ft c's x, y
	Speed = $\underline{34\ \text{ms}^{-1}}$	A1	cao

Q	Solution	Mark	Notes
5(a)	$T = 15g$	B1	si
	Hooke's Law $T = \frac{\lambda x}{l} = \frac{1470 \times x}{0 \cdot 4}$	M1	
	$x = \frac{15 \times 9 \cdot 8 \times 0 \cdot 4}{1470}$ x = <u>0.04 (m)</u>	A1	cao
5(b)	Let PE be zero at the natural length level. PE = mgh	M1	used
	Initial PE = 15 × 9.8 × (-0.16) Initial PE = -23.52 J	A1	
	Initial EE = $\frac{1}{2} \times \frac{\lambda (x)^2}{l}$	M1	used
	Initial EE = $\frac{1}{2} \times \frac{1470 (0 \cdot 16)^2}{0 \cdot 4}$ Initial EE = 47.04 J	A1	
	Final KE = $0.5mv^2$ Final KE = $7.5v^2$	B1	
	Final PE = 15 × 9.8 × -0.05 Final PE = -7.35 J		
	Final EE = $\frac{1}{2} \times \frac{1470 (0 \cdot 05)^2}{0 \cdot 4}$ Final EE = 4.59375 J		
	Conservation of energy	M1	equation, all 3 types
	$7.5v^2 - 7.35 + 4.59375 = 47.04 - 23.52$	A1	all correct, any form
	$v^2 = 3.5035$ v = <u>1.8718</u> = <u>1.87 (ms^{-1})(to 2 d.p.)</u>	A1	

Q	Solution	Mark	Notes
6(a)	Initial $u_H = 35\cos\alpha = (35 \times 0.6 = 21)$ (ms^{-1})	B1	si
	Initial $u_V = 35\sin\alpha = (35 \times 0.8 = 28)$ (ms^{-1})	B1	si
	use of $s = ut+0.5at^2$		
	with s=0,u=28(c),a=(±)9.8	M1	complete method
	$0 = 28t+0.5(-9.8)t^2$	A1	ft u_V
	$t(28 - 4.9t) = 0$		
	$t = (0), \frac{40}{7}$	A1	
	Total distance travelled by ball $= \frac{40}{7} \times 21$		
	$= 120$ (m)		
	Ball will not fall into lake.	A1	
6(b)	time to tree $= \frac{17 \cdot 5}{21} = \frac{5}{6}$	B1	
	Use $v=u+at$ with u=28(c),a=(±)9.8,t=5/6(c)	M1	oe complete method
	$v = 28 - 9.8 \times \frac{5}{6}$	A1	
	$v = \frac{119}{6}$ (= 19.8333)		
	speed $= \sqrt{\left(\frac{119}{6}\right)^2 + (21)^2}$	m1	
	speed = <u>28.89 (ms^{-1})</u>	A1	cao
	$\theta = \tan^{-1}\left(\frac{119}{6 \times 21}\right)$	m1	
	θ = <u>43.36º</u>	A1	cao

Q	Solution	Mark	Notes
7			



Resolve vertically	M1	equation, dim correct No extra force
$R\cos 12^\circ = 1200g$ $R = \underline{12022.73 \text{ (N)}}$	A1	
N2L towards the centre of motion	M1	dim correct, no extra force
$R\sin 12^\circ = \frac{1200 \times v^2}{80}$	A1	
$v = \underline{12.91}$	A1	cao

Q	Solution	Mark	Notes
8(a)(i)	Conservation of energy	M1	KE and PE
	$0.5\times3\times5^2$ =		
	$3\times9.8\times0.8(1-\cos\theta)+0.5\times3\times v^2$	A1A1	
	$25 = v^2 + 1.6\times9.8 - 1.6\times9.8\cos\theta$		
	$v^2 = \underline{9.32 + 15.68\cos\theta}$	A1	cao
8(a)(ii)	N2L towards centre of motion	M1	dim correct, 3 terms T, $3g\cos\theta$ opposing
	$T - 3g\cos\theta = \frac{3v^2}{0\cdot8}$	A1	
	$T = 3g\cos\theta + 3.75(9.32 + 15.68\cos\theta)$	m1	ft v^2 of form a±bsin/cosθ
	$T = \underline{34.95 + 88.2\cos\theta}$	A1	cao
8(b)	Greatest value of θ occurs when T=0		
	$34.95 + 88.2\cos\theta = 0$	M1	ft $T$of form a±bsin/cosθ
	$\cos\theta = -\frac{34\cdot95}{88\cdot2}$		
	$\theta = \underline{113.34^\circ}$	A1	ft a+bcosθ
	Motion stops being circular when θ = 113.34º as string cannot support negative tension. P moves under the action of gravity only.	E1	ft θ>90º