

GCE AS/A level

1 hour 30 minutes

0974/01



MATHEMATICS – C2 Pure Mathematics

A.M. WEDNESDAY, 25 May 2016

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- · a Formula Booklet;
- · a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer all questions.

Sufficient working must be shown to demonstrate the mathematical method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for the integral

$$\int_{3}^{6} \frac{7 - \sqrt{x}}{7 + \sqrt{x}} dx.$$

Show your working and give your answer correct to three decimal places.

[4]

(a) Find all values of θ in the range 0° ≤ θ ≤ 360° satisfying

$$6\sin^2\theta + 1 = 2(\cos^2\theta - \sin\theta).$$
 [6]

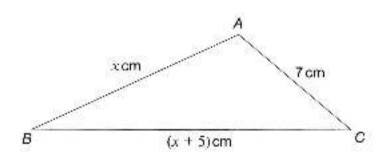
(b) Find all values of x in the range 0° ≤ x ≤ 180° satisfying

$$\tan (3x - 57^{\circ}) = -0.81.$$
 [4]

(c) Without carrying out any calculations, explain why there are no values of φ which satisfy the equation

$$2\sin\phi + 4\cos\phi = -7.$$
 [1]

3. The diagram below shows a sketch of the triangle ABC with AB = x cm, BC = (x + 5) cm, AC = 7 cm and cos $\angle BAC = -\frac{3}{5}$.



- (a) Write down an equation satisfied by x. Hence show that x = 15, [3]
- (b) Find the exact value of the area of triangle ABC. [3]
- (c) The point D lies on BC and is such that AD is perpendicular to BC. Find the length of AD.
 [2]

- 4. (a) Gwilym has decided to run in a half marathon race. In order to get himself fit, he devises a training programme whereby he runs around his local track each day, gradually increasing the distance he runs from day to day. On the first day, he runs 6 laps of the track and subsequently, on any given day, he runs 2 laps further than he did on the day before.
 - (i) How many laps does he run on the 20th day of his programme?
 - (ii) After how many days will the total number of laps he has run since the beginning of his training programme be equal to 750?
 [6]
 - (b) The nth term of an arithmetic series is denoted by t_n. It is known that

$$t_{12} + t_{13} = 50.$$

Find the sum of the first twenty-four terms of this arithmetic series.

- (i) Write down the value of t₁₁ + t₁₄.
 - + 14. [1]

[2]

[3]

 (a) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is given by

$$S_n = \frac{a(1-r^n)}{1-r} . ag{3}$$

- (b) The sum of the first five terms of a geometric series is 275. The sum to infinity of the series is 243. Find the common ratio and the first term of the geometric series. [6]
- **6.** (e) Find $\int \left(\frac{3}{\sqrt[4]{x}} 9x^{\frac{5}{2}}\right) dx$. [2]
 - (b) The region R is bounded by the curve $y = 2x^2 + \frac{6}{x^2}$, the x-axis and the lines x = 1, x = 4. Find the area of R. [5]
- 7. (a) Given that x > 0, show that

(ii)

$$\log_{\sigma} x^{n} = n \log_{\sigma} x. \tag{3}$$

(b) Solve the equation

$$4^{3x+1} = 22$$

Show your working and give your answer correct to two decimal places.

(c) Given that

$$\log_d z = 2\log_d 6 - \log_d 9 - 1,$$

express z in terms of d, giving your answer in a form **not** involving logarithms. [4]

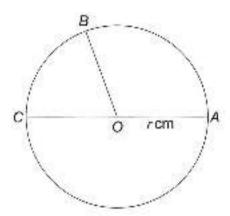
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The circle C₁ has centre A and equation

$$x^2 + y^2 + 6x - 20y + 59 = 0.$$

- (a) (i) Find the coordinates of A and the radius of C1.
 - (ii) Find the shortest distance from the origin to the circle C₁. Give your answer correct to two decimal places.
- (b) The line L has equation y = 3x − 1. The line L and the circle C₁ intersect at the points P and Q.
 - (i) Find the coordinates of P and Q.
 - (ii) The circle C₂ has centre B(6, 7) and is such that PQ is the common chord of C₁ and C₂. Find the equation of C₂.

9.



The diagram shows a sketch of a circle with centre O and radius r cm. Three points A, B and C lie on the circle. The line AC is a diameter of the circle and $\widehat{AOB} = 2.15$ radians.

Given that the area of sector BOC is 26 cm² less than the area of sector AOB, find the value of r. Give your answer correct to one decimal place. [5]

END OF PAPER