GCE AS/A level

0981/01

S15-0981-01

MATHEMATICS M2
Mechanics

A.M. FRIDAY, 5 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The vectors $\mathbf{x}$ and $\mathbf{y}$ are given by

$$\mathbf{x} = \sin\theta\,\mathbf{i} + 2\cos 2\theta\,\mathbf{j},$$
$$\mathbf{y} = 2\mathbf{i} - \mathbf{j}.$$

Find the values of θ between 0 and 2π such that $\mathbf{x}$ is perpendicular to $\mathbf{y}$. [6]

2. An object of mass 50 kg moves in a straight horizontal line under the action of a constant horizontal force of magnitude 1600 N acting along the line. The resistance to motion of the object is proportional to time t seconds. At time t seconds, the velocity of the object is v ms^{-1} and at time $t = 2$, it is moving with velocity 41 ms^{-1} and acceleration -4 ms^{-2}.

(a) Show that v satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 32 - 18t.$$ [4]

(b) Find an expression for v in terms of t and determine the times when the velocity of the object is 28 ms^{-1}. [6]

3. A vehicle of mass 6000 kg is moving up a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{6}{49}$. The vehicle's engine exerts a constant power of P W. The constant resistance to motion of the vehicle is R N. At the instant the vehicle is moving with velocity $\frac{16}{5}$ ms^{-1}, its acceleration is 2 ms^{-2}.

The maximum velocity of the vehicle is $\frac{16}{3}$ ms^{-1}.

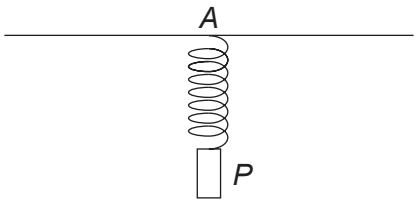
Determine the value of P and the value of R. [9]

4. A particle of mass 0·5 kg is moving under the action of a single force $\mathbf{F}$ N, where $\mathbf{F} = (4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j}$.

(a) The velocity of the particle at time t s is $\mathbf{v}$ ms^{-1}. When $t = 0$, $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$. Find an expression for $\mathbf{v}$ in terms of t. [5]

(b) When $t = 3$, the particle receives an impulse of $2\mathbf{i} - 9\mathbf{j}$ Ns. Find the speed of the particle immediately after the impulse. [5]

5. The diagram shows a light spring of natural length 0·4 m and modulus of elasticity 1470 N with one end *A* fixed and the other end attached to an object *P* of mass 15 kg.



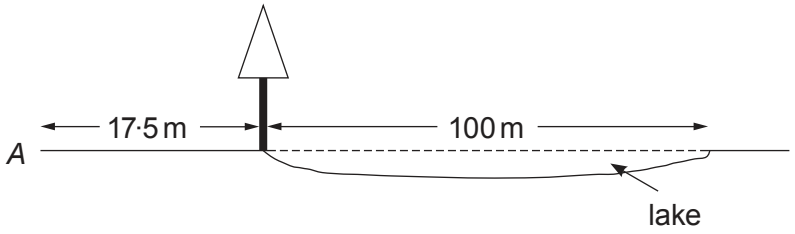
Initially, *P* hangs in equilibrium with the spring vertical.

(a) Determine the extension of the spring. [3]

The object *P* is pulled downwards so that the total length of the spring is 0·56 m. It is then released.

(b) Calculate the speed of *P* when it is at a distance 0·45 m from *A*. [8]

6. A golfer hits a ball from a point *A* with initial velocity of 35 ms^{-1} at an angle α above the horizontal where sin α = 0·8. The ball passes over a tree which is growing in front of a lake. The lake is 100 m wide, as shown in the diagram. The tree is at a horizontal distance of 17·5 m from *A*.



(a) Determine whether or not the golf ball will fall into the lake. [6]

(b) Find the magnitude and direction of the velocity of the ball as it passes over the tree. [7]

7. A car of mass 1200 kg is moving in a horizontal circle of radius 80 m on a road banked at an angle of 12° to the horizontal.
When the car is moving with a constant speed of v ms^{-1}, there is no tendency to sideslip. Calculate the normal reaction of the road on the car and find the value of v. [5]

TURN OVER

8. One end of a light inextensible string of length 0·8 m is attached to a fixed point. The other end of the string is attached to a particle *P* of mass 3 kg. Initially *P* hangs at rest with the string vertical. The particle *P* is then projected horizontally with speed 5 ms^{-1}, so that it starts to describe a vertical circle. When the string is inclined at an angle θ to the downwards vertical, *P* has speed v ms^{-1} and the tension in the string is T N.

(a) Find, in terms of θ,

(i) an expression for v^2,

(ii) an expression for T. [8]

(b) Find the greatest possible value of θ and briefly describe the subsequent motion of *P*. [3]

END OF PAPER

