GCE MARKING SCHEME

SUMMER 2016

Mathematics – C4
0976/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE MATHEMATICS – C4

SUMMER 2016 MARK SCHEME

1. (*a*) $f(x) \equiv \dfrac{A}{(2x-1)} + \dfrac{B}{(x-3)^2} + \dfrac{C}{(x-3)}$ (correct form) M1

$17 + 4x - x^2 \equiv A(x-3)^2 + B(2x-1) + C(x-3)(2x-1)$

(correct clearing of fractions and genuine attempt to find coefficients) m1

$A = 3$, $B = 4$, $C = -2$ (all three coefficients correct) A2

If A2 not awarded, award A1 for at least one correct coefficient

(*b*) $f'(x) = -\dfrac{6}{(2x-1)^2} - \dfrac{8}{(x-3)^3} + \dfrac{2}{(x-3)^2}$ (o.e.)

(f.t. candidate's derived values for *A*, *B*, *C*)

(second term) B1

(**both** the first and third terms) B1

2. (*a*) (i) $(1 + 2x)^{-1/2} = 1 - x + \dfrac{3}{2}x^2$ $(1 - x)$ B1

$(^3/_2 x^2)$ B1

(ii) $|x| < {}^1/_2$ or $-{}^1/_2 < x < {}^1/_2$ B1

(*b*) $6 - 6x + 9x^2 = 4 + 15x - x^2 \Rightarrow 10x^2 - 21x + 2 = 0$

(f.t. only candidate's quadratic expansion in (*a*)) M1

$x = 0\cdot1$ (f.t. only candidate's quadratic expansion in (*a*)) A1

3. (*a*) $4x^3 + 2x^3\dfrac{dy}{dx} + 6x^2y - 12y^3\dfrac{dy}{dx} = 0$ $\left(2x^3\dfrac{dy}{dx} + 6x^2y\right)$ B1

$\left(4x^3 - 12y^3\dfrac{dy}{dx}\right)$ B1

$\dfrac{dy}{dx} = \dfrac{2x^3 + 3x^2y}{6y^3 - x^3}$

(intermediary line required in order to be convincing) B1

(*b*) $2x^3 + 3x^2y = -2(6y^3 - x^3)$ M1

$y(3x^2 + 12y^2) = 0$ A1

$3x^2 + 12y^2 = 0 \Rightarrow x = 0, y = 0$ but not on curve A1

$y = 0 \Rightarrow x = \pm 2 \Rightarrow (2, 0), (-2, 0)$ (both points) A1

4. (*a*) (i) $\dfrac{6\tan x}{1-\tan^2 x} + 16\cot^2 x = 0$ (o.e.)

(correct use of formula for $\tan 2x$) M1

$\dfrac{6\tan x}{1-\tan^2 x} + \dfrac{16}{\tan^2 x} = 0$ (correct use of $\cot^2 x = \dfrac{1}{\tan^2 x}$) M1

$3\tan^3 x - 8\tan^2 x + 8 = 0$

(intermediary line required in order to be convincing) A1

(ii) $3\tan^3 x - 8\tan^2 x + 8 = (\tan x - 2)(3\tan^2 x + a\tan x + b)$

with one of a, b correct M1

$3\tan^3 x - 8\tan^2 x + 8 = (\tan x - 2)(3\tan^2 x - 2\tan x - 4)$ A1

$x = 63{\cdot}4°, 56{\cdot}9°, 139{\cdot}0°$

(rounding off errors are only penalised once) A1 A1 A1

(*b*) $R = 25$ B1

Correctly expanding $\cos(\theta + \alpha)$ and using either $25\cos\alpha = 24$ **or** $25\sin\alpha = 7$ **or** $\tan\alpha = \dfrac{7}{24}$ to find α

(f.t. candidate's value for R) M1

$\alpha = 16{\cdot}26°$ (c.a.o) A1

Use of both critical values -25 and 25

(f.t candidate's derived value for R) M1

$25\cos(\theta + \alpha) = k$ has no solutions if $k < -25$ or $k > 25$

(f.t candidate's derived value for R) A1

5. (*a*) candidate's x-derivative $= -3t^{-2}$ (o.e.)

candidate's y-derivative $= 4$ (at least one term correct)

and use of

$\dfrac{dy}{dx} = \dfrac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\dfrac{dy}{dx} = \dfrac{4}{-3t^{-2}}$ or $-\dfrac{4t^2}{3}$ (c.a.o.) A1

Equation of tangent at P: $y - 4p = -\dfrac{4p^2}{3}\left(x - \dfrac{3}{p}\right)$

(f.t. candidate's expression for $\dfrac{dy}{dx}$) m1

Equation of tangent at P: $3y = -4p^2 x + 24p$

(intermediary line required in order to be convincing) A1

(*b*) Substituting $x = 1$, $y = 9$ in equation of tangent M1

$4p^2 - 24p + 27 = 0$ A1

$p = \dfrac{9}{2}, \dfrac{3}{2}$ (both values, c.a.o.) A1

Points are $(^2/_3, 18)$, $(2, 6)$ (f.t. candidate's values for p) A1

6. (*a*) $u = 2x + 1 \Rightarrow \mathrm{d}u = 2\mathrm{d}x$ (o.e.) B1

$\mathrm{d}v = \mathrm{e}^{-3x}\,\mathrm{d}x \Rightarrow v = -\frac{1}{3}\mathrm{e}^{-3x}$ (o.e.) B1

