



GCE AS/A level

0976/01



S16-0976-01

MATHEMATICS – C4
Pure Mathematics

P.M. FRIDAY, 17 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{17 + 4x - x^2}{(2x - 1)(x - 3)^2}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) **Use your result to part (a)** to find an expression for $f'(x)$. [2]

2. (a) (i) Expand $\frac{1}{\sqrt{1+2x}}$ in ascending powers of x up to and including the term in x^2 .

(ii) State the range of values of x for which your expansion is valid. [3]

(b) Use your expansion in part (a) to find an approximate value for one root of the equation

$$\frac{6}{\sqrt{1+2x}} = 4 + 15x - x^2. \quad [2]$$

3. The curve C has equation

$$x^4 + 2x^3y - 3y^4 = 16.$$

(a) Show that $\frac{dy}{dx} = \frac{2x^3 + 3x^2y}{6y^3 - x^3}$. [3]

(b) Show that there are only two points on C where the gradient of the tangent is -2 . Find the coordinates of each of these two points. [4]

4. (a) The angle x is such that $0^\circ \leq x \leq 180^\circ$, $x \neq 90^\circ$.

Given that x satisfies the equation $3 \tan 2x + 16 \cot^2 x = 0$,

(i) show that $3 \tan^3 x - 8 \tan^2 x + 8 = 0$,

(ii) find all possible values of x , giving each answer in degrees, correct to one decimal place. [8]

(b) Express $24 \cos \theta - 7 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Hence, find the range of values of k for which the equation

$$24 \cos \theta - 7 \sin \theta = k$$

has no solutions. [5]

5. The parametric equations of the curve C are

$$x = \frac{3}{t}, \quad y = 4t.$$

- (a) Show that the tangent to C at the point P with parameter p has equation

$$3y = -4p^2x + 24p. \quad [4]$$

- (b) The tangent to C at the point P passes through the point $(1, 9)$. Show that P can be one of two points. Find the coordinates of each of these two points. [4]

6. (a) Find $\int (2x+1)e^{-3x} dx$. [4]

- (b) Use the substitution $u = 4 + 5 \tan x$ to evaluate

$$\int_0^{\frac{\pi}{4}} \frac{\sqrt{4+5\tan x}}{\cos^2 x} dx. \quad [4]$$

7. The value, $\pounds V$, of a particular car may be modelled as a continuous variable. At time t years, the rate of decrease of V is directly proportional to V^3 .

- (a) Write down a differential equation satisfied by V . [1]

- (b) Given that the initial value of the car is $\pounds A$, show that

$$V^2 = \frac{A^2}{bt+1},$$

where b is a constant. [4]

- (c) When $t = 2$, the value of the car has fallen to a half of its initial value. Find the value of t when the value of the car will have fallen to a quarter of its initial value. [4]

TURN OVER

8. The position vectors of the points A and B are given by

$$\begin{aligned}\mathbf{a} &= \mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \\ \mathbf{b} &= 3\mathbf{i} + 4\mathbf{j} - \mathbf{k},\end{aligned}$$

respectively.

- (a) (i) Write down the vector $\mathbf{AB}$.

- (ii) Find the vector equation of the line AB .

[3]

- (b) The vector equation of the line L is given by

$$\mathbf{r} = -\mathbf{i} + 8\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.

- (i) Given that the lines AB and L intersect, find the value of p .

- (ii) Determine whether or not the line L is perpendicular to the vector $6\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$, giving a reason for your answer.

[7]

9. The region R is bounded by the curve $y = \cos x + \sin x$, the x -axis and the lines $x = \frac{\pi}{5}$, $x = \frac{2\pi}{5}$. Find the volume of the solid generated when R is rotated through four right angles about the x -axis. Give your answer correct to two decimal places.

[6]

10. Prove by contradiction the following proposition.

When x is real and $x \neq 0$,

$$\left| x + \frac{1}{x} \right| \geq 2.$$

The first two lines of the proof are given below.

Assume that there is a real value of x such that

$$\left| x + \frac{1}{x} \right| < 2.$$

Then squaring both sides, we have:

[3]

END OF PAPER