$\int (2x+1)\,\mathrm{e}^{-3x}\,\mathrm{d}x = -\frac{1}{3}\mathrm{e}^{-3x} \times (2x+1) - \int -\frac{1}{3}\mathrm{e}^{-3x} \times 2\mathrm{d}x$ (o.e.) M1

$\int (2x+1)\,\mathrm{e}^{-3x}\,\mathrm{d}x = -\frac{1}{3}\mathrm{e}^{-3x} \times (2x+1) - \frac{2}{9}\mathrm{e}^{-3x} + c$ (c.a.o.) A1

(*b*) $\int \frac{\sqrt{(4 + 5\tan x)}}{\cos^2 x}\,\mathrm{d}x = \int k \times u^{1/2}\,\mathrm{d}u$ ($k = {}^1/_5$ or 5) M1

$\int a \times u^{1/2}\,\mathrm{d}u = a \times \frac{u^{3/2}}{3/2}$ B1

Either: Correctly inserting limits of 4, 9 in candidate's $bu^{3/2}$

or: Correctly inserting limits of 0, $\pi/4$ in candidate's $b(4 + 5\tan x)^{3/2}$ M1

$\int_0^{\pi/4} \frac{\sqrt{(4 + 5\tan x)}}{\cos^2 x}\,\mathrm{d}x = \frac{38}{15} = 2{\cdot}53$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

7. (*a*) $\frac{\mathrm{d}V}{\mathrm{d}t} = -kV^3$ B1

(*b*) $\int \frac{\mathrm{d}V}{V^3} = -\int k\,\mathrm{d}t$ (o.e.) M1

$-\frac{V^{-2}}{2} = -kt + c$ A1

$c = -\frac{A^{-2}}{2}$ (c.a.o.) A1

$2V^2 = \frac{2A^2}{(2A^2k)t + 1} \Rightarrow V^2 = \frac{A^2}{bt + 1}$ (convincing)

where $b = 2A^2k$ A1

(*c*) Substituting $t = 2$ and $V = \frac{A}{2}$ in an expression for V^2 M1

$b = \frac{3}{2}$ $\left(\text{or } k = \frac{3}{4A^2}\right)$ A1

Substituting $V = \frac{A}{4}$ in an expression for V^2 with candidate's value for b or expression for k M1

$t = 10$ (c.a.o) A1

8. (*a*) (i) $\mathbf{AB} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ B1

(ii) Use of $\mathbf{a} + \lambda\mathbf{AB}$, $\mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$, $\mathbf{b} + \lambda\mathbf{AB}$ or $\mathbf{b} + \lambda(\mathbf{b} - \mathbf{a})$ to find vector equation of AB M1

$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ (o.e.)

(f.t. if candidate uses his/her expression for **AB**) A1

(*b*) (i) $1 + 2\lambda = -1 - 2\mu$

$3 + \lambda = 8 + \mu$

$-3 + 2\lambda = p + 3\mu$ (o.e.)

(comparing coefficients, at least one equation correct) M1

(at least two equations correct) A1

Solving the first two equations simultaneously m1

(f.t. for all 3 marks if candidate uses his/her expression for **AB**)

$\lambda = 2$, $\mu = -3$ (o.e.) (c.a.o.) A1

$p = 10$ from third equation (f.t. candidate's derived values for λ and μ provided the third equation is correct) A1

(ii) An attempt to evaluate $(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ M1

$(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}).(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) = -1 \neq 0 \Rightarrow L$ and $(6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})$ not perpendicular A1

9.

Volume $= \pi\int_{\pi/5}^{2\pi/5}(\cos x + \sin x)^2\,dx$ B1

$(\cos x + \sin x)^2 = \cos^2 x + \sin^2 x + 2\sin x\cos x$ B1

$\int(\cos^2 x + \sin^2 x)\,dx = x$ **or** $\left[\frac{x}{2} + \frac{1}{4}\sin 2x\right] + \left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]$ B1

$\int k\sin x\cos x\,dx = -\frac{k}{4}\cos 2x$ **or** $\frac{k}{2}\sin^2 x$ **or** $-\frac{k}{2}\cos^2 x$ B1

Substitution of limits in candidate's integrated expression
(awarded only if at least two of the previous three marks have been awarded) M1

Volume $= 3\cdot73$ (c.a.o.) A1

Note: Answer only with no working earns 0 marks

10. Assume that there is a real value of x such that

$$\left|x + \frac{1}{x}\right| < 2$$

Then squaring both sides, we have:

$x^2 + \frac{1}{x^2} + 2 < 4$ B1

$x^2 + \frac{1}{x^2} - 2 < 0$ B1

$\left(x - \frac{1}{x}\right)^2 < 0$, which is impossible since the square of a real number cannot be negative B1

Alternative Mark Scheme

Assume that there is a real value of x such that

$$\left|x + \frac{1}{x}\right| < 2$$

Then squaring both sides, we have:

$x^2 + \frac{1}{x^2} + 2 < 4$ B1

$x^4 - 2x^2 + 1 < 0$ B1

$(x^2 - 1)^2 < 0$, which is impossible since the square of a real number cannot be negative B1